

1 **Numerical simulation of formation and preservation of**
2 **Ningwu ice cave, Shanxi, China**

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9 **Abstract:** Ice caves exist in locations where annual average temperature ~~is~~ ^{air} higher
10 than 0°C. An example is Ningwu ice cave, Shanxi Province, the largest ice cave in
11 China. In order to quantitatively ~~explain~~ ^{investigate} the mechanism of formation and preservation
12 of the ice cave, we use Finite Element Method to simulate the heat transfer process at
13 this ice cave. There are two major control factors. First, there is the seasonal
14 asymmetric heat transfer. Heat is transferred into the ice cave from outside, very
15 inefficiently by conduction in spring, summer and fall. In winter, thermal convection
16 occurs that transfers heat very efficiently out of the ice cave, thus cooling it down.
17 Secondly, ice-water phase change provides a heat barrier for heat transfer into the
18 cave in summer. The calculation also helps to evaluate effects of global warming,
19 tourists, colored lights, climatic conditions, etc. for sustainable development of ice ^{the}
20 cave as tourism resource. In some other ice caves in China, managers installed
21 air-tight doors at these ice caves entrance intending to “protect” these caves, but this

22 prevent cooling ~~down these caves~~ in winter, and these cave ices will entirely melt
23 within tens of years.

24 1 INTRODUCTION

25 An ice cave is a type of natural cave that contains significant amounts of perennial
26 ice. An ice cave is a rare phenomenon. Among the best known are Eisriesenwelt ice
27 cave, Austria (May et al., 2011; Obleitner and Spötl, 2011; Schöner et al., 2010),
28 Dobšiná ice cave, Slovakia (Bella, 2006; Lalkovič, 1995), Scărisoara ice cave,
29 Romania (Holmlund et al., 2005; Perşoiu et al., 2011) and Monlesi ice cave,
30 Switzerland (Luetscher et al., 2007; Luetscher et al., 2008). Eisriesenwelt ice cave is
31 the largest in the world. Dobšiná ice cave is also huge, with an ice volume of over
32 110,000m³ (Bella, 2006). In China, more than ten ice caves have been found,
33 including Ningwu, Wudalianchi, Taibaishan, Cuihuashan, Baiyizhai and Shennongjia
34 ice caves.

35 Studies of ice caves began as early as 1861 (Peters, 1861). In recent decades, in the
36 context of interest in global climate change, six international conferences on ice caves
37 have been held, with the reconstruction of regional ancient climate change as an
38 important topic for discussion (Laursen, 2010). Several articles reported seasonal air
39 temperature oscillations of several degrees from ventilated cave systems (Roberts et
40 al., 1998; Lacelle et al., 2004; Johnson et al., 2006). Therefore, to evaluate the impact
41 of changing climatic conditions on cave environments, a better explanation of

42 subsurface heat and mass transfers is necessary (Luetscher et al., 2008). Meanwhile,
43 ice caves are tourism resources. A better explanation of subsurface heat and mass
44 transfers could help ~~people~~^{to} manage ice caves more scientifically.

45 In the past, empirical calibrations were performed to determine the spatial and
46 temporal distribution of cave air temperature as a function of the external atmospheric
47 conditions [de Freitas and Littlejohn, 1987; de Freitas et al., 1982). In temperate karst
48 environments, explanation of the survival of subsurface ice accumulations represents
49 probably the most severe test for models of the magnitude and direction of heat and
50 mass transfers induced by cave air circulation (Luetscher et al., 2008). In mathematics
51 and engineering, ~~the~~^{the} Finite Element Method (FEM) and Finite Difference Method (FDM)
52 are popular for finding approximate solutions for partial differential equations. We
53 have not found any study in which these numerical techniques are applied to ice
54 caves.

55 In China, ice cave studies started only recently, after 1998, when Ningwu ice cave
56 was found. Although Ningwu ice cave has been widely reported during the past
57 decade (Gao et al., 2005; Meng et al., 2006), little was known about the processes
58 controlling the formation and preservation of perennial subsurface ice deposits under
59 changing climate conditions (Chen, 2003). We attempt to apply FEM to simulate the
60 energy fluxes of Ningwu ice cave, and then quantitatively interpret the formation and

61 preservation mechanism of ice deposit in Ningwu ice cave. Some suggestions are
62 given to manage Ningwu ice cave.

63 2 Study Site

64 Ningwu ice cave (38°57' N, 112°10' E; 2121 m above sea level ~~(~~Figure 1a~~)~~) is the
65 largest ice cave ever found in China. Located on the northern slopes of Guancen
66 Mountain, Ningwu County, Shanxi Province, it is known to local people as “the ten
67 thousand years ice cave”. The ~~stratum of the cave~~ ^{surrounding rock} consists of Ordovician Majiagou
68 limestone, dolomitic limestone, argillaceous dolomite and thin brecciated limestone
69 which is locally densely fractured (Shao et al., 2007). A geophysical exploration
70 (using magnetotelluric measurement) has been carried out for investigating the ~~spatial~~ ^{shape}
71 ~~form~~ of the ice cave (Shao et al., 2007). They obtained the vertical cross section of the
72 ice cave. The cave space is about 85 m depth. The widest part is in the middle, with a
73 width of 20 m.

74 The ice cave is a major tourist attraction. From May to October, about 1000 visitors
75 enter the cave per day. The ice cave has only one entrance (Figure 1c), and has
76 wooden spiral stairs leading to a bowling ~~shape~~ ^{shaped} room. Ice ~~almost~~ ^{almost} covers the host rock ^{almost}
77 ~~every inch~~ ^{and}. Ice stalactites ^{and} ice stalagmites (Figure 1d) can be seen in all part of the
78 cave.

79 The outside of the ice cave has a temperate climate. The external mean air

80 temperature from June to September is about 14.6 °C, and the mean annual air
81 temperature is 2.3 °C (Meng et al., 2006). The daily temperature from 1957 to 2008 is
82 obtained from Wuzhai meteorological station (about 320m lower than Ningwu ice
83 cave), which is the nearest station to the ice cave. We averaged ~~the same date~~
84 observational air temperature at Wuzhai station to obtain the annual temperature, and
85 then derive the mean annual temperature at Wuzhai station. We calculated ^(J)the
86 difference between the average annual air temperature at Ningwu ice cave and that at
87 Wuzhai station. After reducing the annual temperature at Wuzhai ~~X~~station by the
88 difference, we then obtain the annual temperature variation outside the ice cave
89 (Figure 2).

90 3 Qualitative Analysis

91 There are different hypotheses about the preservation mechanism of ice deposit in
92 Ningwu ice cave. Chen^(J)(2003) proposed that the existence of a “cold source” led to the
93 negative geothermal anomaly which preserves the ice deposit. Meng et al. (2006)
94 ascribed the ice deposit to multiple factors including geographical location, “icehouse
95 effect”, “chimney effect” and “thermal effect” produced by the ice deposit and the
96 “millennial volcano”. But they did not give us more details about these factors. Gao et
97 al. (2005) analyzed two aspects: terrain and climate. Because this region has a long
98 cold winter and a short cool summer, they considered that far more cold air than warm
99 air entered the region and then the ice cave stayed cold over year.

