

**From Doktor
Kurowski's
Schneegrenze to our
modern glacier ELA**

R. J. Braithwaite

From Doktor Kurowski's Schneegrenze to our modern glacier equilibrium line altitude (ELA)

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Abstract

Translated into modern terminology, Kurowski suggested in 1891 that the equilibrium line altitude (ELA) of a glacier is equal to the mean altitude of the glacier when the whole glacier is in balance between accumulation and ablation. Kurowski's method has been widely misunderstood, partly due to inappropriate use of statistical terminology by later workers, and has been little tested except by Braithwaite and Müller in a 1980 paper (for 32 glaciers). I now compare Kurowski's mean altitude with balanced-budget ELA calculated for 103 modern glaciers with measured surface mass balance data. Kurowski's mean altitude is significantly higher (at 95 % level) than balanced-budget ELA for 19 outlet and 42 valley glaciers, but not significantly higher for 34 mountain glaciers. The error in Kurowski mean altitude as a predictor of balanced-budget ELA might be due to generally lower balance gradients in accumulation area compared with ablation areas for many glaciers, as suggested by several workers, but some glaciers have higher gradients, presumably due to precipitation increase with altitude. The relatively close agreement between balanced-budget ELA and mean altitude for mountain glaciers (mean error -8 m with standard deviation 59 m) may reflect smaller altitude ranges for these glaciers such that there is less room for effects of different balance gradients to manifest themselves.

1 Introduction

Ludwig Kurowski was born in 1866 in Napajedl, Moravia (then in the Austrian Empire and now in the Czech Republic) and died in 1912 in Vienna (<http://mahren.germanistika.cz>). For his doctoral-thesis research at the University of Vienna, Kurowski (1891) studied snow line (German: *Schneegrenze*) in the Finsteraarhorn region of the Swiss Alps. He suggested the altitude of snow line on a glacier is equal to the mean altitude of the glacier when snow accumulation and melt are in balance for the whole glacier. A relatively recent definition of snow line (Armstrong et al., 1973) is "The line

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or zone on land that separates areas in which fallen snow disappears in summer from areas in which snow remains throughout the year. The altitude of the snow line is controlled by temperature and the amount of snowfall (cf. *Equilibrium line* and *Firn line*). Students of snow line in the 19th Century would have broadly agreed with this definition before Ratzel (1886) introduced extra terms like *climatic* and *orographic* to qualify snow line. Ratzel (1886) also argued that the material left at the end of the melt season is *firn* rather than *snow* but Kurowski (1891) does not use Ratzel's preferred term *Firngrenze*.

The snow line definition above explicitly refers to the landscape at the end of summer as being snow-covered or snow-free but a mass balance concept is also implicit in the definition, i.e. snow melt equals snow accumulation at the snow line, and this is the aspect of snow line studied by Kurowski (1891).

In early modern mass balance studies in the 1940s and 1950s, the altitude on the glacier where mass balance is zero for a particular year was termed altitude of “firn line”, corresponding to the German *Firngrenze*. However, firn line implies that “firn” is visible on the glacier surface above the zero-balance line while we now know that the lower accumulation zone of some glaciers can consist of ice (“superimposed ice”) formed by refreezing of water from melting snow; see Fig. 2.1 in Paterson (1994). Baird (1952) seems to have been the first to use the term “equilibrium line altitude (ELA)” for the zero-balance altitude, and this usage became accepted as standard by the late 1960s (Anonymous, 1969). The distinction between firn line and equilibrium line is marked for high latitude glaciers, e.g. in Greenland or on Arctic islands, but is quite unimportant for the Alpine glaciers studied by Kurowski and other pioneers. We can therefore translate Kurowski's *Schneegrenze* (where snow melt equals snow accumulation) as equilibrium line (where mass balance is zero) and regard most late 19th Century snow line (a.k.a. firn line) methods as being equally applicable to equilibrium line. The best review in English of models for indirect estimation of firn lines (a.k.a. equilibrium lines) is in an obscure book-chapter by Osmaston (1975), which I only discovered when preparing a late draft of the present paper.

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The simple theory of Kurowski (1891) depends on the assumption that mass balance gradient is constant across the whole altitude range of the glacier. This was criticised by Hess (1904) and Reid (1908), and several modern authors have attempted to account for variations in mass balance gradients by defining a ratio (“balance ratio”) between balance gradients in the ablation and accumulation zones (Furbish and Andrews, 1984; Osmaston, 2005; Rea, 2009). Kurowski himself argued that nonlinearity in the balance-altitude equation need not cause a large error as low and high altitudes on a glacier usually coincide with small areas, and are not weighted heavily in calculating mean altitude. It is surprising that nobody has verified the basic Kurowski theory with observed mass balance data except for Braithwaite and Müller (1980). The main purpose of the present paper is to critically test the original Kurowski (1891) theory with observed mass balance data from more glaciers, and then to discuss the results together with balance ratio data from Rea (2009).

Readers need not share my wish to honour Kurowski’s pioneering work, involving one of the earliest quantitative models in glaciology, but they should agree that the estimation of glacier ELA from topographic proxies is still an active and legitimate area of research in glaciology and quaternary science. Recent ELA-related work includes Benn and Lehmkuhl (2000), Kaser and Osmaston (2002), Cogley and McIntyre (2003), Leonard and Fountain (2003), Carrivick and Brewer (2004), Benn et al. (2005), Osmaston (2005), Dyurgerov et al. (2009), Braithwaite and Raper (2009), Rea (2009), Kern and László (2010), Bakke and Nesje (2011), Ignéczi and Nagy (2013), Loibl and Lehmkuhl (2014) and Heymann (2014), to cite only a few. The possibilities of monitoring year-to-year variations in end-of-summer-snowline (EOSS) from aircraft (Chinn, 1995) or from satellite images (Rabatel et al., 2012; Mathieu et al., 2014) raise similar needs to estimate baseline ELA’s for present-day glaciers for which there are no observed mass balance data.

2 Tutorial on glacier altitudes

Kurowski's work has often been ignored or misquoted, his name is sometimes wrongly spelled following Hess (1904) and Sissons (1974) and Sutherland (1984) re-discovered his method without citing him. Because of many misquotes the reader may not understand Kurowski's method unless he/she has him/herself read the original article. A PDF of the original article (kindly provided by Hans-Dieter Schwartz of the Bavarian Academy of Sciences) is available in the on-line Supplement. One underlying problem is the widespread misuse of statistical terms like *mean* and *median* when applied to glacier altitudes (Cox, 2004). This issue is so central to a discussion of Kurowski (1891) that I give a worked example, using the area-altitude distribution of a well-documented glacier (Hintereisferner in the Austrian Alps in the year 2001), to illustrate concepts; see Sect. 5 for sources of data.

