

Dear Referee,

Thank you very much for reviewing our manuscript.

Please see the revised manuscript for detailed change at the end of this document.

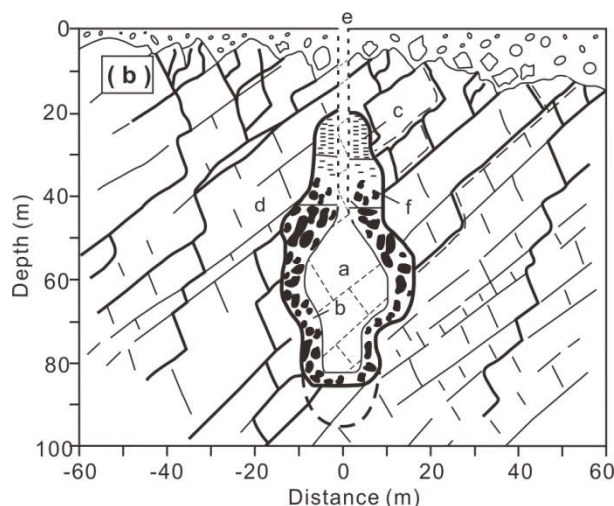
### Response to the comments:

Referee comments are repeated in red.

#### 1) The geometry of the cave

“..., but there is no data provided on the relationship of the passage beneath the cave to the 85 m vertical chamber. The cross-section provided in Figure 1b of Yang and Shi (2015) suggests that there is also a vertical entrance above the chamber to the outside, but no detailed information is provided about its dimensions, nor whether it is open throughout the year. Enlarging the diagram, the upper entrance appears to be about 4-5 m wide and cylindrical, i.e., it is large enough to allow large quantities of summer monsoon rain and winter snow to enter the top of the cave.”

As mentioned in line 80 of our manuscript, the ice cave has only one entrance, and has wooden spiral stairs leading to a bowling room. The following figure is the new version of Figure 1b of our manuscript. E in the figure is the entrance of Ningwu ice cave and the entrance is showed in Figure 1c of our manuscript. The entrance is open throughout the year. In Figure 1c, we can see the entrance is not very vertical. Limited quantities of summer monsoon rain and winter snow enter Ningwu ice cave through the entrance.



#### 2) Mean annual air temperature

“The obvious solution to obtain reliable climate data for the site is to install a weather station at the Ningwa cave site in the natural vegetation cover at an undisturbed site close to the cave, on a similar slope and aspect. The ice cave represents a pocket of sporadic permafrost by definition, but the extent of the pocket needs to be determined. A deep borehole to a depth of about 100 m that is instrumented with thermistors and a data logger could determine whether the adjacent ground contains permafrost. It would also provide basic data for comparison with temperatures measured in the cave itself.”

Thank you very much for giving us these specific instructions. We are glad to perform these works. If we get further support, we will carry out these observations. The current results at least provide a first order approximation for us.

#### 3) Some basic types of ice caves

“Yet another possible source of ice could be by condensation of water from warm air in a humid, subtropical climate (c.f., de Freitas and Schmekal, 2003). This is likely to occur at Ningwu ice cave if there is significant inflow of air into the cold cave in summer.”

Generally, ice caves are classified by their mass and energy exchange process. Section 3

of our manuscript detailed the energy exchange process of Ningwu ice cave. Mass (water and ice) is in dynamic equilibrium state. Water infiltrates into the cave throughout the year, and forms ice. Ice at the bottom of Ningwu ice cave is thawed under geothermal flow, and the water infiltrates into the deeper place. Ice stalactites, ice stalagmites (Fig. 1d of our paper) can be seen in all part of Ningwu ice cave. This can verify the former process. No directly observational evidences support the latter process.

4) Environmental history of the region

“This suggests that the ice in the Ningwu ice cave may only date back about 6 ka instead of 3 million years. The cave may also be a former mass of ice in the cavern that is currently suffering from thawing of the relict ice as a result of a warming climate.”

In this part, referee suggested that the ice in the Ningwu ice cave may only date back about 6ka instead of 3 million years. We agree with you on this point. But, we do not consider the ice cave is currently suffering from thawing of the relict ice. If there is no air convection heat transfer in winter, the ice body in Ningwu ice cave will melt rapidly. According to section 5.2 of our manuscript, to thaw the ice body completely only takes 37 years when the latent heat is considered.

5)

“Is there really no interannual change (ablation or formation) in the cave ice volume? i.e. can you be sure that the cave ice is fossil and that no water infiltrates into the cave throughout the year? If so, where is the water from the catchment area above the cave being directed to? How do these water fluxes affect heat exchanges at the system's boundaries?”

Water and ice are in dynamic equilibrium state. Water infiltrates into the cave throughout the year, and forms ice. Ice at the bottom of Ningwu ice cave is thawed under geothermal flow, and the water infiltrates into the deeper place. Ice stalactites, ice stalagmites (Fig. 1d) can be seen in all part of Ningwu ice cave. This can verify the former process. No directly observational evidences support the latter process which is our supposition. It is difficult to consider these water fluxes in our models, because we could not fix several parameters (porosity, permeability, rainfalls, etc).

6)

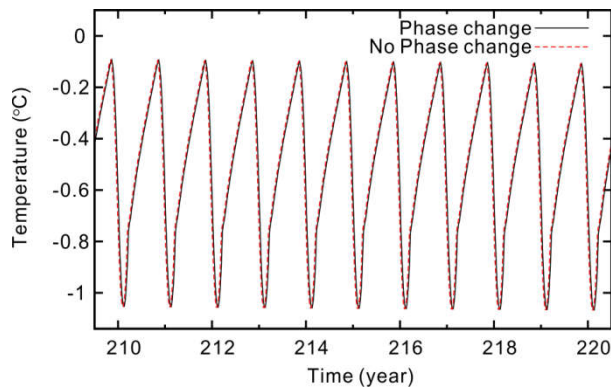
“How would a change in the infiltration regime (i.e. timing and recharge) affect the ice build-up, resp. the preservation of the cave ice?”

The ice build-up process is a self-regulating process. If too much ice was accumulated in Ningwu ice cave, the cavity will become small. Thus, Ra number and Nu number will be reduced. That means the freezing efficiency become low. Some of the cave ice will be thawed, and the cavity will become large. Ra number and Nu number will be increased. That means the freezing efficiency become high. More ice will be accumulated in Ningwu ice cave. This process is always happening.

7)

“Based on the sensitivity study, what would be the minimum climatic conditions required to form the ice cave?”

Based on the current climatic environment, Ningwu ice cave can form when the mean annual temperature increases 3.5 °C. We consider this is the minimum climatic condition required to form Ningwu ice cave. The figure shows the ice cave temperature annual fluctuations when the process has lasted two centuries, long enough to be evolved to a stable cyclic state. We can see the temperature ceiling is -0.1°C.



## Textual comments

Thanks for the comments. Everything was changed accordingly in the revised version. Here are some clarifications.

2370 I.18-21 edit or delete sentence as the formation of hoar frost (i.e. snow crystals) is not a process relevant to your system

We planned to verify that little humidity enters Ningwu ice cave. So these lines should not be deleted.

2372 I.14 please rephrase: do you mean that cold air is trapped inside the cave and leads to a thermal stratification which is not affected by natural convection during summer?

Natural convection occurs due to temperature differences which affect the density. Cold air in Ningwu ice cave is heavier than the environment air during summer and thus will not produce natural thermal convection.

2372 I.27 this assumes an equilibrated energy balance without considering any new formation or ablation of cave ice. Is this true (cf. also general comment)

This question includes two parts: energy and mass (ice or water). As Figure 4 of our paper showed, it takes relative long time to reach a stable cyclic state. The energy could not balance until the cave temperature reaches the stable cyclic state. As mentioned in question 5), water and ice are in dynamic equilibrium state.

2373 I.25 what about the cave ice mass balance? is the formation/ablation strictly limited to the cave entrance zone (l. 10)? If so, does this mean the cave ice in the deeper cave is completely "fossil"? when would it have formed?