Subsurface

100 ~~The~~ temperature usually increases with depth at a geothermal gradient of about 1-3 °C
101 (100m)⁻¹ (Hu et al., 2001), ~~and there have been persistent heat flows from the deep~~
102 ~~crust to the surface~~. The notion that there is a permanent “cold source” underground is
103 unfounded. Even if a cold region had somehow formed, it would be heated up by the
104 geothermal flux from underneath in geological time. Reversal of geotherms can occur
105 in the presence of the advective heat transfer exists due to crustal movement or
106 groundwater flow (Shi and Wang, 1987). A reversal of geotherms can also occur from
107 transient changes in surface temperature and be induced by steep topography (Gruber
108 et al., 2004). But the outside of Ningwu ice cave has a temperate climate. It is hard to
109 preserve an ice cave in a temperate climate without a sustainable cooling mechanism.
110 In presence of a geothermal gradient, the host rock continuously transfers heat to the
111 ice cave, so there must be a sustainable mechanism to remove the heat from
112 underneath and ensure the maintenance of the ice cave.

113 The temperature outside the ice cave undergoes annual cyclic variations: in spring,
114 summer and fall, it is higher than the internal temperature, but in winter it is lower. As
115 Ningwu ice cave is bowl~~ing~~^d shape with only an opening in the upper part, cold air in
116 spring, summer and fall is heavy and sinks into the cave and thus will not produce
117 natural thermal convection. Conduction is the main form of heat transfer from the
118 outside down to the ice cave, and at the same time heat transfers into the cave from
119 the host rock due to the terrestrial heat flows. Thermal conductivities of neither rock

120 nor air are high and the conductive heat transfer efficiency is very low, so the heat
121 transferred to the ice cave in the three seasons is quite limited. In winter, the
122 temperature is low inside the ice cave ^{but} ~~and~~ even lower outside. The air in the ice cave
123 is lighter and air outside the cave entrance is heavier. It could thus become
124 gravitationally ^yunstable, and thermal convection could occur. The external cold air
125 flows into the cave to cool it down, and removes the heat transferred into the cave
126 from the host rock, as well as the heat transferred into the cave through the entrance in
127 spring, summer and fall. Since convective heat transfer is much more efficient than
128 conduction, the heat transferred out of the cave in the winter months is enough to
129 balance the heat that transferred into the cave year-round.

130 Ice melting into water absorbs a lot of latent heat. The melting heat of ice is 334 kJ
131 kg⁻¹ and the specific heat of limestone is 0.84 kJ kg⁻¹ K⁻¹. During summer, much of the
132 heat transferred to cave is consumed to melt the ice to 0 °C water. Therefore, ice-water
133 phase change can reduce the rate of temperature rise. Similarly, when the ambient
134 temperature decreases, ice-water phase change can reduce the rate of temperature
135 decrease. Therefore, ice-water phase change in the ice cave can “buffer” the
136 temperature change ~~and make the temperature change in a small range~~. A small
137 amount of ice melting near the cave entrance effectively prevents the heat from being
138 transferred into the deep cave. When the surface water flows into the ice cave from
139 the entrance, the ice cave temperature will not significantly increase.

140 The calculated energy balance of some cave ice (e.g. Eisriesenwelt ice cave) is largely
141 determined by the input of long-wave radiation originating at the host rock surface
142 (Obleitner and Spötl, 2011). Ice almost covers the host rock in Ningwu cave
143 completely. Therefore, we suggest that long-wave radiation originating at the host
144 rock surface is not ^a predominant factor in the processes of the formation and
145 preservation of ice deposit in Ningwu ice cave.

146 In summary, the air and the host rock transfer heat to the ice cave, making the cave
147 temperature rise in spring, summer and fall. In winter, the heat convection of air
148 makes the heat flow out of the cave, lowering the cave temperature. Meanwhile, four
149 seasons are accompanied by ice-water phase transition effect. The annual heat budget
150 of income and output is balanced, the cave will be in a cyclic state with very small
151 temperature fluctuations and the average temperature is always lower than 0 °C, so ice
152 bodies in the ice cave can persist.

153 *did you mean:*
Hoar frost ??
Snow crystals are single crystals of ice that grow from water vapor. If humidity enters
154 a cave and then form ice deposit, snow crystals could be discovered ~~more or less~~
155 (Kenneth, 2005). Actually, it is hard to find snow crystals in Ningwu ice cave. Any
156 clear traces of water or snow entering the cave through its entrance could not be found.
157 Meanwhile, karstified carbonate rock is heterogeneous, highly fractured, and with a
158 permeability developed such that water movement occurs below the surface (Fairchild

159 and Baker, 2012). In summary, we infer that most of the ice in the cave is formed by
160 freezing of infiltration water.

161 Water and ice are in dynamic equilibrium state. Water infiltrates into Ningwu ice cave
162 throughout the year, and forms ice. Ice at the bottom of Ningwu ice cave is thawed
163 under geothermal flow, and the water infiltrates into the deeper place. Ice stalactites,
164 ice stalagmites (Fig. 1d) can be seen in all part of Ningwu ice cave. This can verify the
165 former process. No directly observational evidences support the latter process.

166 The ice build-up process is a self-regulating process. If too much ice was accumulated
167 in Ningwu ice cave, the cavity will become small. Thus, Ra number and Nu number
168 will be reduced. That means the freezing efficiency become low. Some of the cave ice
169 will be thawed, and the cavity will become large. Ra number and Nu number will be
170 increased. That means the freezing efficiency become high. More ice will be
171 accumulated in Ningwu ice cave. ~~This process is always happening.~~

briefly explain
what these
are (Ra + Nu)

172 4 Principle of Simulation

173 4.1 Basic ideas of Simulation

174 Two heat transmission mechanisms must be taken into account to explain the
175 preservation of ice mass in ice cave, namely, thermal conduction and convection. The
176 phase change must also be considered. The heat conduction equation can be used to
177 describe the heat-conducting process, while for the convection process, due to the
178 complicated geometrical shape structure inside the ice cave and complex varying

179 boundary conditions, the convection pattern of air and its thermal consequences are
180 hard to determine exactly. In view of this, a widely used simplified method is applied
181 in this study: evaluate the Nusselt number (Nu) and solve the conductive equation by
182 introducing an equivalent thermal conductivity of the convecting air. In the case of an
183 upright circular tube, the relation between the temperature difference of the top and
184 the bottom and Nu number can be determined by adopting the experimental relation of
185 natural thermal convection. The enthalpy method can be adopted to calculate the
186 phase change.