The graph of the area-altitude distribution in Fig. 1 looks like a histogram (probability distribution function) of altitudes on Hintereisferner and could have been obtained from a digital elevation model with *area* representing the number of pixels in each altitude interval. The mean altitude for such a distribution is:

$$\bar{h} = H_{\text{mean}} = \left(\sum_{i=1}^{i=N} h_i \times a_i \right) / \sum_{i=1}^{i=N} a_i \quad (1)$$

where a_i is the area of the i th altitude band and h_i is its altitude, and N is the number of altitude bands. For the given altitude-area distribution (Fig. 1) for Hintereisferner, the mean altitude H_{mean} is 3038 m a.s.l. This is the mean altitude of the glacier according to the Kurowski (1891) method and it is obvious from his Table III that he calculates his "*Mittlere Höhe des Gletschers*" from the altitude-area distribution of each glacier according to Eq. (1).

Some authors incorrectly assert that Kurowski (1891) used an accumulation-area ratio (AAR) of 50 % to locate the snow line (Müller, 1980; Kotlyakov and Krenke, 1982) and the guidelines of the World Glacier Inventory (TTS, 1977) incorrectly refer to this

altitude as “mean altitude”. Figure 2 shows the percentage of the area lying above any particular altitude (cumulative distribution function). The median altitude is that altitude dividing the glacier area into equal halves, i.e. it is the altitude (x coordinate) corresponding to a y coordinate of 50 %. For the given altitude-area distribution (Fig. 2), the median altitude H_{50} is 3056 m.a.s.l. This is the altitude giving AAR = 50 %. In a similar way, the altitude H_{60} above which 60 % of the glacier area lies is 2989 m.a.s.l. Kurowski (1891) quotes Brückner (1886) as saying that 75 % of the glacier lies above the snow line (which nobody would believe today), and $H_{75} = 2878$ m.a.s.l. in the present case.

Some authors incorrectly assert that Kurowski (1891) used an average of maximum and minimum glacier altitude to locate the snow line (Cogley and McIntyre, 2003; Leonard and Fountain, 2003). The minimum and maximum altitudes for the glacier are 2400 and 3727 m.a.s.l. respectively and the mid-range altitude of the glacier is:

$$H_{\text{mid}} = (H_{\text{max}} + H_{\text{min}}) / 2. \quad (2)$$

In the present case, the mid-range altitude (H_{mid}) is 3064 m.a.s.l.

Manley (1959) estimated ELA (or snow line or firn line) as mid-range altitude according to Eq. (2) but many authors incorrectly assert that he used the “median” altitude although Manley does not even mention the word. Authors incorrectly using “median” for this mid-range altitude include Porter (1975), Meierding (1982), Hawkins (1985), Benn and Lehmkuhl (2000), Carrivick and Brewer (2004), Benn et al. (2005), Osmaston (2005), Rea (2009), Dobhal (2011), and Bakke and Nesje (2011) to mention only a few. Incorrect use of terminology can be inferred in any book or paper that refers to both “median altitude” and to “AAR” without noting that the correctly-defined median altitude is identical to the altitude with AAR = 50 %, e.g. Nesje and Dahl (2000), and Benn and Evans (2010).

Kurowski’s theory was purely in terms of mean altitude, correctly defined in Eq. (1), but median and mid-range altitudes for glaciers are generally close to the mean altitude and would be identical to it if the area-altitude distribution were symmetric. The area-altitude distribution of this glacier (Fig. 1) is only slightly asymmetric, being somewhat

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skewed to higher altitudes, but a wide variety can be found for other glaciers and it is important not to conflate the various altitudes.

3 Snow line before Kurowski

The scientific concept of snow line was discovered by the French geophysicist Pierre Bouguer (1698–1758) on an expedition to tropical South America (Klengel, 1889). Up to the early 19th Century, the snow line had been observed in many areas so that Alexander von Humboldt could start to compile a global picture of snowline variations. A version of von Humboldt's snow line table is given in English by Kaemtz (1845, 228–229) with snow line altitudes for 34 regions from all over the world. Heim (1885, 18–21) gives a greatly extended table, and Hess (1904, Map 1) plots a world map of glacier cover and snow line. Paschinger (1912) makes the first climatological analysis of snow line in various climatic regions.

Most of this snow line data was based on observations of an apparently sharp delineation between snow-covered and snow-free areas as seen from a distance of a few kilometres, typically by an observer in a valley or on a mountain pass, looking upwards into the high mountains. It was known very early that snow line fluctuates with season, and from one year to the next, with large local spatial variations due to topography and aspect, and that the apparent sharp delineation between snow-covered and snow-free landscape disappears on closer examination to be replaced by a broad zone of snow patches, slowly morphing into a continuous snow cover (Mousson, 1854, p. 3; Heim, 1885, 9–21; Ratzel, 1886; Klengel, 1889; Kurowski, 1891, p. 120). To overcome these problems, snowline has sometimes been defined as the boundary between > 50 % snow cover and < 50 % snow cover on a flat surface (Escher, 1970). All of these problems can be overcome with modern technology of regular remote sensing and image processing (Gafurov et al., 2014) but would have been nearly impossible with 19th Century methods. In this sense, much of the early work on snow line as a measure of snow-covered landscape was premature.

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Ratzel (1886) was very critical of snow line observations based on “travellers’ tales” (this was obviously a poke at Alexander von Humboldt’s table) and introduced much of our modern armoury of regional, climatic, temporary and orographic snow line although these were not easy to measure at the time. More fruitfully, a number of 19th Century workers recognized that glacier accumulation areas occupy most of the region above the snow line so that the year-on-year accumulation of snow is offset by ice flow to lower elevations. More attention was then focussed on glaciers which were then being mapped in some detail for the first time in the Alps. One of the resulting map-based methods to determine glacier snow line was by Kurowski (1891).

4 Kurowski’s work

Kurowski (1891) developed a simple theory for the altitude of snow line on a glacier, which may be one of the first theories in glaciology. I translate his theory into modern mass-balance terminology (Anonymous, 1969) in the present paper although we must remember that glacier mass balance in its modern sense was not measured in the 19th Century. In essence, Kurowski (1891) assumed that specific mass balance b_{it} at any altitude and year is proportional to the height above or below the ELA_0 for which the whole glacier is in balance:

$$b_{it} = k \times (h_i - ELA_0), \quad (3)$$

where k is balance gradient on the glacier and ELA_0 is the balanced-budget ELA. Some people use the term *steady-state* to qualify this ELA but this implies zero change in a multitude of factors rather than just the mass balance, see comments by M. F. Meier in the discussion following the papers by Braithwaite and Müller (1980) and Radok (1980). I have a similar objection to the term *steady-state* AAR used by Kern and Laszlo (2010) and would prefer the term *equilibrium* AAR of Dyrurgerov et al. (2009) if not *balanced-budget* AAR.