As mentioned in question 5), it is difficult to consider these water fluxes in our models. In our numerical model, when the air temperature of Ningwu ice cave is below  $-0.1^{\circ}\text{C}$  (cf. 2376 I.5 of our paper), the air will become ice at numerical simulation. We consider the cave ice in the deeper cave is not completely "fossil". In further study, we will try to model the water fluxes.

2375 I.8-9 Eq. 3 and 4 are not necessary but  $T_s$ ,  $T_I$  and  $T$  need to be better defined

The ( $T_s$ ,  $T_I$ ) is phase change range. Water (or ice) phase change occurs at  $0^{\circ}\text{C}$ . But in numerical model, it is necessary to give a phase change range, for example,  $(-0.01, 0.01)$ .  $T$  is temperature, unknown number.

2378 I. 7 a gradient of  $2^{\circ}\text{C}/100\text{m}$  would be surprisingly high for a mature karst system surrounding the ice cave. A gradient of ca.  $0.5^{\circ}\text{C}/100\text{m}$  would probably be more realistic; cf. discussion in Luetscher and Jeannin (2004).

Fig. 4 in Luetscher and Jeannin (2004) showed the temperature in a borehole of the Swiss Jura Mountains. We can see the observed gradient of the main karst conduits is  $0.55^{\circ}\text{C}/100\text{m}$ . However, in our model, the temperature boundary conditions are assigned to both sides of the model. The both sides of the model are about 150m away from the cave ice (Figure. 3 in our paper). Therefore, we use the normal temperature gradient.

2379 I.11 “increasing rate” do you mean the “positive trend”?

The heat conduction in spring, summer and fall is much less efficient than convective heat transfer in winter. Therefore, the air temperature of Ningwu ice cave decrease rapidly in winter, but the temperature increase slowly in spring, summer and fall.

Sincerely,  
S. Yang and Y. Shi

References

De Freitas, C. R. and Schmekel, A., 2003. Condensation as a microclimatic process: Measurement, numerical simulation and prediction in the glowworm cave, New Zealand. *International Journal of Climatology* 23: 557-575.

Luetscher M., Jeannin P.-Y., 2004. Temperature distribution in karst systems: the role of air and water fluxes. *Terra Nova*, 16, 344-350. doi: 10.1111/j.1365-3121.2004.00572.x

Yang, S. and Shi, Y., 2015. Numerical simulation of formation and preservation of Ningwu ice cave, Shanxi, China. *The Cryosphere Discuss* 9: 2367-2395.

# Numerical simulation of formation and preservation of

## Ningwu ice cave, Shanxi, China

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**Abstract:** Ice caves exist in locations where annual average temperature is higher than 0°C. An example is Ningwu ice cave, Shanxi Province, the largest ice cave in China. In order to quantitatively explain the mechanism of formation and preservation of the ice cave, we use Finite Element Method to simulate the heat transfer process at this ice cave. There are two major control factors. First, there is the seasonal asymmetric heat transfer. Heat is transferred into the ice cave from outside, very inefficiently by conduction in spring, summer and fall. In winter, thermal convection occurs that transfers heat very efficiently out of the ice cave, thus cooling it down. Secondly, ice-water phase change provides a heat barrier for heat transfer into the cave in summer. The calculation also helps to evaluate effects of global warming, tourists, colored lights, climatic conditions, etc. for sustainable development of ice

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22 cave as tourism resource. In some other ice caves in China, managers installed  
23 air-tight doors at these ice caves entrance intending to “protect” these caves, but this  
24 prevent cooling down these caves in winters and these cave ices will entirely melt  
25 within tens of years.

## 26 1 INTRODUCTION

27 An ice cave is a type of natural cave that contains significant amounts of perennial ice.  
28 ~~Therefore some parts of an ice cave must have a temperature below 0°C all year round,~~  
29 ~~and some water must have traveled into the cave’s cold zone.~~ An ice cave is a rare  
30 phenomenon. Among the best known are Eisriesenwelt ice cave, Austria (May et al.,  
31 2011; Obleitner and Spötl, 2011; Schöner et al., 2010), Dobšináice cave, Slovakia  
32 (Bella, 2006; Lalkovič, 1995), Scărisoara ice cave, Romania (Holmlund et al., 2005;  
33 Perşoiu et al., 2011) and Monlesiice cave, Switzerland (Luetscher et al., 2007;  
34 Luetscher et al., 2008). Eisriesenwelt ice cave is the largest in the world. Dobšiná ice  
35 cave is also huge, with an ice volume of over 110,000m<sup>3</sup> (Bella, 2006). In China, more  
36 than ten ice caves have been found, including Ningwu, Wudalianchi, Taibaishan,  
37 Cuihuashan, Baiyizhai and Shennongjia ice caves.

38 Studies of ice caves began as early as 1861 (Peters, 1861). In recent decades, in the  
39 context of interest in global climate change, six international conferences on ice caves  
40 have been held, with the reconstruction of regional ancient climate change as an  
41 important topic for discussion (Laursen, 2010). ~~Paleoenvironmental reconstructions~~

42 ~~from cave deposits are largely based on the assumption of a stable subsurface climate~~  
43 ~~system(Hendy, 1971). However, s~~Several articles reported seasonal air temperature  
44 oscillations of several degrees from ventilated cave systems\_(Roberts et al., 1998;  
45 Lacelle et al., 2004; Johnson et al., 2006). Therefore, to evaluate the impact of  
46 changing climatic conditions on cave environments, a better explanation of subsurface  
47 heat and mass transfers is necessary\_(Luetscher et al., 2008). Meanwhile, ice caves are  
48 tourism resources. A better explanation of subsurface heat and mass transfers could  
49 help people manage ice caves more scientifically.

50 In the past, empirical calibrations were ~~enabled~~performed to determine the spatial and  
51 temporal distribution of cave air temperature as a function of the external atmospheric  
52 conditions(de Freitas and Littlejohn, 1987; de Freitas et al., 1982). In temperate karst  
53 environments, explanation of the survival of subsurface ice accumulations represents  
54 probably the most severe test for models of the magnitude and direction of heat and  
55 mass transfers induced by cave air circulation\_(Luetscher et al., 2008).\_In mathematics  
56 and engineering, Finite Element Method (FEM) and Finite Difference Method (FDM)  
57 are popular for finding approximate solutions for partial differential equations. We  
58 have not found any study in which these numerical techniques are applied to ice  
59 caves.

60 In China, ice cave studies started only recently, after 1998, when Ningwu ice cave

61 was found. Although Ningwu ice cave has been widely reported during the past  
62 decade (Gao et al., 2005; Meng et al., 2006), little was known about the processes  
63 controlling the formation and preservation of perennial subsurface ice deposits under  
64 changing climate conditions (Chen, 2003). We attempt to apply FEM to simulate the  
65 energy fluxes of Ningwu ice cave, and then quantitatively interpret the formation and  
66 preservation mechanism of ice deposit in Ningwu ice cave. Some suggestions are  
67 given to manage Ningwu ice cave.

## 68 **2 Study Site**

69 Ningwu ice cave (38°57' N, 112°10' E; 2121 m above sea level (Figure 1a)) is the  
70 largest ice cave ever found in China. Located on the northern slopes of Guancen  
71 Mountain, Ningwu County, Shanxi Province, it is known to local people as “the ten  
72 thousand years ice cave”. The stratum of the cave consists of Ordovician Majiagou  
73 limestone, dolomitic limestone, argillaceous dolomite and thin brecciated limestone  
74 which is locally densely fractured (Shao et al., 2007). A geophysical exploration  
75 (using magnetotelluric measurement) has been carried out for investigating the spatial  
76 form of the ice cave (Shao et al., 2007). They obtained the vertical cross section of the  
77 ice cave (Figure 1b). The cave space is about 85 m depth. The widest part is in the  
78 middle, with a width of 20 m.