187 In every time step of our modeling process, it is judged if air convection occurs based
188 on the temperature difference between the top and the bottom of the cave. If no
189 convection, the simple conduction problem will be solved, while if the convection
190 occurs, an effective conductivity is used in the thermal equation.

191 4.2 Equation and Physical Parameters

192 The heat conduction equation is

$$193 \quad c\rho \frac{\partial T}{\partial t} = k\nabla^2 T \quad (1)$$

194 where c is the specific heat, ρ is density, T is temperature (unknown number), t is time
195 and k is thermal conductivity. For the convective heat transfer process, an equivalent
196 thermal conductivity is used in Equation (1) based on the Nu number. Details of the Nu will be
197 discussed in the next section.

198 The enthalpy method is used to calculate the phase change process. A physical
 199 quantity enthalpy H is introduced in Equation (2), where T_r is an arbitrary lower
 200 temperature limit. For phase change, enthalpy H can be determined by Equations
 201 (3)-(5) (Lewis, 1996), in particular, the (T_s, T_l) is phase change range. Water-ice phase
 202 change occurs at 0 °C. But in numerical model, it is necessary to give a phase change
 203 range.

$$204 \quad H(T) = \int_{T_r}^T \rho c(T) dT \quad (2)$$

$$205 \quad H(T) = \int_{T_r}^T \rho c_s(T) dT \quad T \leq T_s \quad (3)$$

$$206 \quad H(T) = \int_{T_r}^{T_s} \rho c_s(T) dT + \int_{T_s}^T [\rho \left(\frac{dL}{dT} \right) + \rho c_f(T)] dT \quad T_s < T < T_l \quad (4)$$

$$207 \quad H(T) = \int_{T_r}^{T_s} \rho c_s(T) dT + \rho L + \int_{T_s}^{T_l} \rho c_f(T) dT + \int_{T_l}^T \rho c_l(T) dT \quad T \geq T_l \quad (5)$$

208 c_s is the specific heat in solid phase, c_l is the specific heat in liquid phase, c_f is the
 209 specific heat in solid-liquid mixing state and L is the latent heat. There are many ways
 210 to calculate heat capacity (Lewis and Roberts, 1987). The simple and accurate
 211 backward differentiation formula (Lewis and Roberts, 1987; Morgan et al., 1978) is
 212 adopted here, as expressed in Equation (6), where (n) and $(n-1)$ stand for time step.
 213 Equation (6) can be substituted into the heat equation along with the relevant material
 214 parameters for calculation.

215
$$(c\rho)^{(n)} = \left(\frac{dH}{dT}\right)^{(n)} = \frac{H^{(n)} - H^{(n-1)}}{T^{(n)} - T^{(n-1)}} \quad (6)$$

216 Relevant materials include limestone, ice, ice-limestone mixture, air and water.
217 Parameters of these materials are listed in Table 1. The physical parameter of
218 ice-limestone mixture is taken as the arithmetic mean of those of ice and limestone.
219 We assume that the ice body exists when temperature is below $-0.1\text{ }^{\circ}\text{C}$, and ice-water
220 mixture exists between $-0.1\text{ }^{\circ}\text{C}$ and $0.1\text{ }^{\circ}\text{C}$, and this becomes water when temperature
221 exceeds $0.1\text{ }^{\circ}\text{C}$. The ratio of ice and water in the mixture is linear to the temperature
222 within the phase change range, and so are the physical parameters. The latent heat L
223 of ice-water phase change is 334 kJ kg^{-1} .

224 4.3 Nu and equivalent thermal conductivity

225 When the convection occurs, heat transfer is Nu times greater than the conductive heat
226 transfer at the same conditions. Nu , the Nusselt number, is a dimensionless number,
227 which is defined as the ratio of convection heat transfer to pure conduction heat
228 transfer under the same conditions. In other words, an equivalent thermal conductivity
229 can be introduced, which is Nu times greater than the air thermal
230 conductivity (Schmeling and Marquart, 2014). Nu is related to the temperature
231 difference of air at the top and the bottom of the cave, physical properties (e.g.
232 viscosity and conductivity of air) and also the geometry of the cave. Ningwu ice cave
233 can be approximated by an upright circular tube. For such a tube, Nu can be calculated

explain earlier
in text, before
 Nu is first
used

234 based on fluid thermodynamics studies. When Equation (7) is satisfied (Sparrow and
235 Gregg, 1956; Yang and Tao, 2006), which is the case for Ningwu ice cave, the natural
236 convection heat transfer experimental relation (Sparrow and Gregg, 1956; Incropera et
237 al., 2011) is expressed as equation (8).

$$238 \quad d/h \geq 35 / Gr^{1/4} \quad (7)$$

$$239 \quad Nu_m = C(Gr \cdot Pr)_m^n \quad (8)$$

240 In Equations (7) and (8), d is the diameter of circular tube and h is the height of
241 circular tube; Nu_m is the Nusselt number, subscript m represents for the arithmetic
242 mean temperature of the boundary layer, Gr is the Grashof number, which
243 approximates the ratio of the buoyancy to viscous force acting on a fluid, Pr is the
244 Prandtl number; C and n are constants, the values of which are shown in Table2.

245 The Prandtl number, a dimensionless number, is defined as the ratio of momentum
246 diffusivity to thermal diffusivity. Pr is dependent only on the fluid material. For air,
247 Pr is 0.7. The Gr number is

$$248 \quad Gr = g\beta\Delta Tl^3 / \nu^2 \quad (9)$$

249 where g is the acceleration of gravity, β is the coefficient of cubical expansion, ΔT is
250 a temperature difference, l is a characteristics length and ν is the coefficient of

251 kinematic viscosity. The values are $g=9.8 \text{ m s}^{-2}$, $\beta=3.67 \times 10^{-3} \text{ K}^{-1}$, $l=80 \text{ m}$, ν
252 $=13.30 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and are substituted into equation (9) to obtain

253
$$Gr = 1.041 \times 10^{14} \Delta T \quad (10)$$

254 According to Equation (10), when the temperature difference is only $10^{-3} \text{ }^\circ\text{C}$, the Gr
255 number can reach 1.041×10^{11} . According to Table 2, we infer that natural convection
256 will occur and the flow state of air is a turbulent flow when the temperature is higher
257 inside than outside the ice cave. Equation (11), relating Nu to the temperature
258 difference, can be obtained when relevant parameters are substituted into equation (8).

259
$$Nu = 11000(0.0740\Delta T)^{1/3} \quad (11)$$

260 Even if equation (7) is not satisfied, corresponding experimental relations can also be
261 found in literatures (Cebeci, 1974; Minkowycz and Sparrow, 1974; Yang and Tao,
262 2006).