Kurowski (1891) assumes that balance gradient k is constant over the whole glacier, and for all time. Using modern terminology (Anonymous, 1969), the mean specific balance \bar{b}_t of the whole glacier is the area-weighted sum of specific balances:

$$\bar{b}_t = \left(\sum_{i=1}^{i=N} a_i \times b_{it} \right) / \sum_{i=1}^{i=N} a_i. \quad (4)$$

5 Area-weighted averaging of both sides of Eq. (3) gives:

$$\bar{b}_t = 0 = (k \times \bar{h}) - (k \times \text{ELA}_0), \quad (5)$$

where \bar{h} is the mean altitude of the glacier, defined by Eq. (1). As the area a_i is largest at intermediate altitudes on most glaciers, and lowest at high and low altitudes, the mean specific balance of the whole glacier should be close to the specific balance at \bar{h} the mean altitude of the glacier. Re-arranging Eq. (5) and noting that $\bar{b}_t = \text{zero}$ (by assumption) gives:

$$\text{ELA}_0 = \bar{h} = H_{\text{mean}}. \quad (6)$$

Equation (6) expresses the identity between balanced-budget ELA and the mean altitude of the glacier. Kurowski himself did not assume constant balance gradient casually but discussed available evidence (Kurowski, 1891, 126–130), including application of an early version of the degree-day model, to justify a nearly-constant balance gradient. Remarkably, Kurowski (1891, p. 127) suggested a value of $0.0056 \text{ m.w.e. m}^{-1}$ for vertical balance gradient, which is not greatly out of line with modern results for Alpine glaciers (my own unpublished results). He also tested a balance gradient proportional to the square root of altitude (p. 130) and suggested that it does not greatly affect the calculated ELA because of the relatively small proportions of glacier area at the lowest and highest elevations. Osmaston (2005) appears to misunderstand this as he says the

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“AA method” (his name for Kurowski’s method) is based “on the principle of weighting the mass balance in areas far above or below the ELA by more than in those close to it”.

Kurowski (1891) presents his main results in Table III (142–147 pp.) of his paper. The data consist of measured areas for altitude bands of 150 m width from 1050 to 4200 m a.s.l. for 72 glaciers and 27 snow patches (German: *Schneefleck*) in the Finsteraarhorn Group, Switzerland. The work involved planimetric measurements of 744 individual area-elements, covering a total glacierized area of 461.19 km². The smallest snow patch was 0.04 km² and the largest glacier was 115.1 km² (*Großer Aletschgletscher*). Unfortunately, there is no map showing delineations of separate glacial elements, and we would have to guess which areas were included for which glaciers if we wanted to replicate Kurowski’s work (this is beyond the scope of the present paper). According to the WGMS website (<http://www.wgms.ch/fog.html>), the area of the presently-delineated *Gr. Aletschgletscher* is much smaller than given by Kurowski, i.e. only 83.02 km². This smaller area will reflect: (1) a real reduction in glacier area due to climate change since Kurowski’s time; (2) possible separation of the object seen by Kurowski into two or more objects on modern maps, either due to glacier shrinkage or to better map resolution, (3) possible overestimation of glacier-covered areas at higher altitudes due to the oblique angle of observation by the 19th Century surveyors. Effects (1) and (2) are well documented (Abermann et al., 2009; Fischer et al., 2014).

After so much tedious work with the planimeter, Kurowski must have been frustrated that he had no easy way of verifying his snow line results. From Kurowski’s Table III, I can calculate the average altitude for all 99 glaciers and snow patches as 2867 m a.s.l. with a standard deviation of ± 181 m a.s.l., and there is a large range between minimum and maximum altitudes of 2470 and 3211 m a.s.l. for individual glaciers/snow patches. This variability within a single mountain group is in contrast to Heim (1885, 18–21) where the snow line in the Central Alps of Switzerland is represented by the narrow range 2750–2800 m a.s.l.

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Kurowski (1891, 152–155) discussed the influence of aspect on snow line. According to him, glaciers with E and NE aspect have low snow line altitude, glaciers with NW, N and SW aspect have intermediate altitudes, and glaciers with SE, S and W aspect have higher altitudes. Modern studies of the effect of aspect on glacier altitudes (Evans, 1977, 2006) broadly confirm the importance of aspect claimed by Kurowski (1891).

The late 19th Century work on glacier snow line by Kurowski and other workers appeared to be so successful that Hess (1904, p. 68) stated simply that snow line can be determined from maps of glacier regions rather than by direct observation of snow line in nature.

5 Mass balance and equilibrium line altitude

For present purposes, the most important development in 20th Century glaciology was the systematic measurement of surface mass balance on selected glaciers. The first continuing, multi-year, series was started in 1946 on Storglaciären in northern Sweden (Schytt, 1981) and surface mass balance measurements have gradually extended to several hundred glaciers in all parts of the world (Haeberli et al., 2007). The bulk of these surface mass balance data, including ELA and AAR data and various metadata, have been published in the five-yearly series *Fluctuations of Glaciers* (<http://www.wgms.ch/fog.html>) and the less detailed two-yearly series *Mass Balance Bulletin* (<http://www.wgms.ch/gmbb.html>) from the World Glacier Monitoring Service. Jania and Hagen (1995), Dyurgerov (2002), and Dyurgerov and Meier (2005) have published some additional data to those reported in WGMS publications.

I have maintained my own database for surface mass balance since the mid-1990's, consisting of a large data file compiled from the above sources and a FORTRAN program to calculate statistics for the longer series (Braithwaite, 2002, 2009). Updating and correction of data in 2012/2013 involved checking the database against the latest version of the WGMS data (<http://www.wgms.ch/fog.html>). I now have mass-balance data for 371 glaciers, i.e. with ≥ 1 year of mass balance data, for the period 1946–2010.

This figure is volatile as new data can be expected, and the database will be updated as necessary. Of these 371 glaciers, there are some glaciers that do not appear to be in the WGMS database. This includes data published in the first two volumes of *Fluctuations and Glaciers* (in hardcopy) that were never transferred to WGMS's digital database.

As mass balance data became available from an increasing number of glaciers, several workers (Liestøl, 1967; Hoinkes, 1970; Østrem, 1975; Braithwaite and Müller, 1980; Young, 1981; Schytt, 1981) established empirical equations linking the ELA_t , in the year t , to the mean specific balance \bar{b}_t in the same year:

$$ELA_t = \alpha + (\beta \times \bar{b}_t), \quad (7)$$

where α is the intercept and β is the slope of the equation. These parameters can be evaluated by regression analysis for any glacier with a few years of data. By definition, the balanced-budget $ELA_0 = \alpha$. We can therefore calculate balanced-budget ELA from modern mass-balance data, using Eq. (7) as a regression equation as long as we have a few years of parallel data for surface mass balance and ELA to calibrate α and β . In the absence of long data series, Østrem and Liestøl (1961) calculated balanced-budget ELA for a number of glaciers using a balance-altitude curve from a single year of mass balance observations. The two-yearly Glacier Mass Balance Bulletin published by WGMS (<http://www.wgms.ch/gmbb.html>) since 1988 lists balanced-budget ELA and AAR statistics for a steadily increasing number of glaciers, i.e. 29 glaciers in the 1988–1989 bulletin to 77 glaciers in the 2008–2009 bulletin. The selection criterion in the WGMS reports (<http://www.wgms.ch/gmbb.html>) seems to be $N \geq 6$ of record.