79 The ice cave is a major tourist attraction. From May to October, about 1000 visitors  
80 enter the cave per day. The ice cave has only one entrance (Figure 1c), and has



81 wooden spiral stairs leading to a ~~bowling ball like~~ bowling shape room. Ice almost  
82 covers the host rock every inch. Ice stalactites, ice stalagmites (Figure 1d) can be seen  
83 in all part of the cave. ~~According to the classification of ice caves (Luetscher and~~  
84 ~~Jeannin, 2004), we consider that Ningwu ice cave is a static cave with congelation ice.~~  
85 ~~Snow crystals are single crystals of ice that grow from water vapor. If humidity enters~~  
86 ~~a cave and then form ice deposit, snow crystals could be discovered more or~~  
87 ~~less (Kenneth, 2005). Actually, it is hard to find snow crystals in Ningwu ice cave.~~  
88 ~~Any clear traces of water or snow entering the cave through its entrance could not be~~  
89 ~~found. Meanwhile, karstified carbonate rock is heterogeneous, highly fractured, and~~  
90 ~~with a permeability developed such that water movement occurs below the surface~~  
91 ~~(Fairchild and Baker, 2012). In summary, we infer that most of the ice in the cave is~~  
92 ~~formed by freezing of infiltration water.~~

93 The outside of the ice cave has a temperate climate. The external mean air  
94 temperature from June to September is about 14.6 °C, and the mean annual air  
95 temperature is 2.3 °C (Meng et al., 2006). The daily temperature from 1957 to 2008 is  
96 obtained from Wuzhai meteorological station (about 320m lower than Ningwu ice  
97 cave), which is the nearest station to the ice cave. We averaged the same date  
98 observational air temperature at Wuzhai station to obtain the annual temperature, and  
99 then derive the mean annual temperature at Wuzhai station. We use the 51 years of  
100 daily temperature to obtain the average daily temperature and the annual temperature

101 | ~~at Wuzhai meteorological station, and We~~ calculate the difference between the  
102 | average annual air temperature at Ningwu ice cave and ~~that~~ at Wuzhai ~~meteorological~~  
103 | station. After reducing the ~~average dailyannual~~ temperature at Wuzhai ~~meteorological~~  
104 | station by the difference, we then obtain the annual temperature variation outside the  
105 | ice cave (Figure 2).

### 106 | **3 Qualitative Analysis**

107 | There are different hypotheses about the preservation mechanism of ice deposit in  
108 | ~~the~~Ningwu ice cave. Chen(2003) proposed that the existence of a “cold source” led to  
109 | the negative geothermal anomaly which preserves the ice deposit. Meng et al. (2006)  
110 | ascribed the ice deposit to multiple factors including geographical location, “icehouse  
111 | effect”, “chimney effect” and “thermal effect” produced by the ice deposit and the  
112 | “millennial volcano”. But they did not give us more details about these factors. Gao et  
113 | al. (2005) analyzed two aspects: terrain and climate. Because this region has a long  
114 | cold winter and a short cool summer, they considered that far more cold air than warm  
115 | air entered the region and then the ice cave stayed cold over year.

116 | The temperature usually increases with depth at a geothermal gradient of about 1-3 °C  
117 | (100m)<sup>-1</sup> (Hu et al., 2001), and there have been persistent heat flows from the deep  
118 | crust to the surface. The notion that there is a permanent “cold source” underground is  
119 | unfounded. Even if a cold region had somehow formed, it would be heated up by the  
120 | geothermal flux from underneath in geological time. Reversal of geotherms can occur

121 ~~when in the presence of~~ the advective heat transfer exists due to crustal movement or  
122 groundwater flow\_(Shi and Wang, 1987). ~~For example, in subduction zones, when a~~  
123 ~~cold slab dives to the hot mantle, the geotherms can be overturned. The temperature~~  
124 ~~can also be overturned when continental collision with large overthrust faulting at a~~  
125 ~~speed of several centimeters per year sustains for millions of years. However, as long~~  
126 ~~as the motion stops, it will return to normal geothermal gradient under the heating of~~  
127 ~~mantle heat flow. The movement of groundwater can sometimes produces an~~  
128 ~~abnormal geothermal gradient, as long as the movement continues.~~ A reversal of  
129 geotherms can also occur from transient changes in surface temperature and be  
130 induced by steep topography\_(Gruber et al., 2004). But the outside of Ningwu ice cave  
131 has a temperate climate. It is hard to preserve an ice cave in a temperate climate  
132 without a sustainable cooling mechanism. ~~Since the existence~~In presence of a  
133 geothermal gradient, the host rock continuously transfers heat to the ice cave, so there  
134 must be a sustainable mechanism to remove the heat from underneath and ensure the  
135 maintenance of the ice cave.

136 The temperature outside the ice cave undergoes annual cyclic variations: in spring,  
137 summer and fall, it is higher than the internal temperature, but in winter it is lower. As  
138 Ningwu ice cave is bowling ~~ball-shaped~~shape with only an opening in the upper part,  
139 cold air in spring, summer and fall is heavy and sinks into the cave and thus will not  
140 produce natural thermal convection. Conduction is the main form of heat transfer

141 from the outside down to the ice cave, and at the same time heat transfers into the  
142 cave from ~~underground and~~ the host rock due to the terrestrial heat flows. Thermal  
143 conductivities of neither rock nor air are high and the conductive heat transfer  
144 efficiency is very low, so the ~~temperature rise in heat transferred to~~ the ice cave in the  
145 three seasons is quite limited. In winter, the temperature is low inside the ice cave and  
146 even lower outside. The air in the ice cave is lighter and air outside the cave entrance  
147 is heavier. It could thus become gravitational unstable, and thermal convection could  
148 occur. The external cold air flows into the cave to cool it down, and removes the heat  
149 transferred into the cave from ~~underground and~~ the host rock, as well as ~~that the heat~~  
150 transferred into the cave through the entrance in spring, summer and fall. Since  
151 convective heat transfer is much more efficient than conduction ~~heat transfer~~, the heat  
152 transferred out of the cave in the winter months is enough to balance the heat that  
153 transferred into the cave year-round.

154 Ice melting into water absorbs a lot of latent heat. The melting heat of ice is 334 kJ  
155  $\text{kg}^{-1}$  and the specific heat of limestone is  $0.84 \text{ kJ kg}^{-1} \text{ K}^{-1}$ . ~~That is, when the summer~~  
156 ~~heat conduction makes the temperature inside the cave rise, if there is no ice, the 334~~  
157 ~~kJ heat could cause the temperature of 1 kg limestone to rise by 397.6°C, or make~~  
158 ~~397.6 kg of limestone rise by 1°C, but the heat can only make 1 kg of 0°C ice melt to 0°C~~  
159 ~~water. When the ambient temperature rises, During summer,~~ much of the heat  
160 ~~transferred to cave~~ is consumed to melt the ice to  $0^\circ\text{C}$  water. Therefore, ice-water

161 phase change ~~action~~ can reduce the rate of temperature rise. Similarly, when the  
162 ambient temperature decreases, ice-water phase change ~~action~~ can reduce the rate of  
163 temperature decrease. Therefore, ice-water phase change ~~action~~ in the ice cave can  
164 “buffer” the temperature change and make the temperature change in a small range. A  
165 small amount of ice melting near the cave entrance effectively prevents the heat from  
166 being transferred into the deep cave. When the surface water flows into the ice cave  
167 from the entrance, the ice cave temperature will not significantly increase.

168 The calculated energy balance of some cave ice (e.g. Eisriesenwelt ice cave) is largely  
169 determined by the input of long-wave radiation originating at the host rock surface  
170 (Obleitner and Spötl, 2011). Ice almost covers the host rock in Ningwu cave  
171 completely. Therefore, we suggest that long-wave radiation originating at the host  
172 rock surface is not predominant factor in the processes of the formation and  
173 preservation of ice deposit in Ningwu ice cave.