263 4.4 Models and Boundary Conditions

264 The rectangular Eulerian computational domain corresponds to a physical domain of
265 $300 \times 190 \text{ m}$ on the basis of the ice cave cross section (Figure 1b). There are 32825
266 nodes and 64986 elements involved in drawing the FEM grid. The grids for the ice
267 body and the interior air are denser.

268 The mean value of the geothermal gradient of the Lvliang highland area where

269 Ningwu ice cave is located, is $2.02\text{ }^{\circ}\text{C (100m)}^{-1}$ (Li, 1996). The mean value of the
270 geothermal gradient of the low-lying Linxian and Liulin areas in Shanxi Province is
271 $2.20\text{ }^{\circ}\text{C (100m)}^{-1}$ (Hu et al., 2001). We take the normal geothermal gradient value of
272 $2.0^{\circ}\text{C (100m)}^{-1}$ in the model. The temperature boundary conditions are assigned to
273 both sides of the model, with the annual average temperature at the surface and
274 increase with depth following the geothermal gradient. The heat flow boundary
275 condition is assigned for the bottom boundary. The terrestrial heat flow value is the
276 product of the geothermal gradient times the thermal conductivity of the limestone
277 host rock. According to Figure 2, we prescribe the variation temperature to the top
278 boundary.

279 The initial thermal structure is calculated assuming the surface temperature remained
280 constant at the annual average (Figure 3).

281 During the simulation, models with phase transition included and phase transition
282 neglected are both calculated for comparison. When phase change is considered,
283 latent heat and the material property variation are considered.

284 **5 Simulated Result and Analysis**

285 **5.1 Evolution of an ice deposit forming model**

286 Because of the periodic change of the ambient air temperature, the temperature in the
287 ice cave will show periodic variation corresponding⁹ly to conduction and convective
288 heat transfer. Figure 4a shows the evolution of the temperature at the bottom of the ice

289 cave. It can be interpreted as the process of formation of the cave ice. If a cave was
290 formed but not connected with the outside, it may have a temperature distribution
291 similar to Figure 3. If the cave became connected to the outside, i.e. collapsed at its
292 top and produced an entrance to the cave, an ice cave would then form within a
293 decade due to the winter convective cooling and stabilize in a century. Figure 4b shows
294 the details of first two decades and shows that the calculated results with phase
295 change considered (black line) do not differ significantly from those without
296 considering phase change (red line) in the cooling process. Starting from normal
297 ground temperature, the internal temperature of the ice cave drops rapidly in the first
298 decade, then drops more gradually and finally tends to become stable.

299 Figure 4b shows the details of temperature evolving in the ice cave during its initial
300 16 years of formation. It is seen that the cave ice can be maintained below 0°C all year
301 round after winter cooling for about 5 years. The cave temperature increases in spring,
302 summer and fall and decreases in winter, presenting annually periodic variation. The
303 air temperature of Ningwu ice cave decrease rapidly in winter, but the temperature
304 increase slowly in spring, summer and fall. Because the heat conduction in spring,
305 summer and fall is much less efficient than convective heat transfer in winter. With
306 phase change considered (black line), the increased rate of temperature in summer is
307 smaller than that without phase change (red line), because latent heat is required to
308 melt ices near the cave entrance, thus delaying the conduction of heat to the bottom of

309 the cave. In winter, the convective cooling is so effective that the difference is
310 minimized.

311 Figure 4c shows the cave temperature annual fluctuations when the process has lasted
312 two centuries, long enough to be evolved to a stable cyclic state. The amplitude of the
313 temperature variation is about 1 °C (from -3.9 °C to -2.9 °C). Ningwu ice cave has
314 been open to tourists, so the cave temperature has been disturbed. According to our
315 measurement on 5 June 2012, the lowest internal temperature of the ice cave was
316 -1.5 °C. Through the record in literature, the actually measured internal temperature of
317 the ice cave ranges between -1.0 °C (Meng et al., 2006), -4 °C and -6 °C (Gao et al.,
318 2005). The difference in measured results may be caused by different measuring
319 methods and different measuring time and positions. Similar to Figure 4b, the cave
320 temperature presents annual periodic variation, and the overall increasing rate of cave
321 temperature is smaller than its decreasing rate, because the heat transfer efficiency of
322 conduction is much lower than that of heat convection. The variation of cave
323 temperature for model with phase change considered (black line) is basically the same
324 with that without phase change considered (red line). The reason is that although we
325 considered phase change during calculation, the temperature of the ice body in the
326 cave is always kept below 0 °C when it reaches a stable cyclic state and no phase
327 change actually occurs.

328 Figures 5a and 5b show the spatial temperature distribution around the ice cave in
329 winter and summer respectively under the stable stage. Both figures show that a small
330 portion of rock at the top of the ice cave presents a negative geothermal gradient and
331 most of the host rock presents a normal positive geothermal gradient. Beneath the
332 bottom of the cave, however, geothermal gradients are much higher than normal. The
333 ice body temperature is always kept below 0 °C, although the external temperature is
334 completely different. In Figure 5a, the temperature of the shallow ground is lower
335 than 0 °C, corresponding to a frozen zone in winter. In Figure 5b, the temperature of
336 shallow parts of ground is higher than 0 °C, indicating that the frozen part is melted
337 and there is no permafrost. These features agree with actual conditions.

338 **5.2 Evolution of an ice deposit melting model**

339 The ice body in the ice cave will melt if there is no air convection heat transfer in
340 winter. Taking the temperature shown in Figure 5a as an initial temperature, the
341 evolution of temperature distribution will be calculated with or without phase change
342 effect considered. The results are shown in Figure 6 by a black line and a red line
343 respectively. They are the same when temperature does not reach the phase change
344 temperature. The ice body takes much longer to thaw when the latent heat of melting
345 is taken into consideration than when it is not. To thaw the ice body completely takes
346 23 years when the latent heat of phase change is not considered, compared with 37
347 years when it is considered.

348 **5.3 Sensitivity to model parameters**

349 The external air temperature, Nu and the number of tourists could directly affect the
350 energy transfer in Ningwu ice cave. Therefore, it needs sensitivity experiments on
351 these factors. With respect to the external air temperature, we consider two aspects: 1)
352 the mean annual temperature; 2) the amplitude of annual temperature. When the mean
353 annual temperature increases (respectively decreases) 1.0 °C, the computing results
354 are shown as Figure 7a and 7g (or Figure 7b and 7h). When the amplitude of external
355 temperature increases (respectively decreases) 5.0 °C, the computing results are shown
356 as Figure 7c and 7i (or Figure 7d and 7j). For Nu increases (respectively decreases)
357 10%, the computing results are shown as Figure 7e and 7k (or Figure 7f and 7l).
358 About 1000 visitors enter the cave per day from May to October. A person could
359 release 840 J. We assume that every person spend 1hour in Ningwu ice cave.
360 Meanwhile, there are 200 15^W-lightbulbs. When we consider the number of tourists
361 and bulbs, the computing result is shown as Figure 7m.

