

1           **Numerical simulation of formation and preservation of**  
2                           **Ningwu ice cave, Shanxi, China**

3   S. Yang<sup>1,2</sup>, Y. Shi<sup>2</sup>

4           <sup>1</sup> *State Key Laboratory of Continental Tectonics and Dynamics, Institute of Geology,*  
5                           *Chinese Academy of Geological Sciences, Beijing 100037, China*

6           <sup>2</sup> *Key Laboratory of Computational Geodynamics of Chinese Academy of Sciences,*  
7                           *University of Chinese Academy of Sciences, Beijing 100049, China*

8   *Correspondence to: Y. Shi (shiyl@ucas.ac.cn)*

9   **Abstract:** Ice caves exist in locations where annual average air temperature is  
10 higher than 0 °C. An example is Ningwu ice cave, Shanxi Province, the largest ice  
11 cave in China. In order to quantitatively investigate the mechanism of formation and  
12 preservation of the ice cave, we use the Finite Element Method to simulate the heat  
13 transfer process at this ice cave. There are two major control factors. First, there is the  
14 seasonal asymmetric heat transfer. Heat is transferred into the ice cave from outside,  
15 very inefficiently by conduction in spring, summer and fall. In winter, thermal  
16 convection occurs that transfers heat very efficiently out of the ice cave, thus cooling  
17 it down. Secondly, ice-water phase change provides a heat barrier for heat transfer  
18 into the cave in summer. The calculation also helps to evaluate effects of global  
19 warming, tourists, colored lights, climatic conditions, etc. for sustainable development  
20 of the ice cave as tourism resource. In some other ice caves in China, managers  
21 installed air-tight doors at these ice caves entrance intending to “protect” these caves,

22 but this prevent cooling in winter and these cave ices will entirely melt within tens of  
23 years.

## 24 **1 Introduction**

25 An ice cave is a type of natural cave that contains significant amounts of perennial ice.

26 An ice cave is a rare phenomenon. Among the best known are Eisriesenwelt ice cave,  
27 Austria (May et al., 2011; Obleitner and Spötl, 2011; Schöner et al., 2010),  
28 Dobšináice cave, Slovakia (Bella, 2006; Lalkovič, 1995), Scărisoara ice cave,  
29 Romania (Holmlund et al., 2005; Perşoiu et al., 2011) and Monlesi ice cave,  
30 Switzerland (Luetscher et al., 2007; Luetscher et al., 2008). Eisriesenwelt ice cave is  
31 the largest in the world. Dobšiná ice cave is also huge, with an ice volume of over  
32 110,000m<sup>3</sup> (Bella, 2006). In China, more than ten ice caves have been found,  
33 including Ningwu, Wudalianchi, Taibaishan, Cuihuashan, Baiyizhai and Shennongjia  
34 ice caves.

35 Studies of ice caves began as early as 1861 (Peters, 1861). In recent decades, in the  
36 context of interest in global climate change, six international conferences on ice caves  
37 have been held, with the reconstruction of regional ancient climate change as an  
38 important topic for discussion (Laursen, 2010). Several articles reported seasonal air  
39 temperature oscillations of several degrees from ventilated cave systems (Roberts et  
40 al., 1998; Lacelle et al., 2004; Johnson et al., 2006). Therefore, to evaluate the impact  
41 of changing climatic conditions on cave environments, a better explanation of

42 subsurface heat and mass transfers is necessary (Luetscher et al., 2008). Meanwhile,  
43 ice caves are tourism resources. A better explanation of subsurface heat and mass  
44 transfers could help to manage ice caves more scientifically.

45 In the past, empirical calibrations were performed to determine the spatial and  
46 temporal distribution of cave air temperature as a function of the external atmospheric  
47 conditions (de Freitas and Littlejohn, 1987; de Freitas et al., 1982). In temperate karst  
48 environments, explanation of the survival of subsurface ice accumulations represents  
49 probably the most severe test for models of the magnitude and direction of heat and  
50 mass transfers induced by cave air circulation (Luetscher et al., 2008). In mathematics  
51 and engineering, the Finite Element Method (FEM) and the Finite Difference Method  
52 (FDM) are popular for finding approximate solutions for partial differential equations.  
53 We have not found any study in which these numerical techniques are applied to ice  
54 caves.

55 In China, ice cave studies started only recently, after 1998, when Ningwu ice cave  
56 was found. Although Ningwu ice cave has been widely reported during the past  
57 decade (Gao et al., 2005; Meng et al., 2006), little was known about the processes  
58 controlling the formation and preservation of perennial subsurface ice deposits under  
59 changing climate conditions (Chen, 2003). We attempt to apply FEM to simulate the  
60 energy fluxes of Ningwu ice cave, and then quantitatively interpret the formation and

61 preservation mechanism of ice deposit in Ningwu ice cave. Some suggestions are  
62 given to manage Ningwu ice cave.

## 63 **2 Study Site**

64 Ningwu ice cave (38°57' N, 112°10' E; 2121 m above sea level, Figure 1a) is the  
65 largest ice cave ever found in China. Located on the northern slopes of Guancen  
66 Mountain, Ningwu County, Shanxi Province, it is known to local people as “the ten  
67 thousand years ice cave”. The surrounding rock consists of Ordovician Majiagou  
68 limestone, dolomitic limestone, argillaceous dolomite and thin brecciated limestone  
69 which is locally densely fractured (Shao et al., 2007). A geophysical exploration  
70 (using magnetotelluric measurement) has been carried out for investigating the shape  
71 of the ice cave (Shao et al., 2007). They obtained the vertical cross section of the ice  
72 cave. The cave space is about 85 m depth. The widest part is in the middle, with a  
73 width of 20 m.

74 The ice cave is a major tourist attraction. From May to October, about 1000 visitors  
75 enter the cave per day. The ice cave has only one entrance (Figure 1c), and has  
76 wooden spiral stairs leading to a bowling pin shaped room. Ice covers the host rock  
77 almost completely. Ice stalactites and ice stalagmites (Figure 1d) can be seen in all  
78 part of the cave.

79 The outside of the ice cave has a temperate climate. The external mean air

80 temperature from June to September is about 14.6 °C, and the mean annual air  
81 temperature is 2.3 °C (Meng et al., 2006). The daily temperature from 1957 to 2008 is  
82 obtained from Wuzhai meteorological station (about 320m lower than Ningwu ice  
83 cave), which is the nearest station to the ice cave. We averaged observational air  
84 temperature at Wuzhai station to obtain the annual temperature, and then derive the  
85 mean annual temperature at Wuzhai station. We calculated the difference between the  
86 average annual air temperature at Ningwu ice cave and that at Wuzhai station. After  
87 reducing the annual temperature at Wuzhai station by the difference, we then obtain  
88 the annual temperature variation outside the ice cave (Figure 2).

### 89 **3 Qualitative Analysis**

90 There are different hypotheses about the preservation mechanism of ice deposit in  
91 Ningwu ice cave. Chen (2003) proposed that the existence of a “cold source” led to  
92 the negative geothermal anomaly which preserves the ice deposit. Meng et al. (2006)  
93 ascribed the ice deposit to multiple factors including geographical location, “icehouse  
94 effect”, “chimney effect” and “thermal effect” produced by the ice deposit and the  
95 “millennial volcano”. But they did not give us more details about these factors. Gao et  
96 al. (2005) analyzed two aspects: terrain and climate. Because this region has a long  
97 cold winter and a short cool summer, they considered that far more cold air than warm  
98 air entered the region and then the ice cave stayed cold over year.

99 Subsurface temperature usually increases with depth at a geothermal gradient of about

100 1.0-3.0 °C (100m)<sup>-1</sup> (Hu et al., 2001) . The notion that there is a permanent “cold  
101 source” underground is unfounded. Even if a cold region had somehow formed, it  
102 would be heated up by the geothermal flux from underneath in geological time.  
103 Reversal of geotherms can occur in the presence of the advective heat transfer exists  
104 due to crustal movement or groundwater flow (Shi and Wang, 1987). A reversal of  
105 geotherms can also occur from transient changes in surface temperature and be  
106 induced by steep topography (Gruber et al., 2004). But the outside of Ningwu ice cave  
107 has a temperate climate. It is hard to preserve an ice cave in a temperate climate  
108 without a sustainable cooling mechanism. In presence of a geothermal gradient, the  
109 host rock continuously transfers heat to the ice cave, so there must be a sustainable  
110 mechanism to remove the heat from underneath and ensure the maintenance of the ice  
111 cave.