174 In summary, the air and the host rock transfers heat to the ice cave, making the cave  
175 temperature rise in spring, summer and fall. In winter, the heat convection of air  
176 makes the heat flow out of the cave, lowering the cave temperature. Meanwhile, four  
177 seasons are accompanied by ice-water phase transition effect. The annual heat budget  
178 of income and output is balanced, the cave will be in a cyclic state with very small  
179 temperature fluctuations and the average temperature is always lower than 0 °C, so ice

180 bodies in the ice cave can persist.

181 According to the classification of ice caves (Luetscher and Jeannin, 2004), we  
182 consider that Ningwu ice cave is a static cave with congelation ice. Snow crystals are  
183 single crystals of ice that grow from water vapor. If humidity enters a cave and then  
184 form ice deposit, snow crystals could be discovered more or less (Kenneth, 2005).  
185 Actually, it is hard to find snow crystals in Ningwu ice cave. Any clear traces of water  
186 or snow entering the cave through its entrance could not be found. Meanwhile,  
187 karstified carbonate rock is heterogeneous, highly fractured, and with a permeability  
188 developed such that water movement occurs below the surface (Fairchild and Baker,  
189 2012). In summary, we infer that most of the ice in the cave is formed by freezing of  
190 infiltration water.

191 Water and ice are in dynamic equilibrium state. Water infiltrates into Ningwu ice cave  
192 throughout the year, and forms ice. Ice at the bottom of Ningwu ice cave is thawed  
193 under geothermal flow, and the water infiltrates into the deeper place. Ice stalactites,  
194 ice stalagmites (Fig. 1d) can be seen in all part of Ningwu ice cave. This can verify the  
195 former process. No directly observational evidences support the latter process.

196 The ice build-up process is a self-regulating process. If too much ice was accumulated  
197 in Ningwu ice cave, the cavity will become small. Thus, Ra number and Nu number  
198 will be reduced. That means the freezing efficiency become low. Some of the cave ice

199 will be thawed, and the cavity will become large. Ra number and Nu number will be  
200 increased. That means the freezing efficiency become high. More ice will be  
201 accumulated in Ningwu ice cave. This process is always happening.

## 202 **4 Principle of Simulation**

### 203 **4.1 Basic ideas of Simulation**

204 Two heat transmission mechanisms must be taken into account to explain the  
205 preservation of ice mass in ice cave, namely, thermal conduction and convection. The  
206 phase change must also be considered. The heat conduction equation can be used to  
207 describe the heat-conducting process, while for the convection process, due to the  
208 complicated geometrical shape structure inside the ice cave and complex varying  
209 boundary conditions, the convection pattern of air and its thermal consequences are  
210 hard to determine exactly. In view of this, a widely used simplified method is applied  
211 in this study: evaluate the Nusselt number( $Nu$ ) and solve the conductive equation by  
212 introducing an equivalent thermal conductivity of the convecting air. In the case of an  
213 upright circular tube, the relation between the temperature difference of the top and  
214 the bottom and  $Nu$  number can be determined by adopting the experimental relation of  
215 natural thermal convection. ~~This relation can be applied to the ice cave simulation by~~  
216 ~~introducing an equivalent thermal conductivity into the conduction equation based on~~  
217 ~~the  $Nu$  number.~~ The enthalpy method can be adopted to calculate the phase change.

218 In every time step of our modeling process, it is judged if air convection occurs based

219 on the temperature difference between the top and the bottom of the cave. If no  
220 convection, the simple conduction problem will be solved, while if the convection  
221 occurs, an effective conductivity is used in the thermal equation.

## 222 4.2 Equation and Physical Parameters

223 The heat conduction equation is

224 
$$c\rho\frac{\partial T}{\partial t} = k\nabla^2 T \quad (1)$$

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225 where  $c$  is the specific heat,  $\rho$  is density,  $T$  is temperature (unknown number),  $t$  is time  
226 and  $k$  is thermal conductivity. For the convective heat transfer process, an equivalent  
227 thermal conductivity is used in equation(1) based on the  $Nu$ . Details of the  $Nu$  will be  
228 discussed in the next section.

229 The enthalpy method is used to calculate the phase change process. A physical  
230 quantity enthalpy  $H$  is introduced in equation (2), where  $T_r$  is an arbitrary lower  
231 temperature limit. For phase change, enthalpy  $H$  can be determined by equations  
232 (3)-(5)(Lewis, 1996), in particular, the  $(T_s, T_l)$  ~~are~~ is phase change ranges. Water-ice  
233 phase change occurs at 0 °C. But in numerical model, it is necessary to give a phase  
234 change range.

235 
$$H(T) = \int_{T_r}^T \rho c(T) dT \quad (2)$$

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236

$$H(T) = \int_{T_r}^T \rho c_s(T) dT \quad T \leq T_s \quad (3)$$

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237

$$H(T) = \int_{T_r}^{T_s} \rho c_s(T) dT + \int_{T_s}^T [\rho \left( \frac{dL}{dT} \right) + \rho c_f(T)] dT \quad T_s < T < T_l \quad (4)$$

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238

$$H(T) = \int_{T_r}^{T_s} \rho c_s(T) dT + \rho L + \int_{T_s}^{T_l} \rho c_f(T) dT + \int_{T_l}^T \rho c_l(T) dT \quad T \geq T_l \quad (5)$$

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239  $c_s$  is the specific heat in solid phase,  $c_l$  is the specific heat in liquid phase,  $c_f$  is the  
 240 specific heat in solid-liquid mixing state and  $L$  is the latent heat. There are many ways  
 241 to calculate heat capacity (Lewis and Roberts, 1987). The simple and accurate  
 242 backward differentiation formula (Lewis and Roberts, 1987; Morgan et al., 1978) is  
 243 adopted here, as expressed in equation (6), where  $(n)$  and  $(n-1)$  stand for time step.  
 244 Equation (6) can be substituted into the heat equation along with the relevant material  
 245 parameters for calculation.

246

$$(c\rho)^{(n)} = \left( \frac{dH}{dT} \right)^{(n)} = \frac{H^{(n)} - H^{(n-1)}}{T^{(n)} - T^{(n-1)}} \quad (6)$$

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247 Relevant materials include limestone, ice, ice-limestone mixture, air and water.  
 248 Parameters of these materials are listed in Table 1. The physical parameter of  
 249 ice-limestone mixture is taken as the arithmetic mean of those of ice and limestone.  
 250 We assume that the ice body exists when temperature is below  $-0.1 \text{ } ^\circ\text{C}$ , and ice-water  
 251 mixture exists between  $-0.1 \text{ } ^\circ\text{C}$  and  $0.1 \text{ } ^\circ\text{C}$ , and this becomes water when temperature  
 252 exceeds  $0.1 \text{ } ^\circ\text{C}$ . The ratio of ice and water in the mixture is linear to the temperature

253 within the phase change range, and so are the physical parameters. The latent heat  $L$   
254 of ice-water phase change is  $334 \text{ kJ kg}^{-1}$ .

### 255 4.3 $Nu$ and equivalent thermal conductivity

256 When the convection occurs, heat transfer is  $Nu$  times greater than the conductive heat  
257 transfer at the same conditions.  $Nu$ , the Nusselt number, is a dimensionless number,  
258 which is defined as the ratio of convection heat transfer to pure conduction heat  
259 transfer under the same conditions. In other words, an equivalent thermal conductivity  
260 can be introduced, which is  $Nu$  times greater than the air thermal  
261 conductivity(Schmeling and Marquart, 2014).  $Nu$  is related to the temperature  
262 difference of air at the top and the bottom of the cave, physical properties (e.g.  
263 viscosity and conductivity of air) and also the geometry of the cave. Ningwu ice cave  
264 can be approximated by an upright circular tube. For such a tube,  $Nu$  can be calculated  
265 based on fluid thermodynamics studies. When equation (7) is satisfied(Sparrow and  
266 Gregg, 1956; Yang and Tao, 2006), which is the case for Ningwu ice cave, the natural  
267 convection heat transfer experimental relation (Sparrow and Gregg, 1956; Incropera et  
268 al., 2011) is expressed as equation (8).