362 Similar to Figure 4b, Figure 7a-7f show the details of first two decades and represent
363 that ice deposit would be formed in Ningwu ice cave within first two decades in these
364 different experiments. Figure 7g-7l correspond to Figures 7a-7f respectively. As
365 showed in Figure 4c, Figure 7m-7n depict the cave temperature annual fluctuations
366 when the process has lasted two centuries, long enough to be evolved to a stable
367 cyclic state. Compared with Figure 4c, Figure 7m represents that the current density

368 of tourists and number of light bulbs in Ningwu ice cave could not melt the ice
369 deposit in it. Figure 7n shows the ice cave temperature annual fluctuations when the
370 mean annual temperature increases 3.5 °C. We can see the temperature ceiling is
371 -0.1°C. We consider this is the minimum climatic condition required to form Ningwu
372 ice cave.

373 **6 Discussion**

374 The age of the cave and that of the ice body are different. Formation of the cave
375 cavity could be old and have taken place in a warmer climate. The formation of the
376 ice body in the cave is a much later process that took place when the bowling-ball-like
377 cave was formed and the climate became cold enough. In the present climate, our
378 numerical modeling suggests that the year-round ice body can be formed within a
379 decade.

380 In spring, summer and fall, air and host rock transmit heat to the ice cave by thermal
381 conduction, increasing the temperature in the ice cave only slightly since the
382 conduction efficiency is low. In winter, heat is transmitted out of the ice cave by
383 natural thermal convection of air, efficiently decreasing the temperature in the ice
384 cave. Phase change accompanies the thermal processes. Considering these
385 mechanisms, the results show that (1) starting from a normal ground temperature, a
386 year-round ice body will be formed in the cave in less than a decade, about 5 years in
387 our model (Figure4b), and the ice cave temperature will decrease continuously for

388 more than a century. (2) The ice cave will finally reach a stable cyclic state, and its
389 temperature will fluctuate within a certain range, less than 1°C (from -3.9 °C to -2.9 °C)
390 for Ningwu ice cave. At this stage, the annual total heat transferred to the cave by
391 thermal conduction and the heat removed from the cave by convection are balanced.

392 It would be interesting to further investigate the possibility of imitating nature and
393 constructing a new kind of air conditioning system. At locations with similar climate
394 conditions, people may construct a basement more than 10 m deep, using natural air
395 convection to freeze ice in the basement in winter, and circulate air to the basement
396 for air conditioning in summer.

397 Setting an air-tight door at a cave entrance, as one park has done in China to “protect”
398 the ice cave at night during the tourist season and for the entire winter when the cave
399 is closed to tourist, actually blocks air convection in winter. As a result, cold air
400 cannot bring out heat from the cave, and accumulation of heat flow from the surface
401 and the deep crust will finally lead to melting of the ice body in the cave. Our
402 computation shows that it takes less than 40 years to completely melt the whole ice
403 body in the cave. This implies that Ningwu ice cave probably is not currently suffering
404 from thawing of the relict ice. This also suggests that scientific management is
405 important for sustainable usage of natural tourism resources. Otherwise, well-meaning
406 acts such as installing a trap door to completely seal the entrance for protection will

407 actually destroy the natural wonder in a few decades.

408 **7 Conclusion**

409 This paper has focused on quantitative analysis of the formation and preservation
410 mechanism of an ice body in Ningwu ice cave, a static ice cave. The finite element
411 modeling leads to the following conclusion: The controlling factor for forming and
412 sustaining the ice body in the cave is effective cooling of the cave in winter by natural
413 air convection. Heat conduction in spring, summer and fall is very ineffective to warm
414 up the cave. Ice-water phase change further prevents melting of ice in summer. The
415 formation of the cave may take a long geological time, but the formation of the
416 perennial ice body in the cave only takes decades of years under the current
417 temperature and geothermal gradient in the Ningwu area by winter air convection.
418 Once formed, the cave temperature will keep a stable cyclic state. At this time, the
419 amplitude of annual temperature variation in the Ningwu ice cave is within 1°C.
420 Environmental warming even up to 1 °C in in Ningwu area will increase the cave
421 temperature, but not melt the perennial ice body. The present heat from electric
422 lighting and visitors will not melt the ice body either. However, if the air convective
423 heat transfer is stopped in the winter as happened in some other Chinese ice caves, the
424 ice body in the cave could be completely melted within about 40 years. This analysis
425 is important for sustainable management of the ice cave as a tourism resource. The
426 mechanism of ice cave formation may be adopted for construction of energy-saving

427 buildings; ice may be produced in winter in basement and used for air conditioning in
428 summer.

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Table 1. Relative Material Parameters

Material	Heat Thermal Conductivity W/(m·k)	Density kg/m ³	Specific Heat kJ/(kg·K)
Limestone	2.7	2500	0.84
Ice	2.23	916.5	2.05
Mixture	2.465	1708.25	1.445
Air	0.0243	1.293	1.005
Water	0.58	1000	4.2

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Table 2. Gr Number and Constant for Different Flow Types (Yang and Tao, 2006)

Flow State	Coefficient C	Index n	Gr Application Range
Laminar Flow	0.59	1/4	$1.43 \times 10^4 \sim 3 \times 10^9$
Transitional Flow	0.0292	0.39	$3 \times 10^9 \sim 2 \times 10^{10}$
Turbulent Flow	0.11	1/3	$> 2 \times 10^{10}$