112 The temperature outside the ice cave undergoes annual cyclic variations: in spring,  
113 summer and fall, it is higher than the internal temperature, but in winter it is lower. As  
114 Ningwu ice cave is bowling pin shaped with only an opening in the upper part, cold  
115 air in spring, summer and fall is heavy and sinks into the cave and thus will not  
116 produce natural thermal convection. Conduction is the main form of heat transfer  
117 from the outside down to the ice cave, and at the same time heat transfers into the  
118 cave from the host rock due to the terrestrial heat flows. Thermal conductivities of  
119 neither rock nor air are high and the conductive heat transfer efficiency is very low, so

120 the heat transferred to the ice cave in the three seasons is quite limited. In winter, the  
121 temperature is low inside the ice cave but even lower outside. The air in the ice cave  
122 is lighter and air outside the cave entrance is heavier. It could thus become  
123 gravitationally unstable, and thermal convection could occur. The external cold air  
124 flows into the cave to cool it down, and removes the heat transferred into the cave  
125 from the host rock, as well as the heat transferred into the cave through the entrance in  
126 spring, summer and fall. Since convective heat transfer is much more efficient than  
127 conduction, the heat transferred out of the cave in the winter months is enough to  
128 balance the heat that transferred into the cave year-round.

129 Ice melting into water absorbs a lot of latent heat. The melting heat of ice is 334 kJ  
130  $\text{kg}^{-1}$  and the specific heat of limestone is about  $0.84 \text{ kJ kg}^{-1} \text{ K}^{-1}$ . During summer, much  
131 of the heat transferred to cave is consumed to melt the ice to 0 °C water. Therefore,  
132 ice-water phase change can reduce the rate of temperature rise. Similarly, when the  
133 ambient temperature decreases, ice-water phase change can reduce the rate of  
134 temperature decrease. Therefore, ice-water phase change in the ice cave can “buffer”  
135 the temperature change. A small amount of ice melting near the cave entrance  
136 effectively prevents the heat from being transferred into the deep cave. When the  
137 surface water flows into the ice cave from the entrance, the ice cave temperature will  
138 not significantly increase.

139 The calculated energy balance of some cave ice (e.g. Eisriesenwelt ice cave) is largely  
140 determined by the input of long-wave radiation originating at the host rock surface  
141 (Obleitner and Spötl, 2011). Ice covers the host rock in Ningwu cave almost  
142 completely. Therefore, we suggest that long-wave radiation originating at the host  
143 rock surface is not a predominant factor in the processes of the formation and  
144 preservation of ice deposit in Ningwu ice cave.

145 In summary, the air and the host rock transfer heat to the ice cave, making the cave  
146 temperature rise in spring, summer and fall. In winter, the heat convection of air  
147 makes the heat flow out of the cave, lowering the cave temperature. Meanwhile, four  
148 seasons are accompanied by ice-water phase transition effect. The annual heat budget  
149 of income and output is balanced, the cave will be in a cyclic state with very small  
150 temperature fluctuations and the average temperature is always lower than 0 °C, so ice  
151 bodies in the ice cave can persist.

152 Snow crystal (or hoar frost) is single crystals of ice that grow from water vapor. If  
153 humidity enters a cave and then form ice deposit, snow crystal could be discovered  
154 (Kenneth, 2005). Actually, it is hard to find snow crystals in Ningwu ice cave. Any  
155 clear traces of water or snow entering the cave through its entrance could not be found.  
156 Meanwhile, karstified carbonate rock is heterogeneous, highly fractured, and with a  
157 permeability developed such that water movement occurs below the surface (Fairchild



158 and Baker, 2012). In summary, we infer that most of the ice in the cave is formed by  
159 freezing of infiltration water.

160 Water and ice are in dynamic equilibrium state. Water infiltrates into Ningwu ice cave  
161 throughout the year, and forms ice. Ice at the bottom of Ningwu ice cave is thawed  
162 under geothermal flow, and the water infiltrates into the deeper place. Ice stalactites,  
163 ice stalagmites (Figure 1d) can be seen in all part of Ningwu ice cave. This can verify  
164 the former process. No directly observational evidences support the latter process.

165  $Ra$ , the Rayleigh number, is a dimensionless number associated with buoyancy-driven  
166 flow. When  $Ra$  is below a critical value for that fluid, heat transfer is primarily in the  
167 form of conduction; when it exceeds the critical value, heat transfer is primarily in the  
168 form of convection.  $Nu$ , the Nusselt number, is a dimensionless number, which is  
169 defined as the ratio of convection heat transfer to pure conduction heat transfer under  
170 the same conditions. The ice of Ningwu ice cave build-up process is a self-regulating  
171 process. If too much ice was accumulated in Ningwu ice cave, the cavity will become  
172 small. Thus,  $Ra$  and  $Nu$  will be reduced. That means the freezing efficiency become  
173 low. Some of the cave ice will be thawed, and the cavity will become large.  $Ra$  and  
174  $Nu$  will be increased. That means the freezing efficiency become high. More ice will  
175 be accumulated in Ningwu ice cave.

## 176 **4 Principle of Simulation**

### 177 **4.1 Basic ideas of simulation**

178 Two heat transmission mechanisms must be taken into account to explain the  
179 preservation of ice mass in ice cave, namely, thermal conduction and convection. The  
180 phase change must also be considered. The heat conduction equation can be used to  
181 describe the heat-conducting process, while for the convection process, due to the  
182 complicated geometrical shape structure inside the ice cave and complex varying  
183 boundary conditions, the convection pattern of air and its thermal consequences are  
184 hard to determine exactly. In view of this, a widely used simplified method is applied  
185 in this study: evaluate  $Nu$  and solve the conductive equation by introducing an  
186 equivalent thermal conductivity of the convecting air. In the case of an upright  
187 circular tube, the relation between the temperature difference of the top and the  
188 bottom and  $Nu$  can be determined by adopting the experimental relation of natural  
189 thermal convection. The enthalpy method can be adopted to calculate the phase  
190 change.

191 In every time step of our modeling process, it is judged if air convection occurs based  
192 on the temperature difference between the top and the bottom of the cave. If no  
193 convection, the simple conduction problem will be solved, while if the convection  
194 occurs, an effective conductivity is used in the thermal equation.

### 195 **4.2 Equation and physical parameters**

196 The heat conduction equation is

197 
$$c\rho\frac{\partial T}{\partial t} = k\nabla^2 T \quad (1)$$

198 where  $c$  is the specific heat,  $\rho$  is density,  $T$  is temperature (unknown number),  $t$  is time  
 199 and  $k$  is thermal conductivity. For the convective heat transfer process, an equivalent  
 200 thermal conductivity is used in Equation (1) based on the  $Nu$ .

201 The enthalpy method is used to calculate the phase change process. A physical  
 202 quantity enthalpy  $H$  is introduced in Equation (2), where  $T_r$  is an arbitrary lower  
 203 temperature limit. For phase change, enthalpy  $H$  can be determined by Equations  
 204 (3)-(5) (Lewis, 1996), in particular, the  $(T_s, T_l)$  is phase change range. Water-ice phase  
 205 change occurs at 0 °C. But in numerical model, it is necessary to give a phase change  
 206 range.