269 
$$d / h \geq 35 / Gr^{1/4} \quad (7)$$

270 
$$Nu_m = C(Gr \cdot Pr)_m^n \quad (8)$$

271 In equations (7) and (8),  $d$  is the diameter of circular tube and  $h$  is the height of

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272 circular tube;  $Nu_m$  is the Nusselt number, subscript  $m$  represents for the arithmetic  
273 mean temperature of the boundary layer,  $Gr$  is the Grashof number, which  
274 approximates the ratio of the buoyancy to viscous force acting on a fluid,  $Pr$  is the  
275 Prandtl number;  $C$  and  $n$  are constants, the values of which are shown in Table2.

276 The Prandtl number, a dimensionless number, is defined as the ratio of momentum  
277 diffusivity to thermal diffusivity.  $Pr$  is dependent only on the fluid material. For air,  
278  $Pr$  is 0.7. The  $Gr$  number is

279 
$$Gr = g\beta\Delta T l^3 / \nu^2 \quad (9)$$

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280 where  $g$  is the acceleration of gravity,  $\beta$  is the coefficient of cubical expansion,  $\Delta T$  is  
281 a temperature difference,  $l$  is a characteristics length and  $\nu$  is the coefficient of  
282 kinematic viscosity. The values are  $g=9.8 \text{ m/s}^2$ ,  $\beta=3.67\times 10^{-3} \text{ k}^{-1}$ ,  $l=80 \text{ m}$ ,  $\nu$   
283  $=13.30\times 10^{-6} \text{ m}^2 \text{ s}^{-1}$  and are substituted into equation (9) to obtain

284 
$$Gr = 1.041\times 10^{14} \Delta T \quad (10)$$

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285 According to equation (10), when the temperature difference is only  $10^{-3} \text{ }^\circ\text{C}$ , the  $Gr$   
286 number can reach  $1.041\times 10^{11}$ . According to Table 2, we infer that natural convection  
287 will occur and the flow state of air is a turbulent flow when the temperature is higher  
288 inside than outside the ice cave. Equation (11), relating  $Nu$  to the temperature  
289 difference, can be obtained when relevant parameters are substituted into equation (8).

290 
$$Nu = 11000(0.0740\Delta T)^{1/3} \quad (11)$$

291 Even if equation (7) is not satisfied, corresponding experimental relations can also be  
292 found in literatures (Cebeci, 1974; Minkowycz and Sparrow, 1974; Yang and Tao,  
293 2006).

#### 294 4.4 Models and Boundary Conditions

295 The rectangular Eulerian computational domain corresponds to a physical domain of  
296  $300 \times 190$  m on the basis of the ice cave cross section (Figure 1b). There are 32825  
297 nodes and 64986 elements involved in drawing the FEM grid. The grids for the ice  
298 body and the interior air are denser.

299 The mean value of the geothermal gradient of the Lvliang highland area where  
300 Ningwu ice cave is located, is  $2.02 \text{ } ^\circ\text{C} (100\text{m})^{-1}$  (Li, 1996). The mean value of the  
301 geothermal gradient of the low-lying Linxian and Liulin areas in Shanxi Province is  
302  $2.20 \text{ } ^\circ\text{C} (100\text{m})^{-1}$  (Hu et al., 2001). We take the normal geothermal gradient value of  
303  $2.0 \text{ } ^\circ\text{C} (100\text{m})^{-1}$  in the model. The temperature boundary conditions are assigned to  
304 both sides of the model, with the annual average temperature at the surface and  
305 increase with depth following the geothermal gradient. The heat flow boundary  
306 condition is assigned for the bottom boundary. The terrestrial heat flow value is the  
307 product of the geothermal gradient times the thermal conductivity of the limestone  
308 host rock. According to Figure 2, we prescribe the variation temperature to the top

309 boundary.

310 The initial thermal structure is calculated assuming the surface temperature remained  
311 constant at the annual average (Figure 3).

312 During the simulation, models with phase transition included and phase transition  
313 neglected are both calculated for comparison. When phase change is considered,  
314 latent heat and the material property variation are considered.

## 315 **5 Simulated Result and Analysis**

### 316 **5.1 Evolution of an ice deposit forming model**

317 Because of the periodic change of the ambient air temperature, the temperature in the  
318 ice cave will show periodic variation correspondingly to conduction and convective  
319 heat transfer. Figure 4a shows the evolution of the temperature at the bottom of the ice  
320 cave. It can be interpreted as the process of formation of the cave ice. If a cave was  
321 formed but not connected with the outside, it may have a temperature distribution  
322 similar to Figure 3. If the cave became connected to the outside, i.e. collapsed at its  
323 top and produced an entrance to the cave, an ice cave would then form within a  
324 decade due to the winter convective cooling and stabilize in a century. Figure4bshows  
325 the details of first two decades and shows that the calculated results with phase  
326 change considered (black line) do not differ significantly from those without  
327 considering phase change (red line) in the cooling process. Starting from normal  
328 ground temperature, the internal temperature of the ice cave drops rapidly in the first

329 decade, then drops more gradually and finally tends to become stable.

330 Figure 4b shows the details of temperature evolving in the ice cave during its initial  
331 16 years of formation. It is seen that the cave ice can be maintained below 0°C all year  
332 round after winter cooling for about 5 years. The cave temperature increases in spring,  
333 summer and fall and decreases in winter, presenting annually periodic variation. The  
334 air temperature of Ningwu ice cave decrease rapidly in winter, but the temperature  
335 increase slowly in spring, summer and fall. The increasing rate of cave temperature is  
336 smaller than its decreasing rate, because the heat conduction in spring, summer and  
337 fall is much less efficient than convective heat transfer in winter. With phase change  
338 considered (black line), the increased rate of temperature in summer is smaller than  
339 that without phase change (red line), because latent heat is required to melt ices near  
340 the cave entrance, thus delaying the conduction of heat to the bottom of the cave. In  
341 winter, the convective cooling is so effective that the difference is minimized.

342 Figure 4c shows the cave temperature annual fluctuations when the process has lasted  
343 two centuries, long enough to be evolved to a stable cyclic state. The amplitude of the  
344 temperature variation is about 1\_°C (from -3.9\_°C to -2.9\_°C). Ningwu ice cave has  
345 been open to tourists, so the cave temperature has been disturbed. According to our  
346 measurement on 5 June 2012, the lowest internal temperature of the ice cave was  
347 -1.5\_°C. Through the record in literature, the actually measured internal temperature of

348 | ~~at~~the ice cave ranges between  $-1.0\text{ }^{\circ}\text{C}$  (Meng et al., 2006),  $-4\text{ }^{\circ}\text{C}$  and  $-6\text{ }^{\circ}\text{C}$  (Gao et al.,  
349 | 2005). The difference in measured results may be caused by different measuring  
350 | methods and different measuring time and positions. Similar to Figure 4b, the cave  
351 | temperature presents annual periodic variation, and the overall increasing rate of cave  
352 | temperature is smaller than its decreasing rate, because the heat transfer efficiency of  
353 | conduction is much lower than that of heat convection. The variation of cave  
354 | temperature for model with phase change considered (black line) is basically the same  
355 | with that without phase change considered (red line). The reason is that although we  
356 | considered phase change during calculation, the temperature of the ice body in the  
357 | cave is always kept below  $0\text{ }^{\circ}\text{C}$  when it reaches a stable cyclic state and no phase  
358 | change actually occurs.

359 | Figures 5a and 5b show the spatial temperature distribution around the ice cave in  
360 | winter and summer respectively under the stable stage. Both figures show that a small  
361 | portion of rock at the top of the ice cave presents a negative geothermal gradient and  
362 | most of the host rock presents a normal positive geothermal gradient. Beneath the  
363 | bottom of the cave, however, geothermal gradients are much higher than normal. The  
364 | ice body temperature is always kept below  $0\text{ }^{\circ}\text{C}$ , although the external temperature is  
365 | completely different. In Figure 5a, the temperature of the shallow ground is lower  
366 | than  $0\text{ }^{\circ}\text{C}$ , corresponding to a frozen zone in winter. In Figure 5b, the temperature of  
367 | shallow parts of ground is higher than  $0\text{ }^{\circ}\text{C}$ , indicating that the frozen part is melted

368 and there is no permafrost. These features agree with actual conditions.