ELA varies greatly from year to year on any glacier. Figure 3 illustrates ELA variations on Hintereisferner as an example. This large year-to-year variation, with a standard deviation of ± 129 m for Hintereisferner, means that at least a few years of ELA measurement are needed to calculate a reliable mean ELA. The mean ELA for the 25 years of record in Fig. 3 is 3037 m a.s.l. This mean ELA is slightly biased as a cli-

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matological index because it excludes the years (warmest years?) when the ELA was above the maximum altitude of the glacier.

There is an obvious secular variation in ELA for Hintereisferner with a slight downward trend until the late 1970s followed by a rising trend up to the year 2010, with an increasing number of single years with ELA above the maximum altitude of the glacier. The mean ELA for the whole record (3037 m.a.s.l.) is therefore too high for the first three decades and too low for the last three decades. The mean ELA does not itself say much about the overall “health” of the glacier over the nearly six decades of record. A more meaningful index is the deviation of ELA from the balanced-budget ELA, i.e. the ELA needed to keep the glacier (with its current area distribution) in an overall condition of zero mass balance. The latter concept is illustrated in Fig. 4 for Hintereisferner where yearly values of ELA are plotted against mean specific balance.

Figure 4 shows a strong negative correlation between ELA and mass balance for Hintereisferner (correlation coefficient $r = -0.93$ with sample size 55). The ELA-balance relation in this case is represented by the regression line, whose reliability is expressed by the 95 % confidence interval. The balanced-budget ELA is 2923 m.a.s.l. where the regression line coincides with zero mass balance, and the associated 95 % confidence interval here has a width of ± 35 m. From Fig. 4, the observed ELA for 1953–1980 is often lower than the balanced-budget ELA while it is never lower after 1980, suggesting that the present altitude-area distribution of Hintereisferner is increasingly out of equilibrium with climate.

Values of the various altitude concepts for Hintereisferner, discussed in Sect. 2 or above, are summarized in Table 1, clearly showing that they are clustered near the middle reaches of the glacier, i.e. around 3050 m.a.s.l., while balanced-budget ELA is somewhat lower. The clustering of the topographic parameters will occur for any other glacier that is somehow “fat in the middle”, although topographic “anomalies” can occur for other glaciers.

Of the total of 371 glaciers in my database with ≥ 1 year of mass balance data, there are 137 glaciers (37 % of total) with no ELA data ($N = 0$), either because ELA mea-



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surements are not part of the observation programme or because ELA was above the glacier ($ELA \geq h_{max}$) for the whole period of record. There are a further 84 glaciers (23 % of total) with less than five years of record for both ELA and balance ($5 > N \geq 1$). This means that data from only 150 glaciers (40 % of total) are potentially available to calculate balanced-budget ELA if we regard $N \geq 5$ as sufficient for calculating reliable statistics (reduced to 85 glaciers if we use the stricter criterion $N \geq 10$ years). For these 150 glaciers with the necessary data, there are generally high correlations between ELA and mass balance (Fig. 5). For example, there are 143 glaciers with “good correlations” (correlation coefficient $r \leq -0.71$). There are, however, seven glaciers with “poor correlations” ($r > -0.7$). For a correlation coefficient approaching zero, the slope of the regression equation will also approach zero as both slope and correlation coefficient depend on the covariance of mass balance and ELA. As the slope of the regression equation approaches zero the intercept approaches the mean of the ELA. Although low correlations between mass balance and ELA will cause errors I did not exclude results for these 7 glaciers for further analysis because I wanted to see their effects on final results (discussed in Sect. 6).

Rea (2009) calculated balanced-budget ELA for 66 glaciers but only includes glaciers with at least 7 years of record ($N \geq 7$) up to 2003, and excludes very small glaciers ($< 1 \text{ km}^2$). The agreements between my estimates of balanced-budget ELA and his are very close for the 66 glaciers common to both studies, i.e. with mean and standard deviation of +3 and ± 25 m for the differences between the two studies.

Glacial geomorphologists like to claim that their single-glacier results “represent” conditions in a wide region around the glacier. The question of spatial representativeness of the surface mass balance data is beyond the scope of the present paper, e.g. see Gardner et al. (2013), but it is also important to note that the available mass-balance data include relatively few glaciers with heavy debris-cover or with tongues calving into lakes or oceans. We may also doubt whether anybody chooses to measure the surface mass balance of a glacier fed by frequent avalanches onto the accumulation area, so the available data are biased against this type of glacier. The available data

cannot therefore be completely representative of conditions in the real world where debris cover, calving, and substantial accumulation by snow avalanching are common, especially in the high mountain environments of Benn and Lehmkuhl (2000).

6 Balanced-budget ELA and Kurowski mean altitude

Liestøl (1967) calculated balanced-budget ELA by regression of ELA on measured mass balance and compared it with mean altitude for one glacier (Storbreen, Norway), and Braithwaite and Müller (1980) did the same for 32 glaciers in different parts of the world.

According to Sect. 5, balanced-budget ELA's are available for 150 glaciers in the 371-glacier dataset and the Kurowski mean altitude should be estimated for as many of these glaciers as possible. Detailed area-altitude data were identified in the published metadata for 148 out of the 371 glaciers. For most of these glaciers, area-altitude tables are given for every year of record (together with mass balance as a function of altitude) and area-altitude data for 2001 were selected, if available, for the calculation of Kurowski mean altitude. Otherwise, data for the year closest to 2001 were selected. For a few glaciers, the area-altitude distribution is very out of date but nothing better is available. For Hintereisferner, the Kurowski mean altitude varies from 3010 m a.s.l. in 1965 to 3038 m a.s.l. in 2001 so errors of several decametres can occur if there is a large time difference between area-altitude and mass balance data.

When combining the datasets for ELA_0 (150 out of 371 glaciers) and Kurowski mean altitude H_{mean} (148 out of 371 glaciers), it was found that many glaciers had one kind of data and not the other kind, so there are in total only 103 glaciers with data for both ELA_0 and H_{mean} . Of these 103 glaciers there are now only three with poor correlations ($r > -0.71$).

The data availability is summarized in Table 2. It is sad to see how easily 371 glaciers with some mass balance data has been reduced to only 103 glaciers (28 % of total) with all the information that we need for the present study. There is little that can be done

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about the shortness of most surface mass balance series as such work is generally not well funded or resourced with the honourable exceptions of some studies in the Alps and in Scandinavia. However, the lack of published area-altitude data for some glaciers is less excusable as such data are almost certainly available to the data collectors. I hope that my paper will encourage workers to publish their missing area-altitude data. With more area-altitude data, the number of glaciers in the study could be increased to 40 % of the total. Even the single digits for “primary classification” and “frontal characteristics” are not available for all observed glaciers (<http://www.wgms.ch/fog.html>).