207 
$$H(T) = \int_{T_r}^T \rho c(T) dT \quad (2)$$

208 
$$H(T) = \int_{T_r}^T \rho c_s(T) dT \quad T \leq T_s \quad (3)$$

209 
$$H(T) = \int_{T_r}^{T_s} \rho c_s(T) dT + \int_{T_s}^T [\rho(\frac{dL}{dT}) + \rho c_f(T)] dT \quad T_s < T < T_l \quad (4)$$

210 
$$H(T) = \int_{T_r}^{T_s} \rho c_s(T) dT + \rho L + \int_{T_s}^{T_l} \rho c_f(T) dT + \int_{T_l}^T \rho c_l(T) dT \quad T \geq T_l \quad (5)$$

211  $c_s$  is the specific heat in solid phase,  $c_l$  is the specific heat in liquid phase,  $c_f$  is the  
 212 specific heat in solid-liquid mixing state and  $L$  is the latent heat. There are many ways  
 213 to calculate heat capacity (Lewis and Roberts, 1987). The simple and accurate

214 backward differentiation formula (Lewis and Roberts, 1987; Morgan et al., 1978) is  
215 adopted here, as expressed in Equation (6), where  $(n)$  and  $(n-1)$  stand for time step.  
216 Equation (6) can be substituted into the heat equation along with the relevant material  
217 parameters for calculation.

$$218 \quad (c\rho)^{(n)} = \left(\frac{dH}{dT}\right)^{(n)} = \frac{H^{(n)} - H^{(n-1)}}{T^{(n)} - T^{(n-1)}} \quad (6)$$

219 Relevant materials include limestone, ice, ice-limestone mixture, air and water.  
220 Parameters of these materials are listed in Table 1. The physical parameter of  
221 ice-limestone mixture is taken as the arithmetic mean of those of ice and limestone.  
222 We assume that the ice body exists when temperature is below  $-0.1$  °C, and ice-water  
223 mixture exists between  $-0.1$  °C and  $0.1$  °C, and this becomes water when temperature  
224 exceeds  $0.1$  °C. The ratio of ice and water in the mixture is linear to the temperature  
225 within the phase change range, and so are the physical parameters. The latent heat  $L$   
226 of ice-water phase change is  $334 \text{ kJ kg}^{-1}$ .

### 227 **4.3 Equivalent thermal conductivity**

228 When the convection occurs, heat transfer is  $Nu$  times greater than the conductive heat  
229 transfer at the same conditions. In other words, an equivalent thermal conductivity can  
230 be introduced, which is  $Nu$  times greater than the air thermal conductivity (Schmeling  
231 and Marquart, 2014).  $Nu$  is related to the temperature difference of air at the top and  
232 the bottom of the cave, physical properties (e.g. viscosity and conductivity of air) and

233 also the geometry of the cave. Ningwu ice cave can be approximated by an upright  
234 circular tube. For such a tube,  $Nu$  can be calculated based on fluid thermodynamics  
235 studies. When Equation (7) is satisfied (Sparrow and Gregg, 1956; Yang and Tao,  
236 2006), which is the case for Ningwu ice cave, the natural convection heat transfer  
237 experimental relation (Sparrow and Gregg, 1956; Incropera et al., 2011) is expressed  
238 as Equation (8).

$$239 \quad d / h \geq 35 / Gr^{1/4} \quad (7)$$

$$240 \quad Nu_m = C(Gr \cdot Pr)_m^n \quad (8)$$

241 In Equations (7) and (8),  $d$  is the diameter of circular tube and  $h$  is the height of  
242 circular tube;  $Nu_m$  is the Nusselt number, subscript  $m$  represents for the arithmetic  
243 mean temperature of the boundary layer,  $Gr$  is the Grashof number, which  
244 approximates the ratio of the buoyancy to viscous force acting on a fluid,  $Pr$  is the  
245 Prandtl number;  $C$  and  $n$  are constants, the values of which are shown in Table 2.

246 The Prandtl number, a dimensionless number, is defined as the ratio of momentum  
247 diffusivity to thermal diffusivity.  $Pr$  is dependent only on the fluid material. For air,  
248  $Pr$  is 0.7. The  $Gr$  number is

$$249 \quad Gr = g\beta\Delta Tl^3 / \nu^2 \quad (9)$$

250 where  $g$  is the acceleration of gravity,  $\beta$  is the coefficient of cubical expansion,  $\Delta T$  is  
251 a temperature difference,  $l$  is a characteristics length and  $\nu$  is the coefficient of  
252 kinematic viscosity. The values are  $g=9.8 \text{ m s}^{-2}$ ,  $\beta=3.67\times 10^{-3} \text{ k}^{-1}$ ,  $l=80 \text{ m}$ ,  $\nu$   
253  $=13.30\times 10^{-6} \text{ m}^2 \text{ s}^{-1}$  and are substituted into Equation (9) to obtain

$$254 \quad \quad \quad Gr = 1.041\times 10^{14} \Delta T \quad \quad \quad (10)$$

255 According to Equation (10), when the temperature difference is only  $10^{-3} \text{ }^\circ\text{C}$ , the  $Gr$   
256 number can reach  $1.041\times 10^{11}$ . According to Table 2, we infer that natural convection  
257 will occur and the flow state of air is a turbulent flow when the temperature is higher  
258 inside than outside the ice cave. Equation (11), relating  $Nu$  to the temperature  
259 difference, can be obtained when relevant parameters are substituted into Equation  
260 (8).

$$261 \quad \quad \quad Nu = 11000(0.0740\Delta T)^{1/3} \quad \quad \quad (11)$$

262 Even if Equation (7) is not satisfied, corresponding experimental relations can also be  
263 found in literatures (Cebeci, 1974; Minkowycz and Sparrow, 1974; Yang and Tao,  
264 2006).

#### 265 **4.4 Models and boundary conditions**

266 The rectangular Eulerian computational domain corresponds to a physical domain of  
267  $300 \times 190 \text{ m}$  on the basis of the ice cave cross section (Figure 1b). There are 32825

268 nodes and 64986 elements involved in drawing the FEM grid. The grids for the ice  
269 body and the interior air are denser.

270 The mean value of the geothermal gradient of the Lvliang highland area where  
271 Ningwu ice cave is located, is  $2.02\text{ }^{\circ}\text{C (100m)}^{-1}$  (Li, 1996). The mean value of the  
272 geothermal gradient of the low-lying Linxian and Liulin areas in Shanxi Province is  
273  $2.20\text{ }^{\circ}\text{C (100m)}^{-1}$  (Hu et al., 2001). We take the normal geothermal gradient value of  
274  $2.0\text{ }^{\circ}\text{C (100m)}^{-1}$  in the model. The temperature boundary conditions are assigned to  
275 both sides of the model, with the annual average temperature at the surface and  
276 increase with depth following the geothermal gradient. The heat flow boundary  
277 condition is assigned for the bottom boundary. The terrestrial heat flow value is the  
278 product of the geothermal gradient times the thermal conductivity of the limestone  
279 host rock. According to Figure 2, we prescribe the variation temperature to the top  
280 boundary.

281 The initial thermal structure is calculated assuming the surface temperature remained  
282 constant at the annual average (Figure 3).

283 During the simulation, models with phase transition included and phase transition  
284 neglected are both calculated for comparison. When phase change is considered,  
285 latent heat and the material property variation are considered.

## 286 **5 Simulated Result and Analysis**

### 287 **5.1 Evolution of an ice deposit forming model**

288 Because of the periodic change of the ambient air temperature, the temperature in the  
289 ice cave will show a periodic variation corresponding to conduction and convective  
290 heat transfer. Figure 4a shows the evolution of the temperature at the bottom of the ice  
291 cave. It can be interpreted as the process of formation of the cave ice. If a cave was  
292 formed but not connected with the outside, it may have a temperature distribution  
293 similar to Figure 3. If the cave became connected to the outside, i.e. collapsed at its  
294 top and produced an entrance to the cave, an ice cave would then form within a  
295 decade due to the winter convective cooling and stabilize in a century. Figure 4b  
296 shows the details of first two decades and shows that the calculated results with phase  
297 change considered (black line) do not differ significantly from those without  
298 considering phase change (red line) in the cooling process. Starting from normal  
299 ground temperature, the internal temperature of the ice cave drops rapidly in the first  
300 decade, then drops more gradually and finally tends to become stable.

301 Figure 4b shows the details of temperature evolving in the ice cave during its initial  
302 16 years of formation. It is seen that the cave ice can be maintained below 0 °C all  
303 year round after winter cooling for about 5 years. The cave temperature increases in  
304 spring, summer and fall and decreases in winter, presenting annually periodic  
305 variation. The air temperature of Ningwu ice cave decreases rapidly in winter, but the  
306 temperature increases slowly in spring, summer and fall. Because the heat conduction



307 in spring, summer and fall is much less efficient than convective heat transfer in  
308 winter. With phase change considered (black line), the increased rate of temperature  
309 in summer is smaller than that without phase change (red line), because latent heat is  
310 required to melt ices near the cave entrance, thus delaying the conduction of heat to  
311 the bottom of the cave. In winter, the convective cooling is so effective that the  
312 difference is minimized.