## 369 **5.2 Evolution of an ice deposit melting model**

370 The ice body in the ice cave will melt if there is no air convection heat transfer in  
371 winter. Taking the temperature shown in Figure 5a as an initial temperature, the  
372 evolution of temperature distribution will be calculated with or without phase change  
373 effect considered. The results are shown in Figure 6 by a black line and a red line  
374 respectively. They are the same when temperature does not reach the phase change  
375 temperature. The ice body takes much longer to thaw when the latent heat of melting  
376 is taken into consideration than when it is not. To thaw the ice body completely takes  
377 23 years when the latent heat of phase change is not considered, compared with 37  
378 years when it is considered.

## 379 **5.3 Sensitivity to model parameters**

380 The external air temperature,  $Nu$  and the number of tourists could directly affect the  
381 energy transfer in Ningwu ice cave. Therefore, it ~~is~~ needs to do sensitivity  
382 experiments on these factors. With respect to the external air temperature, we consider  
383 two aspects: 1) the mean annual temperature; 2) the amplitude of annual temperature.  
384 When the mean annual temperature increases (~~or~~ respectively decreases) 1.0 °C, the  
385 computing results are showed as Figure 7a and 7g (or Figure 7be and 7hd). When  
386 the amplitude of external temperature increases (~~or~~ respectively decreases) 5.0 °C, the  
387 computing results are showed as Figure 7ce and 7if (or Figure 7dg and 7jh). For  $Nu$



388 increases (~~or~~ respectively decreases) 10%, the computing results are shown~~ed~~ as  
389 Figure 7~~e~~i and 7~~k~~j (or Figure 7~~k~~i and 7l). About 1000 visitors enter the cave per day  
390 from May to October. A person could release ~~200 cal~~ 840 J. We assume that every  
391 person spend 1hour in Ningwu ice cave. Meanwhile, there are 200 15w-lightbulbs.  
392 When we consider the number of tourists and bulbs, the computing result is shown~~ed~~  
393 as Figure 7m.

394 Similar to Figure 4b, Figure 7a-~~7f, 7e, 7e, 7g, 7i and 7k~~ show the details of first two  
395 decades and represent that ice deposit would be formed in Ningwu ice cave within  
396 first two decades in these different experiments. Figure ~~7g-7l b, 7d, 7f, 7h, 7j and 7l~~  
397 correspond to Figures 7a-7f respectively. As showed in Figure 4c, Figure 7g-7l depict  
398 the cave temperature annual fluctuations when the process has lasted two centuries,  
399 long enough to be evolved to a stable cyclic state. Compared with Figure 4c, Figure  
400 7m represent~~s~~ that the current density of tourists and number of light bulbs in Ningwu  
401 ice cave could not melt the ice deposit in it. Figure 7n shows the ice cave  
402 temperature annual fluctuations when the mean annual temperature increases 3.5 °C.  
403 We can see the temperature ceiling is -0.1°C. We consider this is the minimum  
404 climatic condition required to form Ningwu ice cave.

## 405 **6 Discussion**

406 The age of the cave and that of the ice body are different. Formation of the cave  
407 cavity could be old and have taken place in a warmer climate. The formation of the

408 ice body in the cave is a much later process that took place when the bowling-ball-like  
409 cave was formed and the climate became cold enough. In the present climate, our  
410 numerical modeling suggests that the year-round ice body can be formed within a  
411 decade.

412 In spring, summer and fall, air and host rock transmit heat to the ice cave by thermal  
413 conduction, increasing the temperature in the ice cave only slightly since the  
414 conduction efficiency is low. In winter, heat is transmitted out of the ice cave by  
415 natural thermal convection of air, efficiently decreasing the temperature in the ice  
416 cave. Phase change accompanies the thermal processes. Considering these  
417 mechanisms, the results show that (1) starting from a normal ground temperature, a  
418 year-round ice body will be formed in the cave in less than a decade, about 5 years in  
419 our model (Figure4b), and the ice cave temperature will decrease continuously for  
420 more than a century. (2) The ice cave will finally reach a stable cyclic state, and its  
421 temperature will fluctuate within a certain range, less than 1°C (from -3.9 °C to -2.9 °C)  
422 for Ningwu ice cave. At this stage, the annual total heat transferred to the cave by  
423 thermal conduction and the heat removed from the cave by convection are balanced.

424 It would be interesting to further investigate the possibility of imitating nature and  
425 constructing a new kind of air conditioning system. At locations with similar climate  
426 conditions, people may construct a basement more than 10 m deep, using natural air

427 convection to freeze ice in the basement in winter, and circulate air to the basement  
428 for air conditioning in summer.

429 Setting an air-tight door at a cave entrance, as one park has done in China to “protect”  
430 the ice cave at night during the tourist season and for the entire winter when the cave  
431 is closed to tourist, actually blocks air convection in winter. As a result, cold air  
432 cannot bring out heat from the cave, and accumulation of heat flow from the surface  
433 and the deep crust will finally lead to melting of the ice body in the cave. Our  
434 computation shows that it takes less than 40 years to completely melt the whole ice  
435 body in the cave. This implies that Ningwu ice cave probably is not currently suffering  
436 from thawing of the relict ice. This also suggests that scientific management is  
437 important for sustainable usage of natural tourism resources. Otherwise, well-meaning  
438 acts such as installing a trap door to completely seal the entrance for protection will  
439 actually destroy the natural wonder in a few decades.

## 440 **7 Conclusion**

441 This paper has focused on quantitative analysis of the formation and preservation  
442 mechanism of an ice body in Ningwu ice cave, a static ice cave. The finite element  
443 modeling leads to the following conclusion: The controlling factor for forming and  
444 sustaining the ice body in the cave is effective cooling of the cave in winter by natural  
445 air convection. Heat conduction in spring, summer and fall is very ineffective to warm  
446 up the cave. Ice-water phase change further prevents melting of ice in summer. The

447 formation of the cave may take a long geological time, but the formation of the  
448 perennial ice body in the cave only takes decades of years under the current  
449 temperature and geothermal gradient in the Ningwu area by winter air convection.  
450 Once formed, the cave temperature will keep a stable cyclic state. ~~Under this stable~~ At  
451 this time, the amplitude of annual temperature variation in the Ningwu ice cave is  
452 within 1°C. Environmental warming even up to 1\_°C in in Ningwu area will increase  
453 the cave temperature, but not melt the perennial ice body. The present heat from  
454 electric lighting and visitors will not melt the ice body either. However, if the air  
455 convective heat transfer is stopped in the winter as happened in some other Chinese  
456 ice caves, the ice body in the cave could be completely melted within about 40 years.  
457 This analysis is important for sustainable management of the ice cave as a tourism  
458 resource. The mechanism of ice cave formation may be adopted for construction of  
459 energy-saving buildings; ice may be produced in winter in basement and used for air  
460 conditioning in summer.