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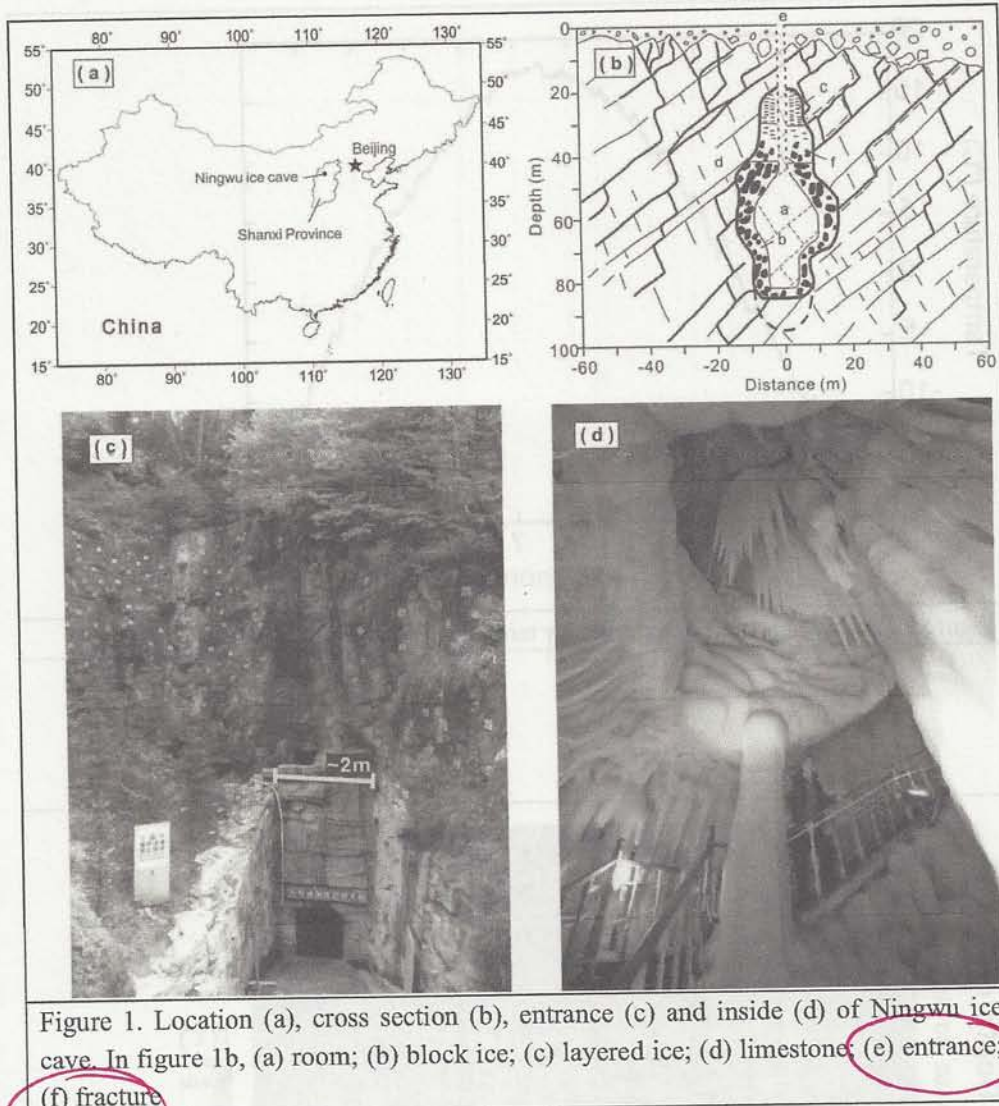
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(e) and (f) do not exist

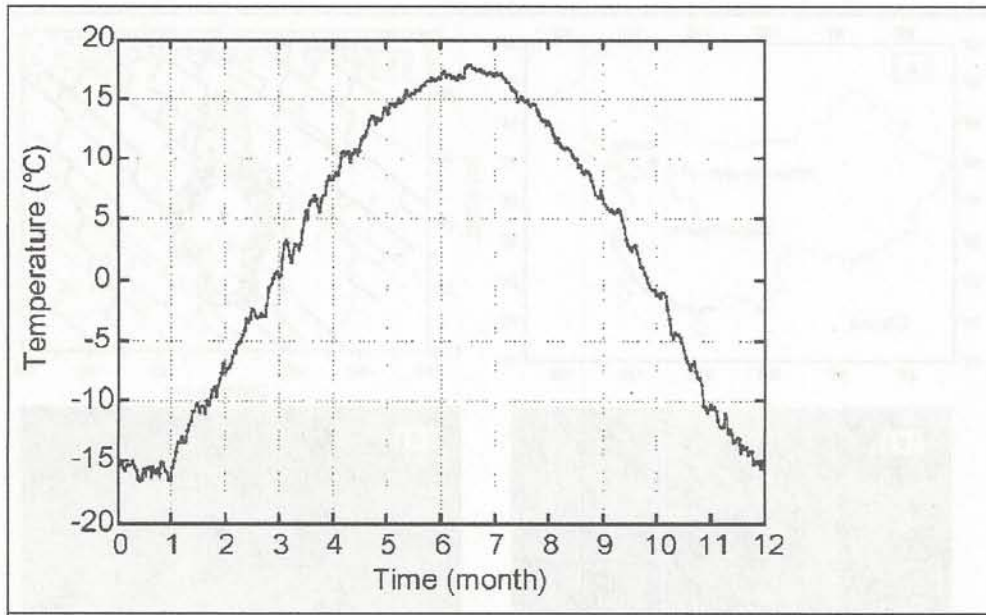


Figure 2. Yearly variation of external air temperature of Ningwu ice cave

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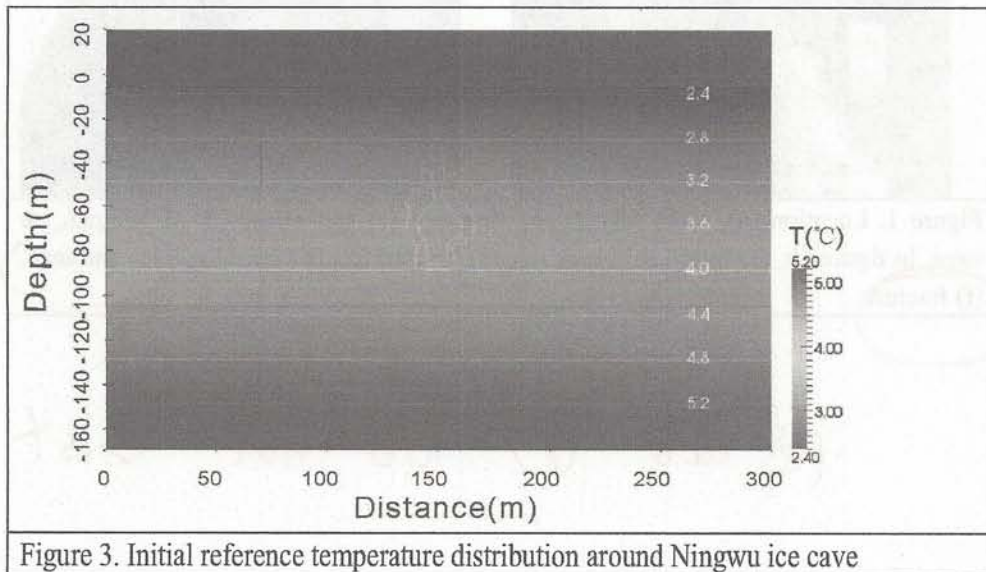