313 Figure 4c shows the cave temperature annual fluctuations when the process has lasted  
314 two centuries, long enough to be evolved to a stable cyclic state. The amplitude of the  
315 temperature variation is about 1.0 °C (from -3.9 °C to -2.9 °C). Ningwu ice cave has  
316 been open to tourists, so the cave temperature has been disturbed. According to our  
317 measurement on 5 June 2012, the lowest internal temperature of the ice cave was  
318 -1.5 °C. Through the record in literature, the actually measured internal temperature of  
319 the ice cave ranges between -1.0 °C (Meng et al., 2006), -4.0 °C and -6.0 °C (Gao et al.,  
320 2005). The difference in measured results may be caused by different measuring  
321 methods and different measuring time and positions. Similar to Figure 4b, the cave  
322 temperature presents annual periodic variation, and the overall increasing rate of cave  
323 temperature is smaller than its decreasing rate, because the heat transfer efficiency of  
324 conduction is much lower than that of heat convection. The variation of cave  
325 temperature for model with phase change considered (black line) is basically the same  
326 with that without phase change considered (red line). The reason is that although we

327 considered phase change during calculation, the temperature of the ice body in the  
328 cave is always kept below 0 °C when it reaches a stable cyclic state and no phase  
329 change actually occurs.

330 Figures 5a and 5b show the spatial temperature distribution around the ice cave in  
331 winter and summer respectively under the stable stage. Both figures show that a small  
332 portion of rock at the top of the ice cave presents a negative geothermal gradient and  
333 most of the host rock presents a normal positive geothermal gradient. Beneath the  
334 bottom of the cave, however, geothermal gradients are much higher than normal. The  
335 ice body temperature is always kept below 0 °C, although the external temperature is  
336 completely different. In Figure 5a, the temperature of the shallow ground is lower  
337 than 0 °C, corresponding to a frozen zone in winter. In Figure 5b, the temperature of  
338 shallow parts of ground is higher than 0 °C, indicating that the frozen part is melted  
339 and there is no permafrost. These features agree with actual conditions.

## 340 **5.2 Evolution of an ice deposit melting model**

341 The ice body in the ice cave will melt if there is no air convection heat transfer in  
342 winter. Taking the temperature shown in Figure 5a as an initial temperature, the  
343 evolution of temperature distribution will be calculated with or without phase change  
344 effect considered. The results are shown in Figure 6 by a black line and a red line  
345 respectively. They are the same when temperature does not reach the phase change  
346 temperature. The ice body takes much longer to thaw when the latent heat of melting

347 is taken into consideration than when it is not. To thaw the ice body completely takes  
348 23 years when the latent heat of phase change is not considered, compared with 37  
349 years when it is considered.

### 350 **5.3 Sensitivity to model parameters**

351 The external air temperature,  $Nu$  and the number of tourists could directly affect the  
352 energy transfer in Ningwu ice cave. Therefore, it needs sensitivity experiments on  
353 these factors. With respect to the external air temperature, we consider two aspects: 1)  
354 the mean annual temperature; 2) the amplitude of annual temperature. When the mean  
355 annual temperature increases (respectively decreases) 1.0 °C, the computing results  
356 are shown as Figure 7a and 7g (or Figure 7b and 7h). When the amplitude of external  
357 temperature increases (respectively decreases) 5.0 °C, the computing results are shown  
358 as Figure 7c and 7i (or Figure 7d and 7j). For  $Nu$  increases ( respectively decreases)  
359 10%, the computing results are shown as Figure 7e and 7k (or Figure 7f and 7l).  
360 About 1000 visitors enter the cave per day from May to October. A person could  
361 release 840 J. We assume that every person spend 1hour in Ningwu ice cave.  
362 Meanwhile, there are 200 15W-lightbulbs. When we consider the number of tourists  
363 and bulbs, the computing result is shown as Figure 7m.

364 Similar to Figure 4b, Figures 7a-7f show the details of first two decades and represent  
365 that ice deposit would be formed in Ningwu ice cave within first two decades in these  
366 different experiments. Figures 7g-7l correspond to Figures 7a-7f respectively. As

367 shown in Figure 4c, Figures 7g-7l depict the cave temperature annual fluctuations  
368 when the process has lasted two centuries, long enough to be evolved to a stable  
369 cyclic state. Compared with Figure 4c, Figure 7m represents that the current density  
370 of tourists and number of light bulbs in Ningwu ice cave could not melt the ice  
371 deposit in it. Figure 7n shows the ice cave temperature annual fluctuations when the  
372 mean annual temperature increases 3.5 °C. We can see the temperature ceiling is  
373 -0.1 °C. We consider this is the minimum climatic condition required to form Ningwu  
374 ice cave.

## 375 **6 Discussion**

376 The age of the cave and that of the ice body are different. Formation of the cave  
377 cavity could be old and have taken place in a warmer climate. The formation of the  
378 ice body in the cave is a much later process that took place when the bowling pin  
379 shaped cave was formed and the climate became cold enough. In the present climate,  
380 our numerical modeling suggests that the year-round ice body can be formed within a  
381 decade.

382 In spring, summer and fall, air and host rock transmit heat to the ice cave by thermal  
383 conduction, increasing the temperature in the ice cave only slightly since the  
384 conduction efficiency is low. In winter, heat is transmitted out of the ice cave by  
385 natural thermal convection of air, efficiently decreasing the temperature in the ice  
386 cave. Phase change accompanies the thermal processes. Considering these

387 mechanisms, the results show that (1) starting from a normal ground temperature, a  
388 year-round ice body will be formed in the cave in less than a decade, about 5 years in  
389 our model (Figure 4b), and the ice cave temperature will decrease continuously for  
390 more than a century. (2) The ice cave will finally reach a stable cyclic state, and its  
391 temperature will fluctuate within a certain range, less than 1.0 °C (from -3.9 °C to  
392 -2.9 °C) for Ningwu ice cave. At this stage, the annual total heat transferred to the  
393 cave by thermal conduction and the heat removed from the cave by convection are  
394 balanced.

395 It would be interesting to further investigate the possibility of imitating nature and  
396 constructing a new kind of air conditioning system. At locations with similar climate  
397 conditions, people may construct a basement more than 10 m deep, using natural air  
398 convection to freeze ice in the basement in winter, and circulate air to the basement  
399 for air conditioning in summer.

400 Setting an air-tight door at a cave entrance, as one park has done in China to “protect”  
401 the ice cave at night during the tourist season and for the entire winter when the cave  
402 is closed to tourist, actually blocks air convection in winter. As a result, cold air  
403 cannot bring out heat from the cave, and accumulation of heat flow from the surface  
404 and the deep crust will finally lead to melting of the ice body in the cave. Our  
405 computation shows that it takes less than 40 years to completely melt the whole ice

406 body in the cave. This implies that Ningwu ice cave probably is not currently  
407 suffering from thawing of the relict ice. This also suggests that scientific management  
408 is important for sustainable usage of natural tourism resources. Otherwise,  
409 well-meaning acts such as installing a trap door to completely seal the entrance for  
410 protection will actually destroy the natural wonder in a few decades.

## 411 **7 Conclusion**

412 This paper has focused on quantitative analysis of the formation and preservation  
413 mechanism of an ice body in Ningwu ice cave, a static ice cave. The Finite Element  
414 modeling leads to the following conclusion: The controlling factor for forming and  
415 sustaining the ice body in the cave is effective cooling of the cave in winter by natural  
416 air convection. Heat conduction in spring, summer and fall is very ineffective to warm  
417 up the cave. Ice-water phase change further prevents melting of ice in summer. The  
418 formation of the cave may take a long geological time, but the formation of the  
419 perennial ice body in the cave only takes decades of years under the current  
420 temperature and geothermal gradient in the Ningwu area by winter air convection.  
421 Once formed, the cave temperature will keep a stable cyclic state. At this time, the  
422 amplitude of annual temperature variation in the Ningwu ice cave is within 1.0 °C.  
423 Environmental warming even up to 1.0 °C in in Ningwu area will increase the cave  
424 temperature, but not melt the perennial ice body. The present heat from electric  
425 lighting and visitors will not melt the ice body either. However, if the air convective

426 heat transfer is stopped in the winter as happened in some other Chinese ice caves, the  
427 ice body in the cave could be completely melted within about 40 years. This analysis  
428 is important for sustainable management of the ice cave as a tourism resource. The  
429 mechanism of ice cave formation may be adopted for construction of energy-saving  
430 buildings; ice may be produced in winter in basement and used for air conditioning in  
431 summer.