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554

**Table 1.** Relative Material Parameters

Material	Heat W/(m·k)	Conductivity	Density kg/m <sup>3</sup>	Specific Heat kJ/(kg·K)
Limestone	2.7		2500	0.84
Ice	2.23		916.5	2.05
Mixture	2.465		1708.25	1.445
Air	0.0243		1.293	1.005
Water	0.58		1000	4.2

555

**Table 2.** *Gr* Number and Constant for Different Flow Types (Yang and Tao, 2006)

Flow State	Coefficient C	Index n	<i>Gr</i> Application Range
Laminar Flow	0.59	1/4	$1.43 \times 10^4 \sim 3 \times 10^9$
Transitional Flow	0.0292	0.39	$3 \times 10^9 \sim 2 \times 10^{10}$
Turbulent Flow	0.11	1/3	$> 2 \times 10^{10}$

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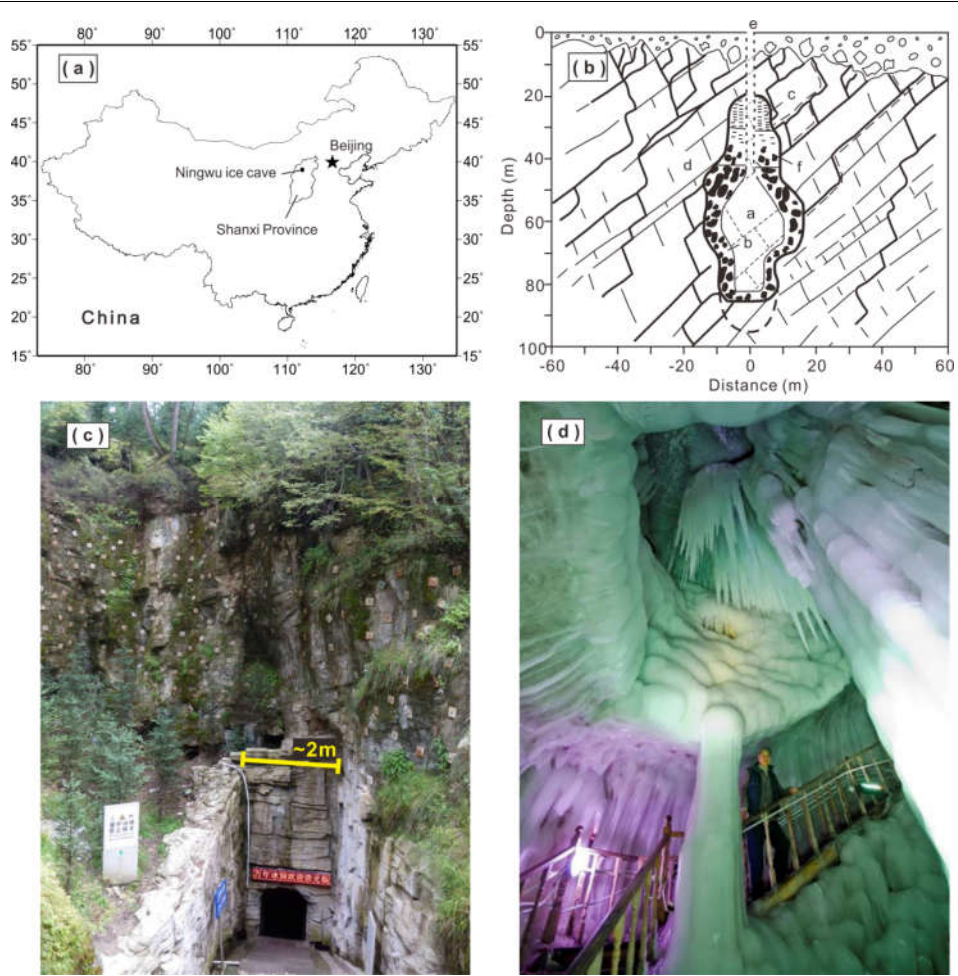


Figure 1. Location (a), cross section (b), entrance (c) and inside (d) of Ningwu ice cave. In figure 1b, (a) room; (b) block ice; (c) layered ice; (d) limestone; (e) entrance; (f) fracture

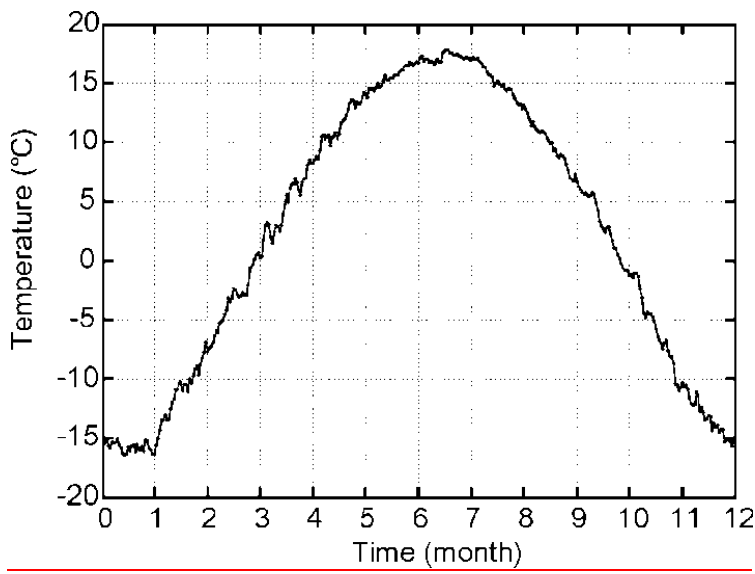


Figure 2. Yearly variation of external air temperature of Ningwu ice cave

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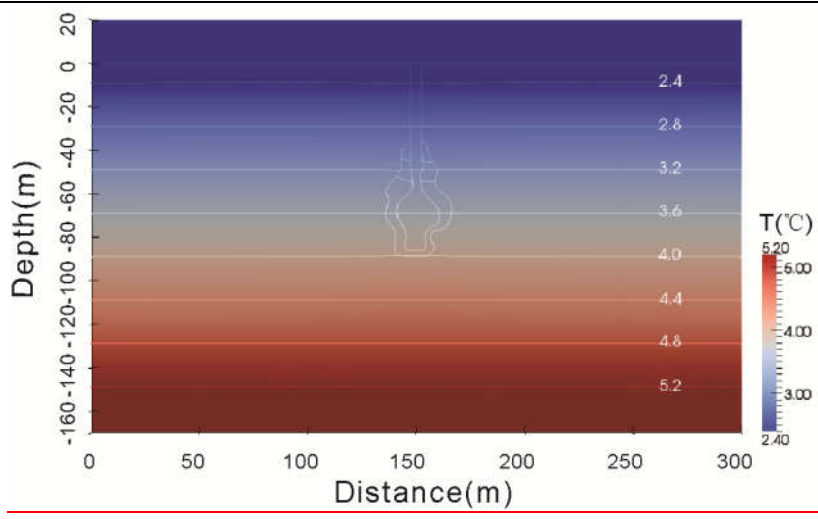


Figure 3. Initial reference temperature distribution around Ningwu ice cave

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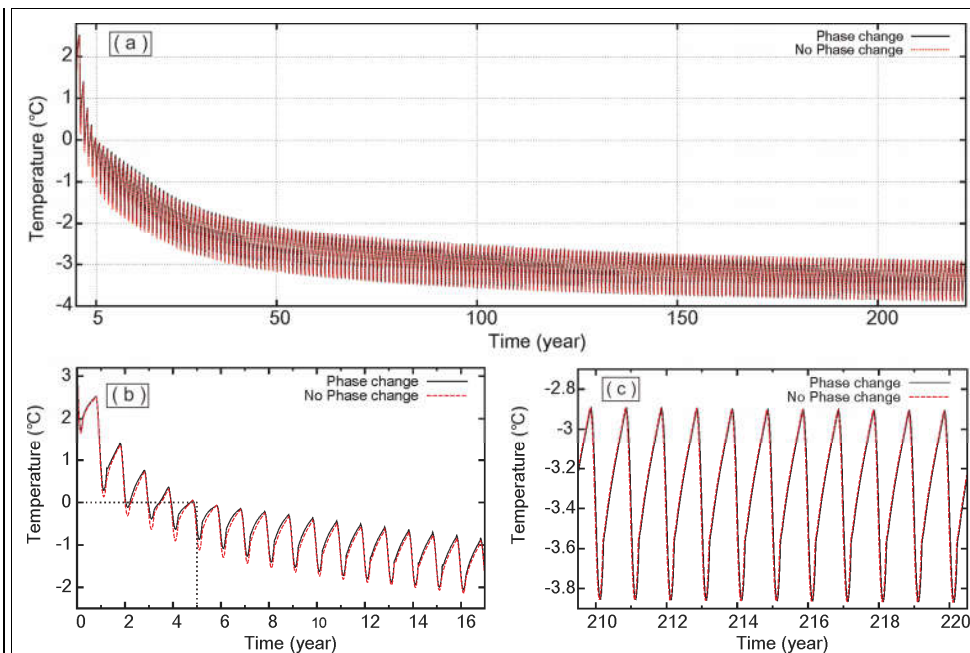