Figure 3. Initial reference temperature distribution around Ningwu ice cave

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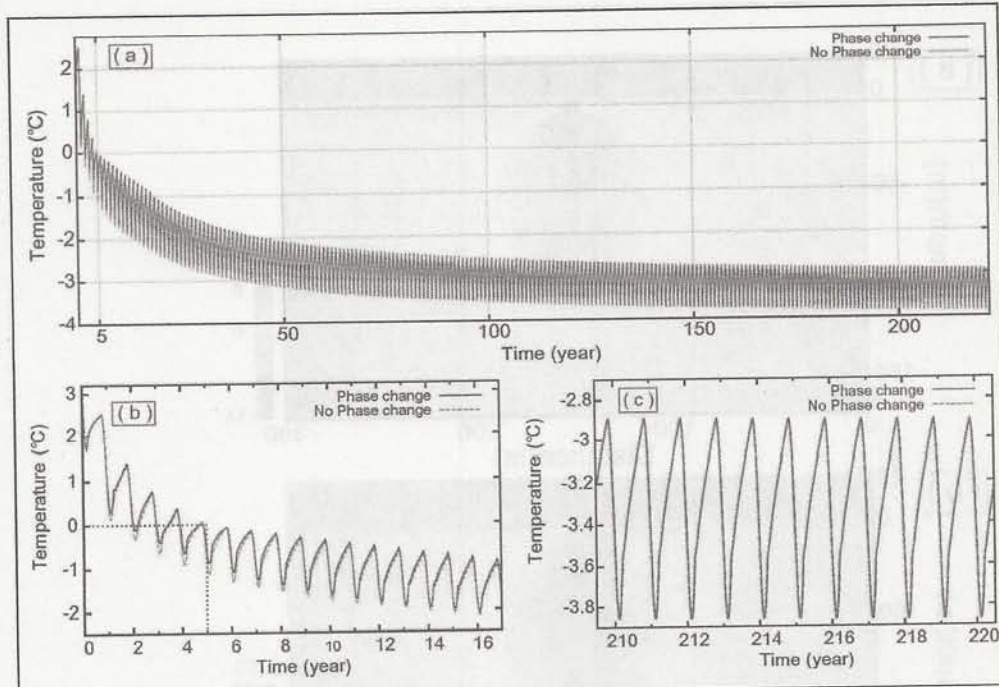
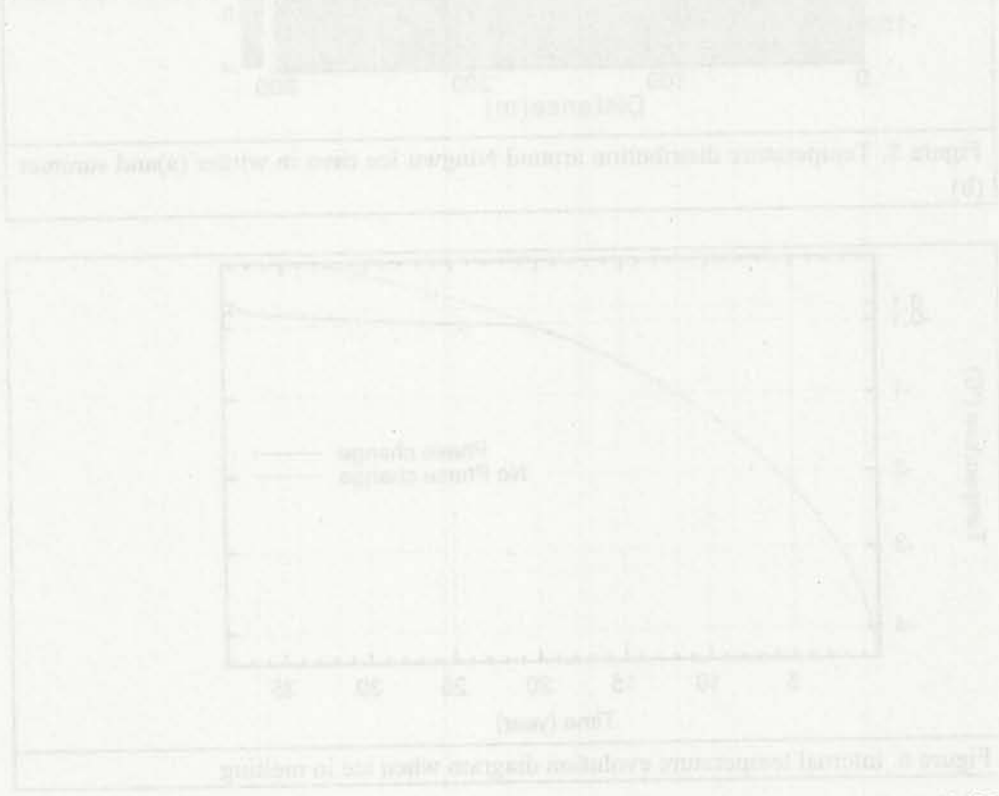


Figure 4. (a) Formation process of Ningwu ice cave; (b) Initial formation process; (c) Quasi stable state

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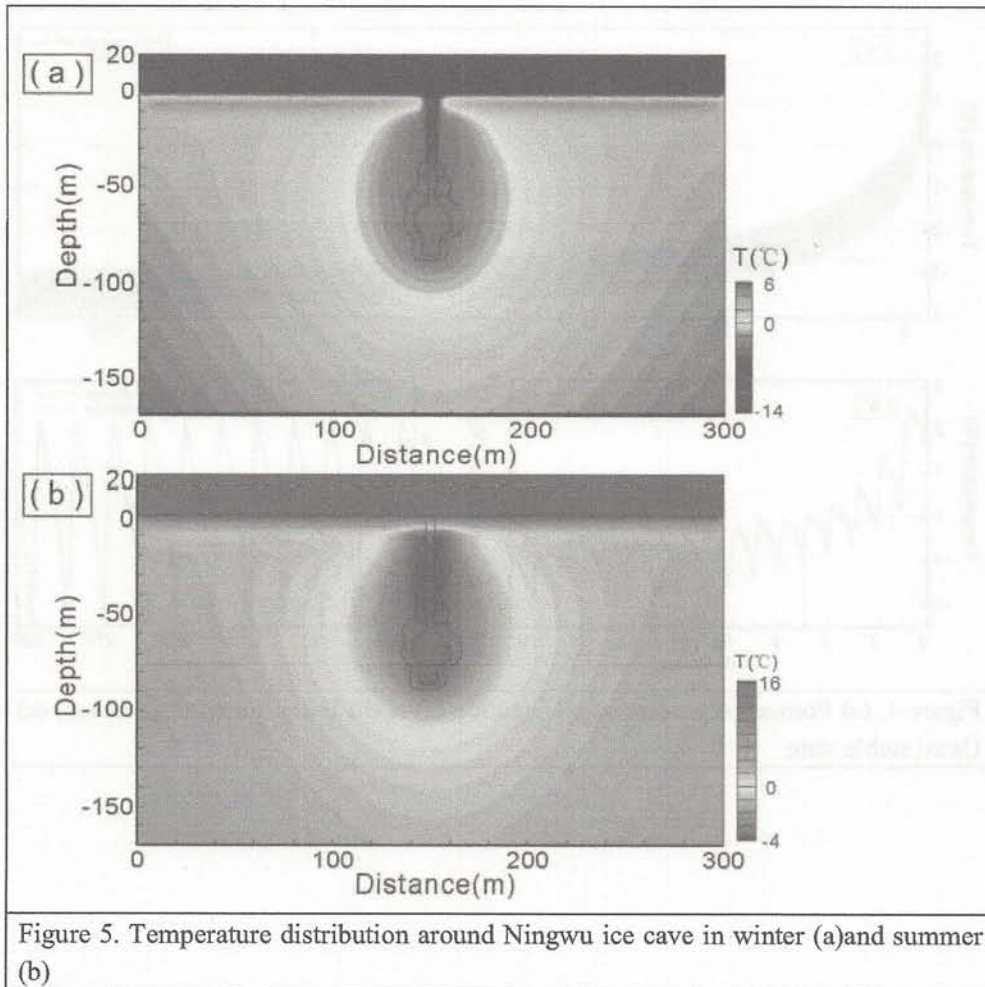