### 432 **Acknowledgements**

433 We thank Yong'en Cai and Bojing Zhu for helpful discussions. Constructive  
434 comments and suggestions from Stuart A. Harris and an anonymous reviewer  
435 significantly improved the quality of this paper. This research is supported by  
436 National Natural Science Foundation of China (NSFC) Project 41174067 and the  
437 CAS/CAFEA international partnership program for creative research teams  
438 (No.KZZD-EW-TZ-19).

### 439 **References**

- 440 Bella, P.: Morphology of ice surface in the Dobšiná Ice Cave, 2nd International Workshop on Ice  
441 Caves, Demänovská dolina, Slovak Republic, 2006,  
442 Cebeci, T.: Laminar-free-convective-heat transfer from the outer surface of a vertical slender circular  
443 cylinder, Proceedings of the 5th International Conference, Tokyo, Japan, 1974, 15-19,  
444 Chen, S.: Cave Tourism Science, Fujian People's Publishing House, Fuzhou, 2003.  
445 de Freitas, C., Littlejohn, R., Clarkson, T., and Kristament, I.: Cave climate: assessment of airflow and  
446 ventilation, *Int. J. Climatol.*, 2, 383-397, 10.1002/joc.3370020408, 1982.  
447 de Freitas, C., and Littlejohn, R.: Cave climate: assessment of heat and moisture exchange, *Journal of*  
448 *Climatology*, 7, 553-569, 10.1002/joc.3370070604, 1987.  
449 Fairchild, I. J., and Baker, A.: *Speleothem science: from process to past environments*, John Wiley &  
450 Sons, 2012.  
451 Gao, L., Wang, X., and Wan, X.: Analysis of Ice Cave Formation in Ningwu Shanxi, *Journal of*

452 Taiyuan university of technology, 36, 455-458, 10.3969/j.issn.1007-9432.2005.04.022, 2005.

453 Gruber, S., King, L., Kohl, T., Herz, T., Haeberli, W., and Hoelzle, M.: Interpretation of geothermal  
454 profiles perturbed by topography: the alpine permafrost boreholes at Stockhorn Plateau, Switzerland,  
455 *Permafr. Periglac. Proc.*, 15, 349-357, 10.1002/ppp.503, 2004.

456 Holmlund, P., Onac, B. P., Hansson, M., Holmgren, K., Mörth, M., Nyman, M., and Persoiu, A.:  
457 Assessing the palaeoclimate potential of cave glaciers: the example of the the Scărișoara Ice Cave  
458 (Romania), *Geografiska Annaler: Series A, Physical Geography*, 87, 193-201,  
459 10.1111/j.0435-3676.2005.00252.x, 2005.

460 Hu, S., He, L., and Wang, J.: Compilation of heat flow data in the China continental area (3rd edition),  
461 *Chinese Journal Geophysics*, 44, 611-626, 10.3321/j.issn:0001-5733.2001.05.005, 2001.

462 Incropera, F. P., Lavine, A. S., and DeWitt, D. P.: *Fundamentals of heat and mass transfer*, 11th ed.,  
463 John Wiley & Sons Incorporated, Hoboken NJ, 2011.

464 Johnson, K. R., Hu, C., Belshaw, N. S., and Henderson, G. M.: Seasonal trace-element and  
465 stable-isotope variations in a Chinese speleothem: The potential for high-resolution paleomonsoon  
466 reconstruction, *Earth Planet. Sci. Lett.*, 244, 394-407, 2006.

467 Kenneth, G. L.: The physics of snow crystals, *Reports on Progress in Physics*, 68, 855, 2005.

468 Lacelle, D., Lauriol, B., and Clark, I. D.: Seasonal isotopic imprint in moonmilk from Caverne de  
469 l'Ours (Quebec, Canada): implications for climatic reconstruction, *Can. J. Earth Sci.*, 41, 1411-1423,  
470 10.1139/e04-080, 2004.

471 Lalkovič, M.: On the problems of the ice filling in the Dobšina Ice Cave, *Acta carsologica*, 24, 313-322,  
472 1995.

473 Laursen, L.: Climate scientists shine light on cave ice, *Science*, 329, 746,  
474 10.1126/science.329.5993.746, 2010.

475 Lewis, R., and Roberts, P.: Finite element simulation of solidification problems, *Applied Scientific*  
476 *Research*, 44, 61-92, 10.1007/BF00412007, 1987.

477 Lewis, R. W.: *The finite element method in heat transfer analysis*, John Wiley & Sons Inc, Hoboken NJ,  
478 1996.

479 Li, Q.: Some characteristics of the geothermal distribution in ShanXi rift zone, *Earthquake research in*  
480 *ShanXi*, 26-30, 1996.

481 Luetscher, M., and Jeannin, P.: A process-based classification of alpine ice caves, *Theoretical and*  
482 *Applied Karstology*, 17, 5-10, 2004.

483 Luetscher, M., Bolius, D., Schwikowski, M., Schotterer, U., and Smart, P. L.: Comparison of  
484 techniques for dating of subsurface ice from Monlesi ice cave, Switzerland, *J. Glaciol.*, 53, 374-384,  
485 10.3189/002214307783258503, 2007.

486 Luetscher, M., Lismonde, B., and Jeannin, P. Y.: Heat exchanges in the heterothermic zone of a karst  
487 system: Monlesi cave, Swiss Jura Mountains, *J. Geophys. Res.*, 113, F02025,  
488 10.1029/2007JF000892, 2008.

489 May, B., Spötl, C., Wagenbach, D., Dublyansky, Y., and Liebl, J.: First investigations of an ice core  
490 from Eisriesenwelt cave (Austria), *The Cryosphere*, 5, 81-93, 10.5194/tc-5-81-2011, 2011.

491 Meng, X., Zhu, D., Shao, Z., Yu, J., Han, J., and Meng, Q.: A discussion on the formation mechanism  
492 of the "Ten-Thousand-Year-Old Ice Cave" in Shanxi Province, *ACTA GEOSCIENTIA SINICA*, 27,



493 163-168, 10.3321/j.issn:1006-3021.2006.02.011, 2006.

494 Minkowycz, W., and Sparrow, E.: Local nonsimilar solutions for natural convection on a vertical  
 495 cylinder, *Journal of Heat Transfer*, 96, 178, 10.1115/1.3450161, 1974.

496 Morgan, K., Lewis, R., and Zienkiewicz, O.: An improved algorithm for heat conduction problems  
 497 with phase change, *International Journal for Numerical Methods in Engineering*, 12, 1191-1195,  
 498 10.1002/nme.1620120710 1978.

499 Obleitner, F., and Spötl, C.: The mass and energy balance of ice within the Eisriesenwelt cave, Austria,  
 500 *The Cryosphere*, 5, 245-257, 10.5194/tc-5-245-2011, 2011.

501 Perşoiu, A., Onac, B. P., Wynn, J. G., Bojar, A. V., and Holmgren, K.: Stable isotope behavior during  
 502 cave ice formation by water freezing in Scărișoara Ice Cave, Romania, *J. Geophys. Res.*, 116,  
 503 D02111, 10.1029/2010JD014477, 2011.

504 Peters, K. F.: *Geologische und mineralogische Studien aus dem südöstlichen Ungarn, insbesondere aus*  
 505 *der Umgegend von Rézbánya*, KK Hof-und Staatsdr, Wien, 1861.

506 Roberts, M. S., Smart, P. L., and Baker, A.: Annual trace element variations in a Holocene speleothem,  
 507 *Earth Planet. Sci. Lett.*, 154, 237-246, 10.1016/S0012-821X(97)00116-7, 1998.

508 Schöner, W., Weyss, G., and Mursch-Radlgruber, E.: Linkage of cave-ice changes to weather patterns  
 509 inside and outside the cave Eisriesenwelt (Tennengebirge, Austria), *The Cryosphere Discuss*, 4,  
 510 1709-1740, 10.5194/tc-5-603-2011, 2010.