Figure 4. (a) Formation process of Ningwu ice cave; (b) Initial formation process; (c) Quasi stable state

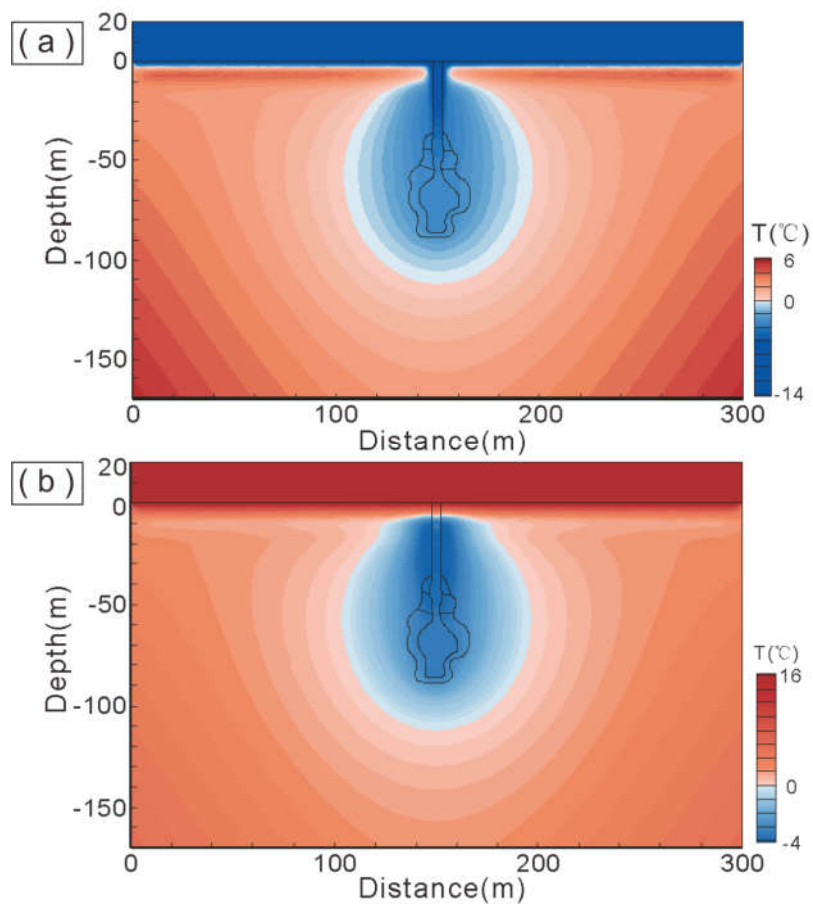


Figure 5. Temperature distribution around Ningwu ice cave in winter (a) and summer (b)

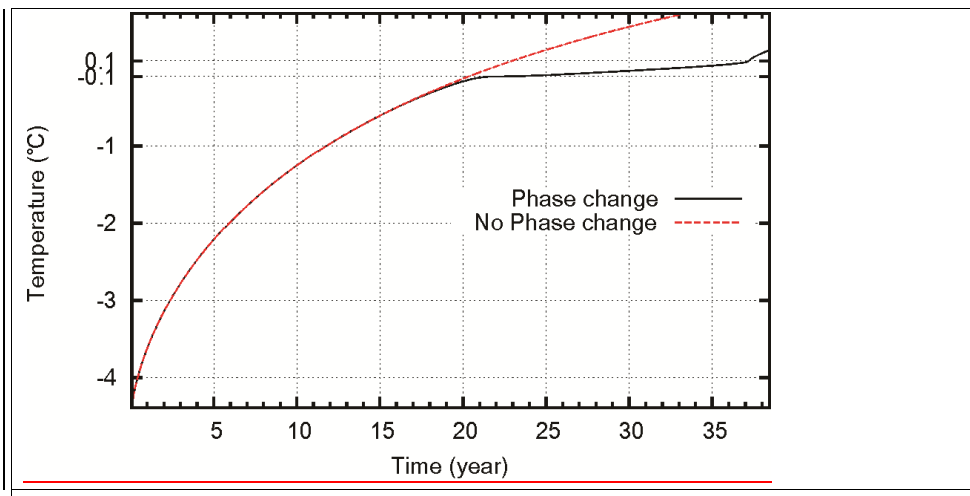


Figure 6. Internal temperature evolution diagram when ice in melting

571

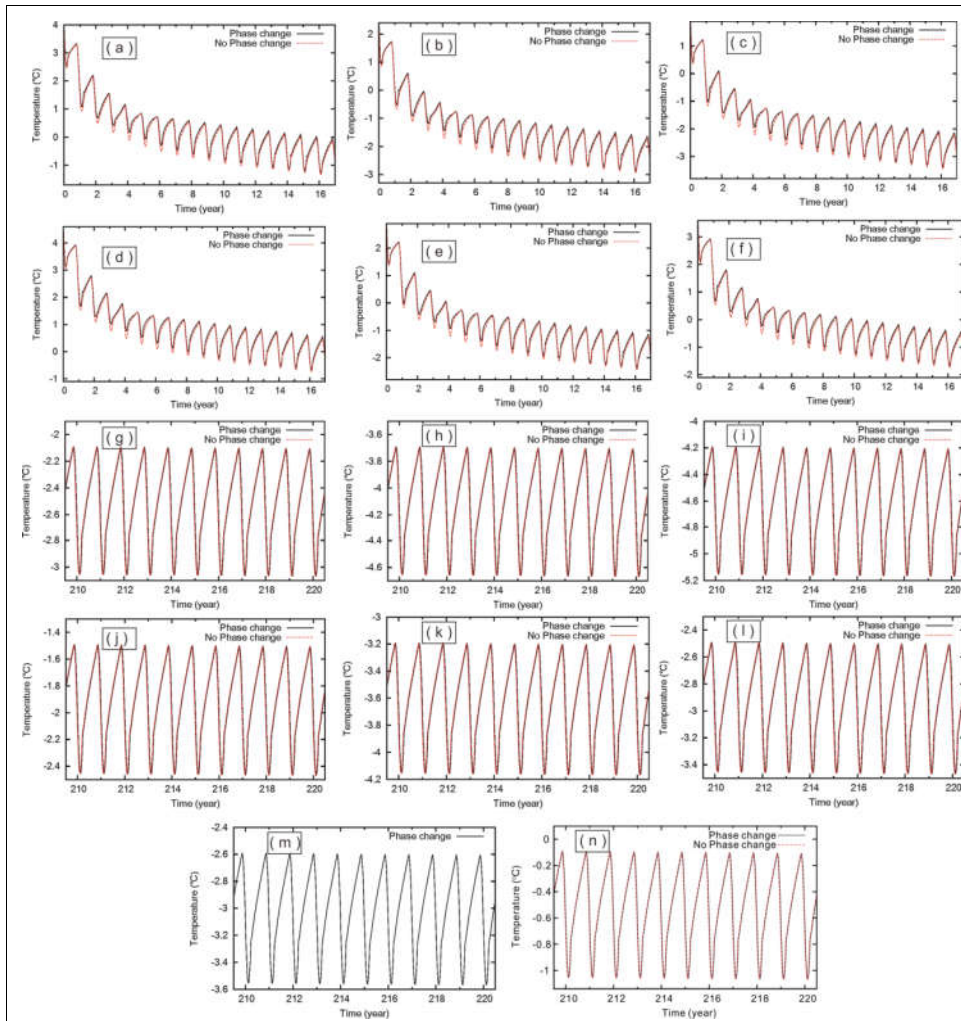


Figure 7. (a)-(f) Initial formation process of Ningwu ice cave in different sensitivity experiments. (g)-(l) Corresponding Quasi stable state. (m) Tourists and bulbs sensitivity experiment. (n) Quasi stable state when the mean annual temperature increases 3.5 °C

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