Figure 5. Temperature distribution around Ningwu ice cave in winter (a) and summer (b)

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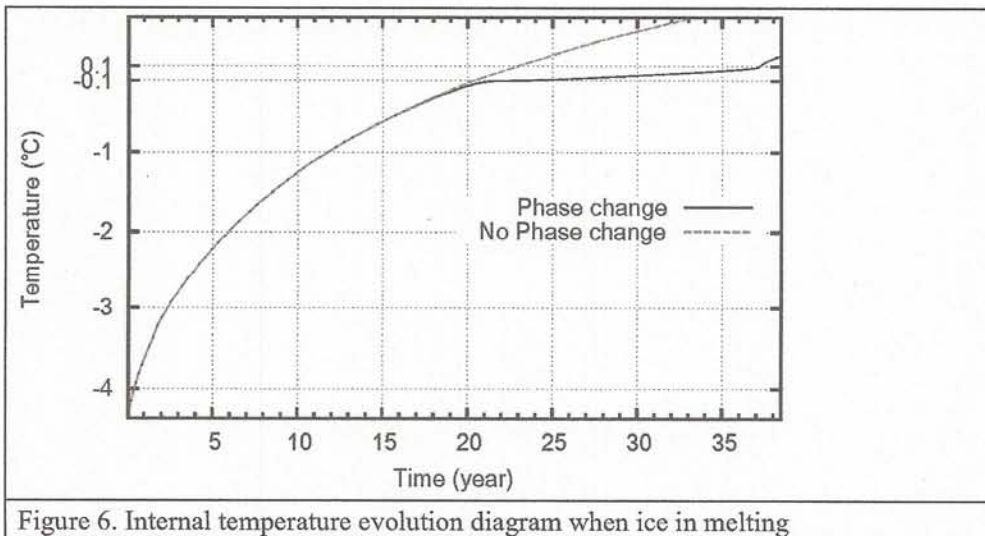


Figure 6. Internal temperature evolution diagram when ice is melting

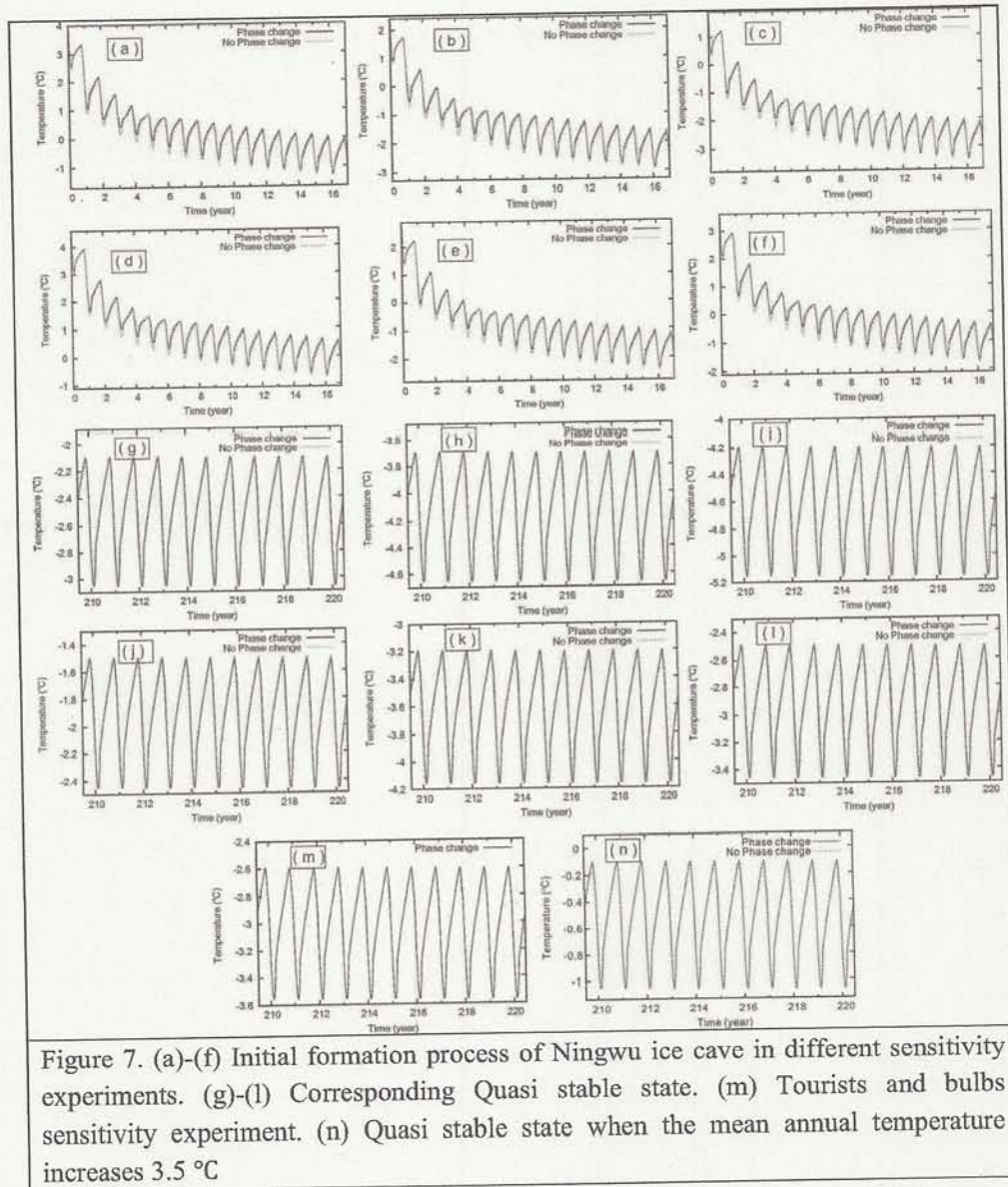


Figure 7. (a)-(f) Initial formation process of Ningwu ice cave in different sensitivity experiments. (g)-(l) Corresponding Quasi stable state. (m) Tourists and bulbs sensitivity experiment. (n) Quasi stable state when the mean annual temperature increases 3.5 °C

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