511 Schmeling, H., and Marquart, G.: A scaling law for approximating porous hydrothermal convection by  
 512 an equivalent thermal conductivity: theory and application to the cooling oceanic lithosphere,  
 513 *Geophys. J. Int.*, 197, 645-664, 10.1093/gji/ggu022, 2014.

514 Shao, Z., Meng, X., Zhu, D., Yu, J., Han, J., Meng, Q., and Lv, R.: Detecation for the spatial  
 515 distribution of "Ten-Thousand Ice Cave" in Ningwu, ShanXi Province, *Journal of Jilin University*  
 516 (Earth Science Edition), 37, 961-966, 10.3969/j.issn.1671-5888.2007.05.019, 2007.

517 Shi, Y., and Wang, C.-Y.: Two-dimensional modeling of the PTt paths of regional metamorphism in  
 518 simple overthrust terrains, *Geology*, 15, 1048-1051, 1987.

519 Sparrow, E., and Gregg, J.: Laminar free convection heat transfer from the outer surface of a vertical  
 520 circular cylinder, *Trans. ASME*, 78, 1823-1829, 1956.

521 Yang, S., and Tao, W.: *Heat transfer*, 4th ed., Higher education press, Beijing, 2006.

522

523

**Table 1.** Relative Material Parameters

Material	Thermal Conductivity W/(m·k)	Density kg/m <sup>3</sup>	Specific Heat kJ/(kg·K)
Limestone	2.7	2500	0.84
Ice	2.23	916.5	2.05
Mixture	2.465	1708.25	1.445
Air	0.0243	1.293	1.005
Water	0.58	1000	4.2

524

**Table 2.** *Gr* Number and Constant for Different Flow Types (Yang and Tao, 2006)

Flow State	Coefficient C	Index n	<i>Gr</i> Application Range
Laminar Flow	0.59	1/4	$1.43 \times 10^4 \sim 3 \times 10^9$
Transitional Flow	0.0292	0.39	$3 \times 10^9 \sim 2 \times 10^{10}$
Turbulent Flow	0.11	1/3	$> 2 \times 10^{10}$

525

526

527

528

529

530

531

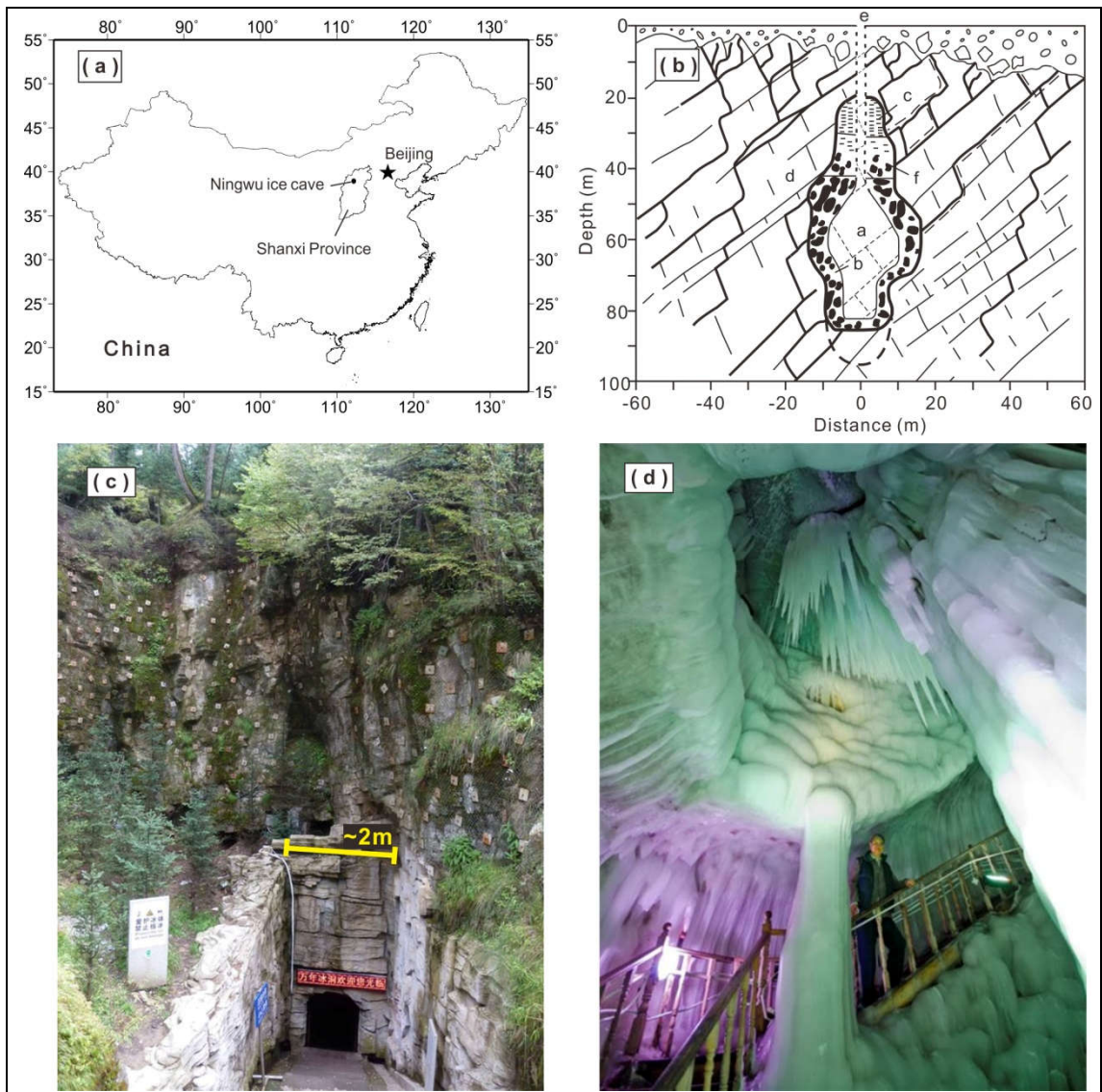


Figure 1. Location (a), cross section (b), entrance (c) and inside (d) of Ningwu ice cave. In Figure 1b, (a) room; (b) block ice; (c) layered ice; (d) limestone; (e) entrance; (f) fracture