The most obvious way of comparing balanced-budget ELA and Kurowski mean altitude is to plot an X - Y scatter graph, and Fig. 6 shows the extremely high correlation between the two variables. The 95 % confidence interval is not plotted here because it is too close to the regression line to make a neat figure. This high correlation is by no means “spurious” (Leonard and Fountain, 2003) but it is not very useful because the scale of variations of the dependant and independent variables is so large compared with differences between the variables. Plotting balanced-budget ELA against other topographic variables also shows extremely high correlations. In an attempt to find a more meaningful correlation, I follow Leonard and Fountain (2003) and Braithwaite and Raper (2009) and “normalize” both variables with respect to the altitude range of the glaciers before re-plotting (Fig. 7). The normalization involves subtraction of H_{\min} from each variable and then dividing by $(H_{\max} - H_{\min})$. The correlations between balanced-budget ELA and Kurowski mean altitude in normalized form (Fig. 7) is lower than in Fig. 6 but is still high enough to show a satisfactory agreement ($r = +0.83$ for 103 glaciers) between the two variables. The regression line in Fig. 7, with its 95 % confidence interval, is slightly lower than the 1 : 1 line expected for $ELA_0 = H_{\text{mean}}$. However, Cox (2004) points out that plots like Fig. 7 may also be misleading because it is the absolute difference (in metres) between the balanced-budget ELA and Kurowski mean altitude that we wish to see. Altitude-bias can be ruled out as the correlation between the difference $(E_0 - H_{\text{mean}})$ and average altitude $(ELA_0 + H_{\text{mean}})/2$ is not significantly different from zero at 95 % probability. Similarly the difference $(ELA_0 - H_{\text{mean}})$

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has negligible correlation with correlation coefficient between ELA and annual balance referred to in Sect. 5, thus justifying the inclusion of several glaciers with poor ELA-balance correlation. The three glaciers with poor ELA-balance correlations ($r > -0.71$) have small differences $ELA_0 - H_{\text{mean}}$ and the glaciers with largest (Goldbergkees) and smallest (Bench) differences have good ELA-balance correlations.

The difference ($ELA_0 - H_{\text{mean}}$) is plotted in the histogram in Fig. 8. Differences of between +212 and -195 m occur but overall the differences have mean -36 m and standard deviation ± 56 m, indicating general agreement within a few decametres. The distribution is somewhat skewed with more negative values than positive values so that extremely negative values in Fig. 8 are perhaps not so noteworthy. The very high positive value in Fig. 8 (for Goldbergkees in the Austrian Alps) is isolated and can therefore be regarded as an “anomaly”. Braithwaite and Müller (1980) found a mean and standard deviation of -40 ± 40 m for the differences for 32 glaciers, not including Goldbergkees, which is not very different from present results.

One might expect the Kurowski mean altitude H_{mean} to perform differently for glaciers of differing morphology. This is tested with the boxplot in Fig. 9 where mean and 95 % confidence intervals for ($ELA_0 - H_{\text{mean}}$) are plotted against primary classification of the glaciers using metadata from the World Glacier Monitoring website (<http://wgms.ch/fog.html>). According to the definitions in TTS (1977) the digits and their definitions are: 3 Ice cap = *Dome-shaped ice mass with radial flow*; 4 Outlet glacier = *Drains an ice-field or ice cap, usually of valley glacier form, the catchment area may not be clearly delineated*; 5 Valley glacier = *Flows down a valley, the catchment area is in most cases well defined*; 6 Mountain glacier = *Any shape, sometimes similar to a valley glacier but much smaller, frequently located in a cirque or niche*.

Primary classification is missing for four out of the 103 glaciers. It is difficult to draw any conclusions for ice caps as there are only four cases and the confidence interval is very large (and unreliable). For the other morphologies, it is clear that the Kurowski mean altitude significantly overestimates (at 95 % level) the balanced-budget ELA_0 for outlet glaciers (mean and standard deviation of -40 and ± 42 m for 19 glaciers) and for

valley glaciers (-50 ± 52 m for 42 glaciers). However, for mountain glaciers the overestimation is insignificant with a mean and standard deviation of -8 ± 59 m for 34 glaciers.

Discrepancies of the magnitudes found here between balanced-budget ELA_0 and Kurowski's mean altitude may be tolerable for some applications, e.g. reconstructions of temperature and precipitation from traces of former glaciers (Hughes and Braithwaite, 2008). In this case, one could simply calculate the Kurowski mean altitude for a reconstruction of the former glacier's topography and then apply the appropriate "correction" according to the primary classification of the glacier. For a "standard" vertical lapse rate of temperature (-0.006 K m^{-1}) and an error of ± 50 m a.s.l. in estimated ELA , the resulting error in estimating summer mean temperature would only be of the order $\pm 0.3 \text{ K}$. This is fairly small compared with the uncertainties in the relation between accumulation and summer mean temperature at the ELA reported by several workers: see Braithwaite (2008) for references to such studies going back to 1924.

7 Discussion

If we return to Kurowski's theoretical treatment, his only real assumption is that balance gradient is constant over the whole glacier. It has long been supposed that this is not exactly correct (Hess, 1904; Reid, 1908; Furbish and Andrews, 1984) although Kurowski (1891) assessed the possible error as small. Osmaston (2005) and Rea (2009) extend the Kurowski method to account for different balance gradients but do not assess the error in the original Kurowski mean altitude.

Equation (6) can be modified to:

$$ELA_0 = H_{\text{mean}} + x, \quad (8)$$

where H_{mean} is the "theoretical" ELA_0 according to Kurowski, and x is the error in the Kurowski theory for the glacier in question. This error is negative for 84 glaciers (82 % of 103 glaciers) and positive for 19 glaciers (18 %) in Fig. 9. In the original theory

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of Kurowski (1891) the vertical gradient of mass balance is constant over the whole glacier:

$$(db/dh)_{\text{glacier}} = \text{Constant} \quad (9)$$

where b is the specific mass balance and h is the elevation above sea level. A recent modification of the Kurowski theory (Osmaston, 2005; Rea, 2009) involves a parameter called the balance ratio BR where:

$$\text{BR} = (db/dh)_{\text{abl}} (db/dh)_{\text{acc}} \quad (10)$$

The subscripts *glacier*, *abl* and *acc* refer respectively to balance gradients for the whole glacier, for the ablation zone, and for the accumulation zone. According to Rea (2009), balance ratio greater than unity would lower the theoretical ELA, i.e. make the error x in Eq. (8) negative, and balance ratio less than unity would make the error positive.

Rea (2009) calculates balance ratio for 66 glaciers using published data for observed surface mass balance vs. altitude, and I can compare his balance ratios with the Kurowski error x . There is a strong correlation (Fig. 10) between Brice Rea's balance ratio and the Kurowski error, i.e. $r = -0.83$. The very high balance ratio in Fig. 10 for Zongo glacier is an obvious anomaly. I took this value from Table 3 in Rea's paper but in a footnote to the table referring to this point and others he notes "*indicates a glacier where either, or both, the net balance accumulation or ablation gradient is not approximated by a linear relationship. AABRs for these glaciers should be treated with caution. These glaciers were not used to calculate the global AABR*". Suroco et al. (2009) report a significant revision of data from Zongo glacier but this would have been too late for the Rea (2009) analysis. Figure 10 confirms the reluctance of Rea (2009) to use Zongo data in his global AABR.