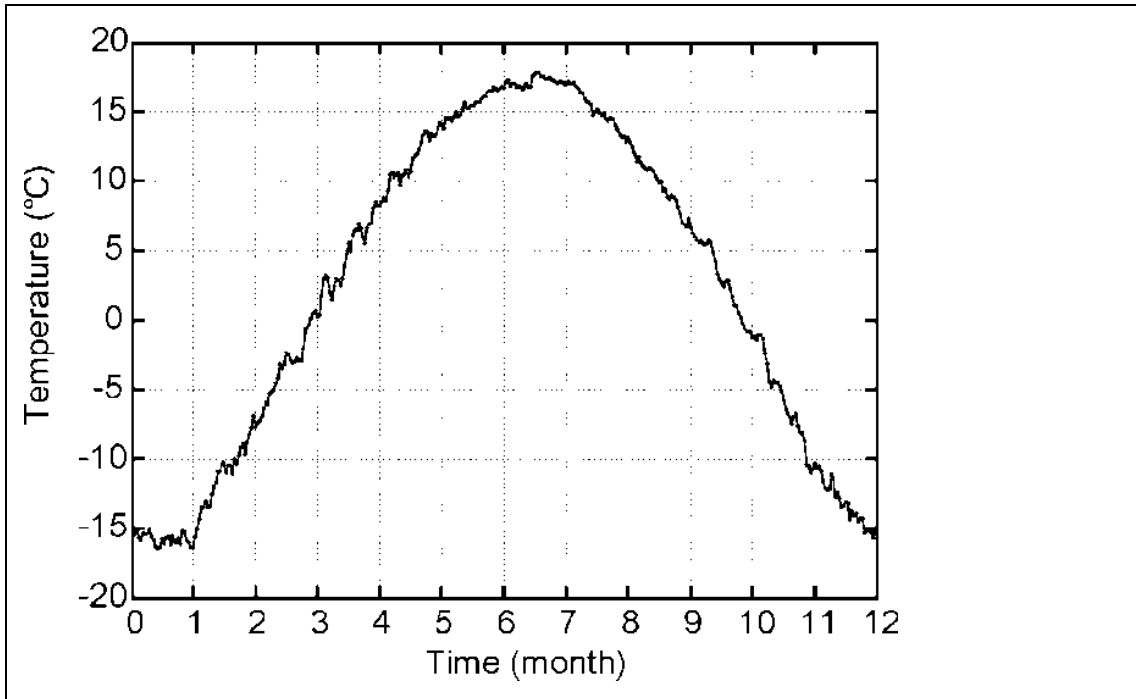


Figure 2. Yearly variation of external air temperature of Ningwu ice cave

533  
534  
535  
536

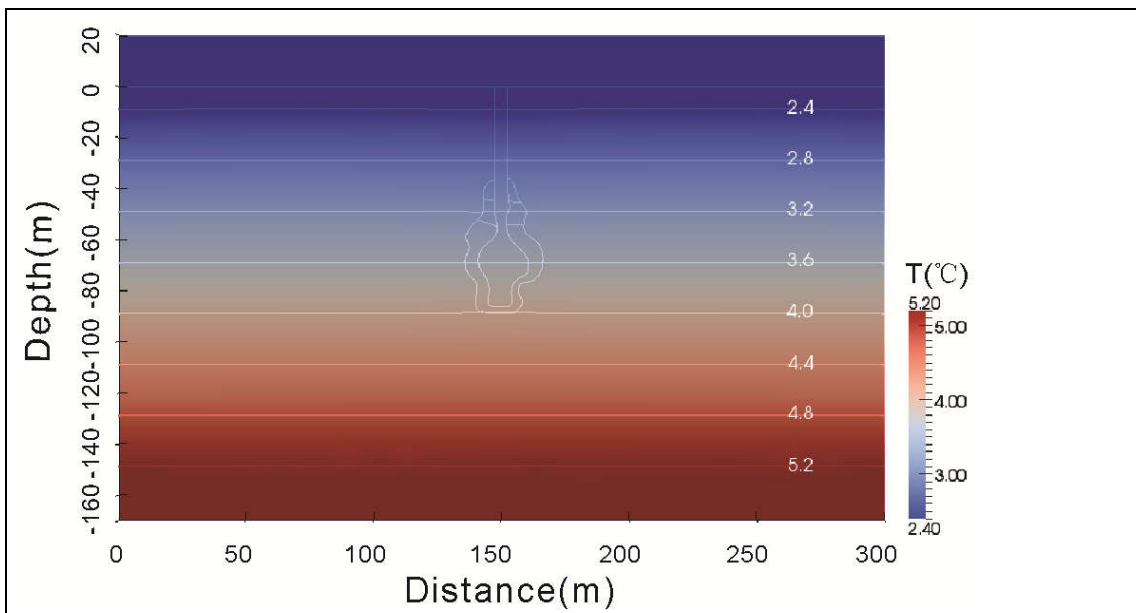


Figure 3. Initial reference temperature distribution around Ningwu ice cave

537

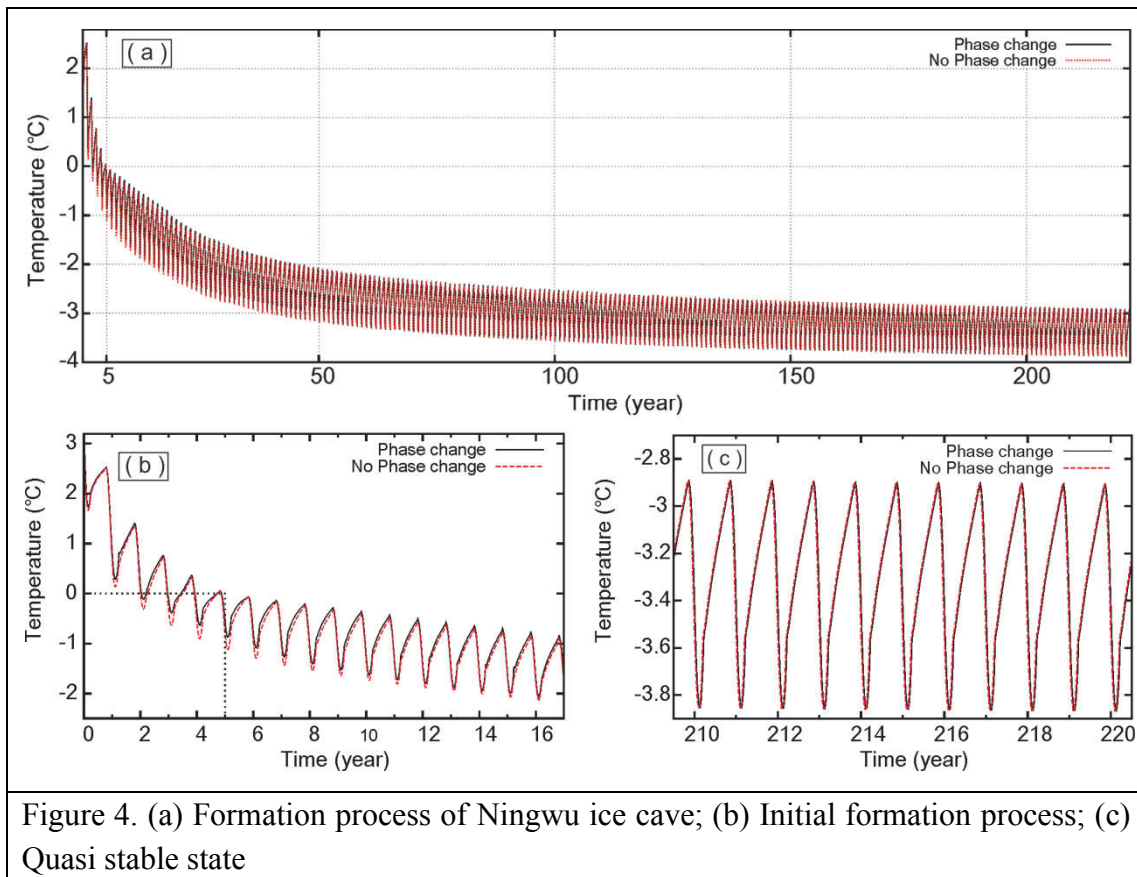


Figure 4. (a) Formation process of Ningwu ice cave; (b) Initial formation process; (c) Quasi stable state

538

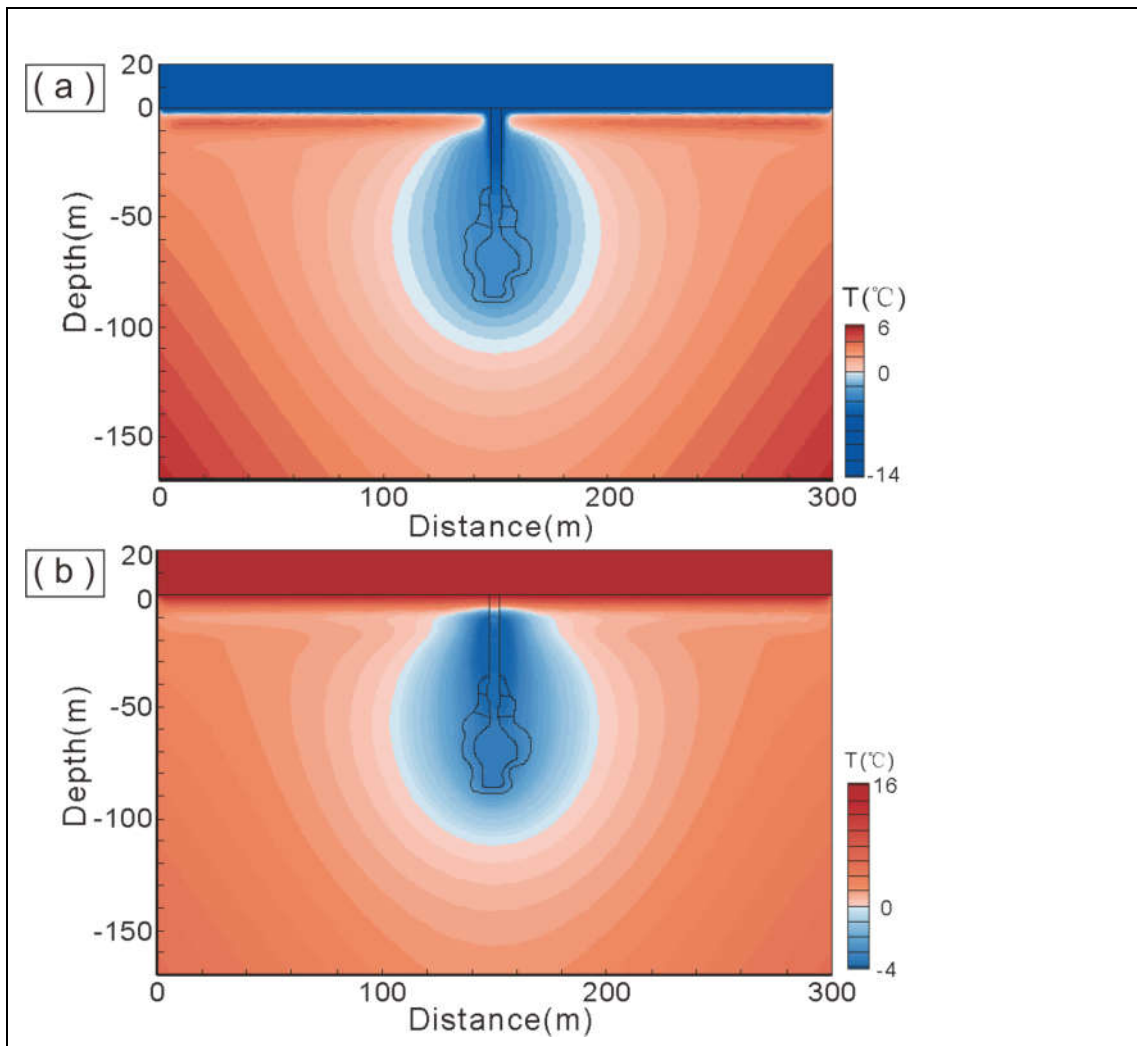


Figure 5. Temperature distribution around Ningwu ice cave in winter (a) and summer (b)

539

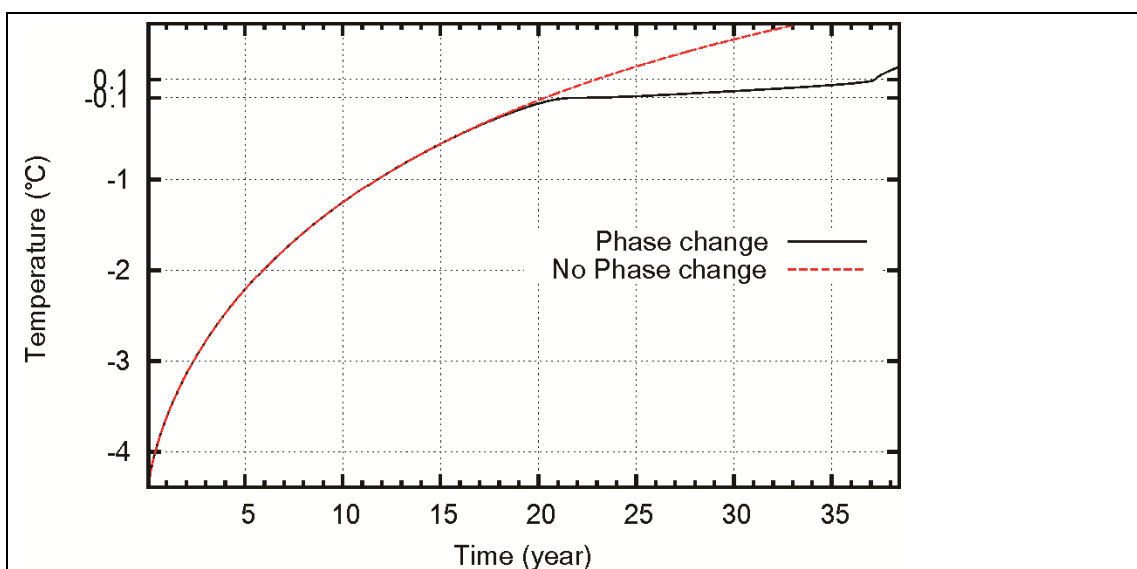


Figure 6. Internal temperature evolution diagram when ice in melting

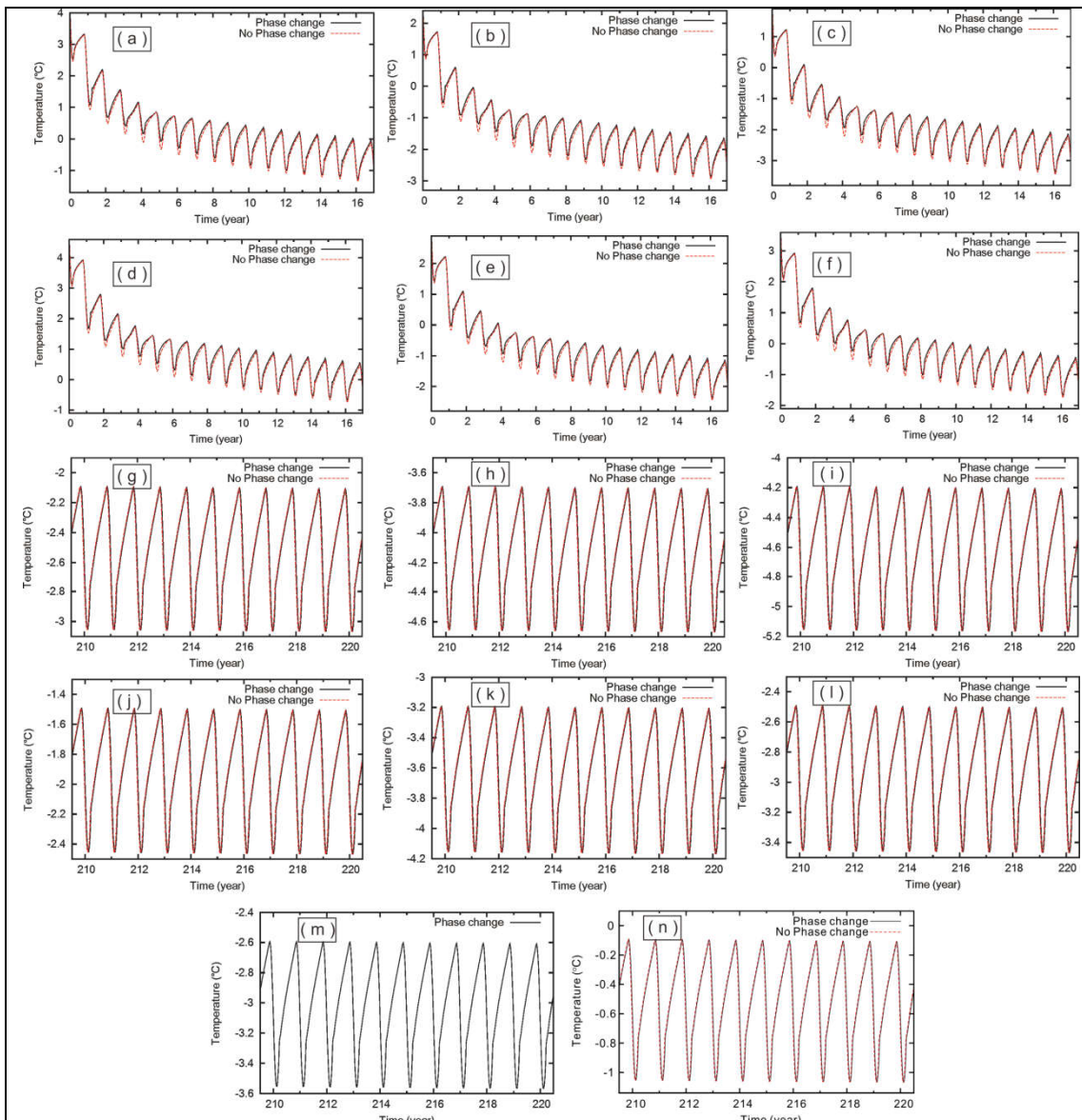


Figure 7. (a)-(f) Initial formation process of Ningwu ice cave in different sensitivity experiments. (g)-(l) Corresponding Quasi stable state. (m) Tourists and bulbs sensitivity experiment. (n) Quasi stable state when the mean annual temperature increases 3.5 °C

541  
 542  
 543  
 544