This strong correlation in Fig. 10 supports the validity of the balance ratio approach. However, it is clear that the 66 glaciers in Fig. 10 show a lower proportion of glaciers with positive Kurowski error than the full dataset of 103 glaciers. The boxplot in Fig. 11

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shows means and 95 % confidence intervals of Brice Rea's balance ratio for different types of glaciers. The solid dots refer to results from the original data (66 glaciers) of Rea (2009) while the open circles refer to an "augmented" data set (103 glaciers) where balance ratio for the 37 excluded glaciers is estimated from the regression equation in

Fig. 10. Leaving aside the unspecified and ice cap classes for which there are too few data, the plots show higher balance ratios for outlet glaciers and valley glaciers (not significantly different from BR = 2 with 95 % confidence), and lower balance ratio (not significantly different from BR = 1) for mountain glaciers. The increased sample size using the regression equation has doubled the number of mountain glaciers from 17 in the original data to 34 and this has reduced the width of the 95 % confidence interval for mean balance ratio for mountain glaciers but still does not exclude BR = 1.

The pattern in Fig. 11 does not support the global validity of a balance ratio of much greater than unity, i.e. 1.75 ± 0.71 according to Rea (2009). Rather, balance ratios are generally greater than unity for outlet glaciers and valley glaciers, consistent with the negative error in equating balanced-budget ELA to Kurowski mean altitude for these glacier types. For mountain glaciers, balance ratios are closer to unity and the average error in the Kurowski altitude is correspondingly less.

Outlet and valley glaciers in the present dataset have average altitude ranges between highest and lowest points of about $960 \text{ m} \pm$ standard deviation 405 and $978 \pm 499 \text{ m}$ respectively, while mountain glaciers have a much smaller average range of about $570 \pm 249 \text{ m}$ (all figures calculated for the 103 glaciers). A larger altitude range might allow enough contrast in balance gradients between accumulation and ablation zones to significantly lower the balanced-budget ELA_0 while a more restricted altitude range might not allow such a large contrast in balance gradients, and ELA_0 will therefore be in better agreement with H_{mean} for these glaciers.

Kern and László (2010) relate their "steady-state accumulation-area ratio" to glacier size but there is no physical reason for this. From present results, I suggest their apparent relation between AAR_0 and glacier size actually reflects the dependence on primary classification of the glacier, as I show here for $(ELA_0 - H_{\text{mean}})$.

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The most likely physical explanation for different balance gradients in ablation and accumulation areas is the vertical variation in precipitation and/or accumulation across glaciers (Jarosch et al., 2012). On real-world glaciers, meteorological precipitation may increase due to orographic or topographic channelling effects, or the “effective” precipitation at the glacier surface may be augmented by snow drifting or avalanching from surrounding topography. These effects may be more important for mountain glaciers that are more constrained by topography than for outlet and valley glaciers. For example, two mountain glaciers in the Polar Ural (IGAN and Obruchevea) have excellent agreement between balanced-budget ELA and Kurowski mean altitude and are known to depend upon topographic augmentation of precipitation (Voloshina, 1988).

Kurowski (1891) is a good example of a glacier-centred approach to snow line avoiding problematic discussions of climatic and orographic snowlines as proposed by Ratzel (1886). Hess (1904, p. 68) suggests that glacier-based snow line refers to climatic snow line but most glaciers are influenced to some degree by local topography so balanced-budget ELA's generally have the nature of orographic rather than climatic snow line. Some glaciers, e.g. many of the mountain glaciers in the present study, may be more affected by local precipitation variations than most of the outlet and valley glaciers in the present study.

8 Conclusions

The estimation of balanced-budget ELA by the mean altitude of a glacier, suggested by Kurowski (1891), has been widely misquoted in the literature but not properly tested. There is a high correlation between balanced-budget ELA and Kurowski mean altitude for the 103 glaciers for which the necessary data are available. There is a relatively small difference between balanced-budget ELA and Kurowski mean altitude for the 103 glaciers, with a mean difference of -36 m with standard deviation ± 56 m. Balanced-budget ELA is significantly lower (at 95 confidence level) than Kurowski mean altitude for outlet and valley glaciers and not significantly lower for mountain glaciers. The

agreement between balanced-budget ELA and Kurowski mean altitude is very impressive for a method proposed more than 120 years ago and now tested against modern mass balance data.

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Table 1. Summary of glacier-altitudes for Hintereisferner, Austrian Alps, based on area-altitude data for the year 2001.

Concept	Symbol	Altitude (m.a.s.l.)
Mid-range altitude	mid	3064
Median (50 %) altitude	H_{50}	3056
Kurowski mean altitude (area weighted mean)	H_{mean}	3038
Mean ELA for 1953–2010	$\overline{\text{ELA}}$	3037
Balance-budget ELA (intercept in ELA-balance re- gression equation)	ELA_0	2923

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Table 2. Available surface mass balance data for the present analysis from WGMS (www.wgms.ch/fog.html) and some other sources.

Nr	Name of variable	Glaciers with data	% of total
1	≥ 1 year of mass balance measurements up to year 2010	371	100 %
2	≥ 1 year of ELA measurements up to year 2010	234	63 %
3	≥ 5 years of mass balance and ELA measurements up to year 2010	150	40 %
4	Hypsographic (area-altitude) data for ≥ 1 year allowing calculation of Kurowski mean altitude	148	40 %
5	Combining Eqs. (3) and (4)	103	28 %

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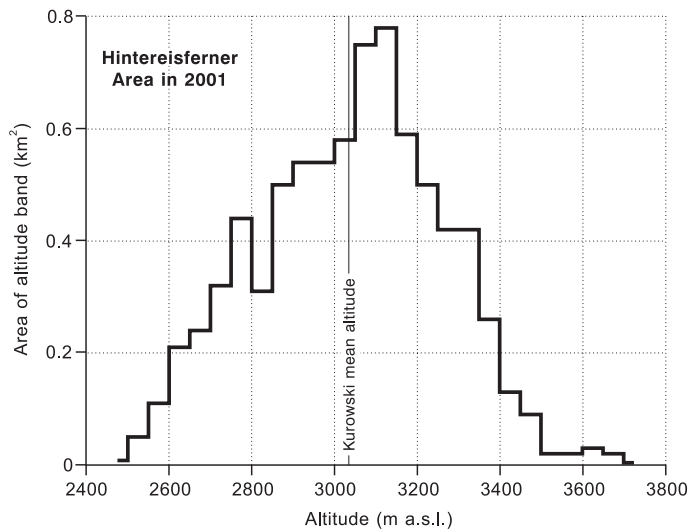


Figure 1. Area-altitude distribution for Hintereisferner, Austran Alps, showing areas of altitude bands vs. their mean altitudes. Area distribution is for year 2001.

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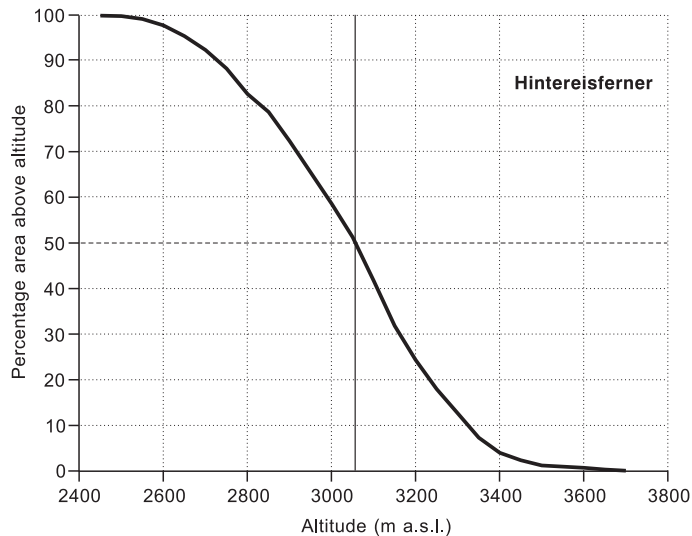


Figure 2. Hypsographic curve for Hintereisferner, Austrian Alps, showing the percentage of glacier area above any particular altitude. Area distribution is for year 2001.

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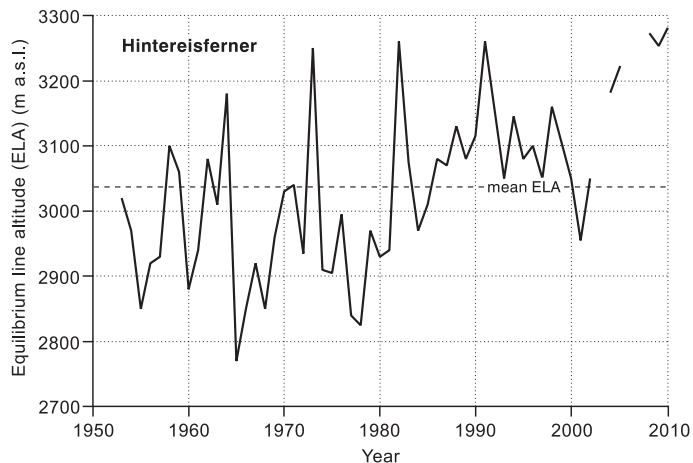


Figure 3. Year-to-year variations in equilibrium line altitude (ELA) at Hintereisferner, Austrian Alps, as measured in a surface mass balance programme for 1953–2010.

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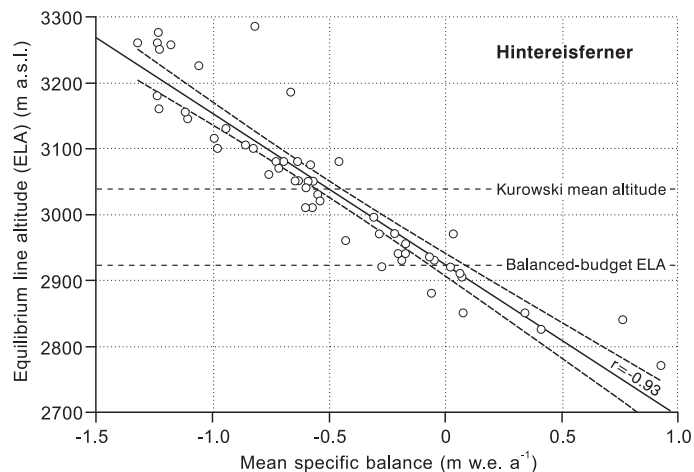


Figure 4. Equilibrium line altitude (ELA) plotted against mean specific mass balance of Hintereisferner, Austrian Alps. Dashed lines denote 95 % confidence interval around the regression line according to Student t statistic (Kreyszig, 1970, p. 302).

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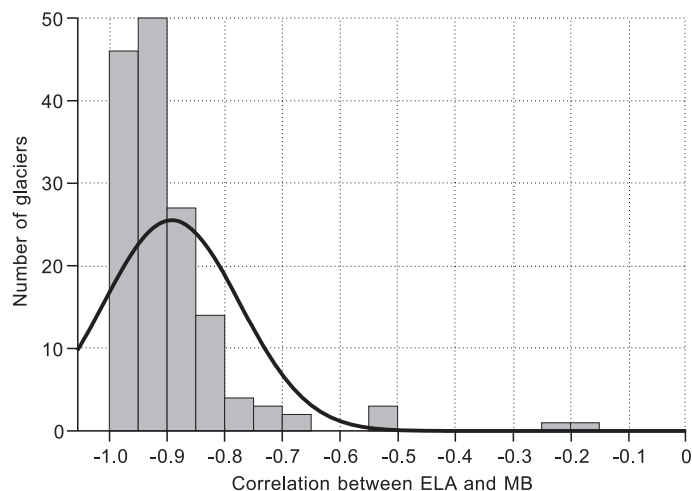


Figure 5. Histogram showing number of glaciers vs. correlation coefficients between equilibrium line altitude (ELA) and mean specific balance. Bold line denotes Gaussian curve with same mean and standard deviation.

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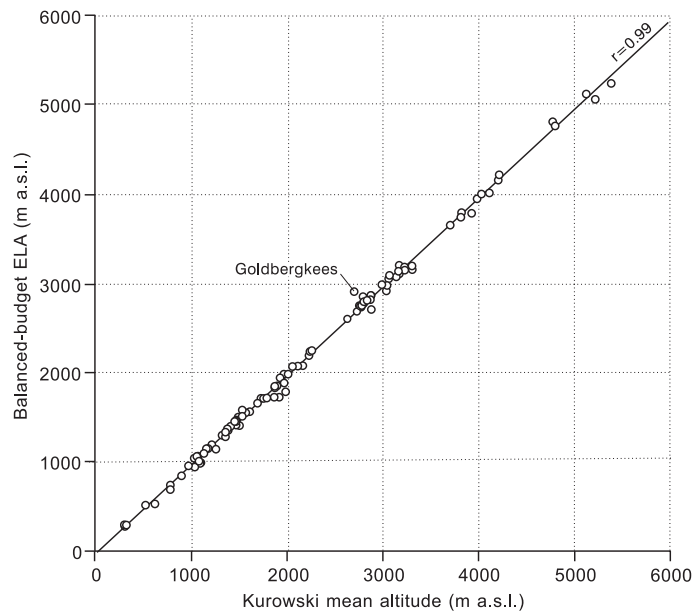


Figure 6. Balanced-budget ELA vs. Kurowski's mean altitude for 103 glaciers.

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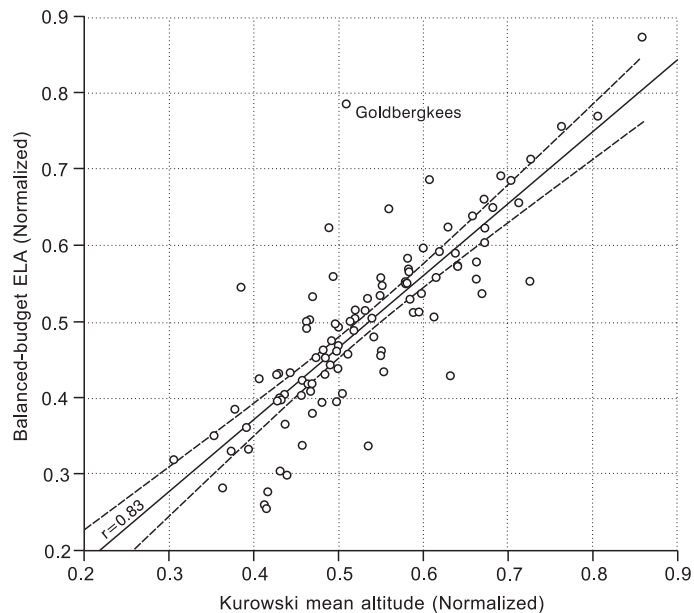


Figure 7. Balanced-budget ELA vs. Kurowski's mean altitude for 103 glaciers with both variables normalized to the altitude range of the glaciers. Normalized $Y = (YH_{\min}) / (H_{\max} - H_{\min})$. Dashed lines denote 95 % confidence interval around the regression line according to Student t statistic (Kreyszig, 1970, p. 302).

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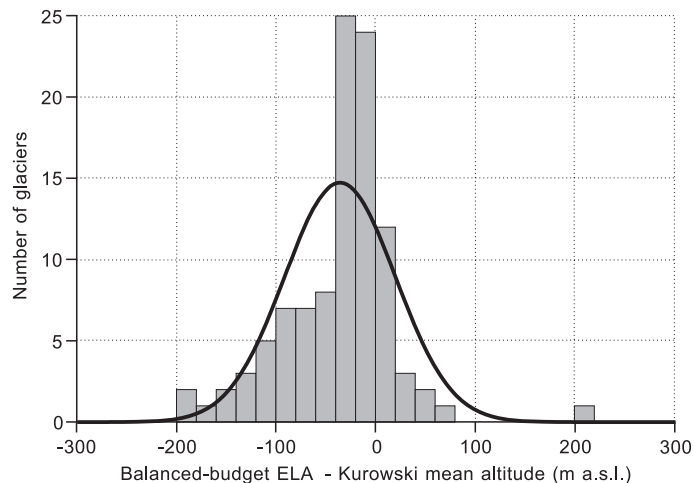


Figure 8. Histogram of difference between balanced-budget ELA and Kurowski mean altitude for 103 glaciers. Bold line denotes Gaussian curve with same mean and standard deviation.

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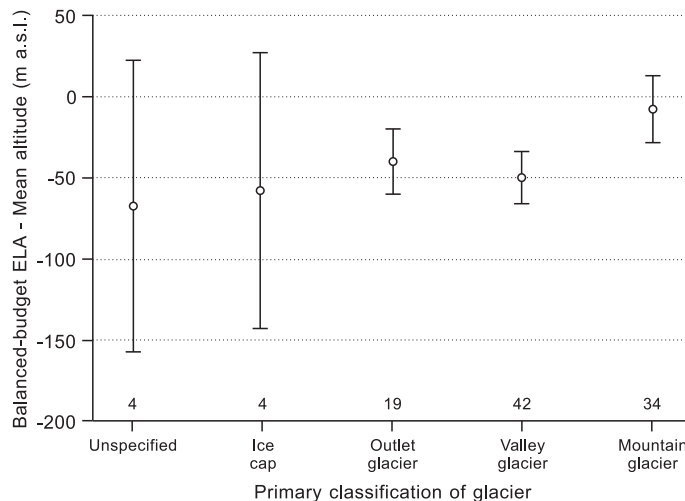


Figure 9. Boxplot of mean balanced-budget ELA minus Kurowski mean altitude ($ELA_0 - H_{\text{mean}}$) vs. primary classification of glaciers. Error bars represent 95 % confidence intervals of the means according to Student t statistic (Kreyszig, 1970, p. 178). Number of glaciers in each group are given in the lower part of the diagram.

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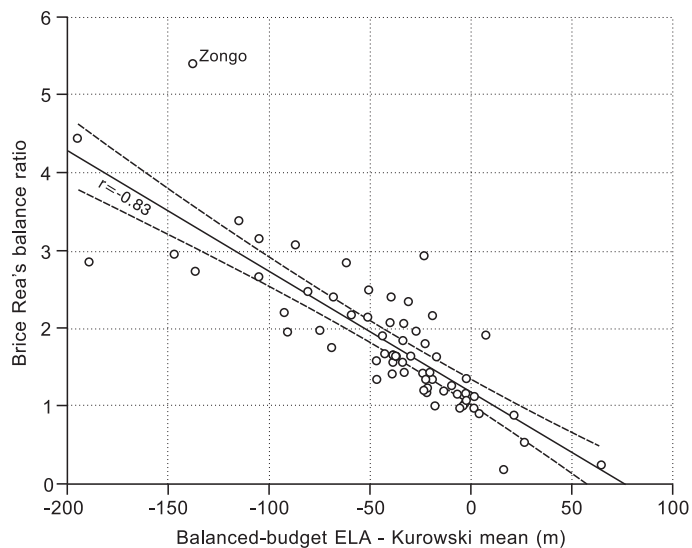


Figure 10. Brice Rea's balance ratio (Rea, 2009) plotted against Kurowski error ($ELA_0 - H_{\text{mean}}$) for 65 glaciers. Dashed lines denote 95 % confidence interval around the regression line according to Student t statistic (Kreyszig, 1970, p. 302).

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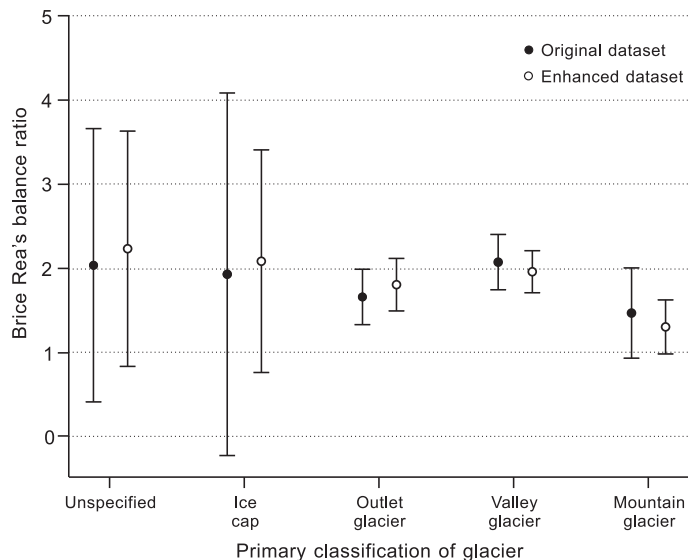


Figure 11. Brice Rea's balance ratio (Rea, 2009) vs. primary classification of glacier. Version 1 is for the original data (66 glaciers) and Version 2 is for an augmented data set (103 glaciers) using regression line in Fig. 10. Error bars represent 95 % confidence intervals of the means according to Student *t* statistic (Kreyszig, 1970, p. 178).

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