

We thank all referees for their interesting and constructive feedback that helped us to improve the paper. The reviewer’s comment is colored in black, our answer is in blue and the subsequent changes in the paper are described in red (see also modifications directly in the paper). The mentioned lines correspond to the latest version of the paper.

## Reply to referee #1

The authors addressing certainly a very important issue in snow modelling, name the influence of micro-structure on snow mechanics. They approach the structure with the Discrete Element Method (DEM). However, following points should be addressed in the article:

**Comment 1** There is no reference as to which temperature the mechanical behaviour is connected to or how the temperature dependence is taken into account.

As investigated in section 3.2.2, the simulated macroscopic behavior of dry snow strongly depends on the microscopic cohesion  $\sigma_{ice}$  and friction coefficient  $\tan \varphi$  at the contacts. The effects of temperature on the mechanical behavior of snow could be taken into account by changing the microscopic contact law from a fixed law to a temperature-dependent law, to the extent that sintering effects remain limited. However, the aim of the article is to propose a mechanical model accounting explicitly for snow microstructure. The final goal of the model is to investigate the link between the geometry of the microstructure and the mechanical behavior of snow for a fixed microscopic contact law. That is why this temperature dependence was not implemented.

To clarify this point, the following text was added to the paper l. 317-320: “In the following, we fixed the microscopic cohesion to 1 MPa. In future studies, effects of temperature on the macroscopic behavior of snow, to the extent that sintering effects remain limited, could also be considered by accounting for the influence of temperature on the microscopic cohesion.”

**Comment 2** The DEM model taking into account the interaction between snow grains is not presented in a clear way: The cohesion model needs a better explanation and presentation in formulas for the reader to understand e.g. what is  $\tan(\phi)$  referring to?

The friction coefficient  $\tan \varphi$  corresponds to the ratio between the normal force and the shear force between two spheres sliding against each other. This notation is commonly used to describe a Mohr-Coulomb model.

To clarify the definition of the microscopic contact law, the following figure (Fig. 1, Fig. 3 in the paper) and a description of the role of each parameters in the contact law (l. 166-169) were added.

**Comment 3** Is bonding or re-bonding taken into account?

As explained in the text (l.142-144), the bonds that exist in the initial microstructure are elastic, cohesive and frictional. The new bonds created during deformation are only elastic and frictional. Sintering during deformation is therefore not taken into account.

The following text was added l.144 “Sintering during deformation is therefore not taken into account.”

**Comment 4** *No validation e.g. comparison with experiments has been presented for the DEM model developed. Therefore, are the material properties chosen meaningful?*

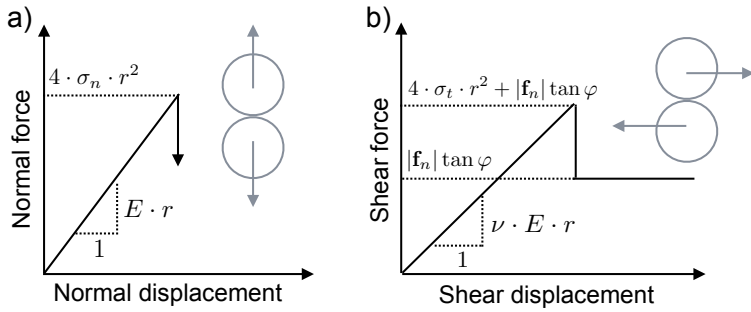


Figure 1: Behavior of a sphere-sphere contact in two simple cases: normal loading (a) and simple shearing under constant normal force  $\mathbf{f}_n$  (b).

The microscopic contact law was set according to typical values of cohesion and friction coefficient found in the literature for ice. We agree that an important scatter on these parameters remains, and, as shown by the sensitivity analysis, that this scatter directly affects the macroscopic compressive behavior of snow. As pointed by reviewer, the model was not evaluated by experiments. This evaluation would also help to adjust precisely the microscopic contact law parameters (e.g. temperature dependence). However, this is beyond the scope of the paper, which focuses on the presentation of a new mechanical model accounting directly for the geometry of the snow microstructure.

This point is now indicated as future work 1.482-485.

## Reply to referee #2 (Mark Hopkins)

This paper presents an interesting discrete element based dynamic microstructural snow model. The model snow microstructure was derived from muCT imagery and implemented by replacing each voxel of the muCT derived structure with a sphere whose diameter equals the width of the cubic voxel. Discrete elements or grains are defined by a preprocessing algorithm that delineates the location of necks or weak points in the structure. Only the surface spheres that cover the elements participate in interactions between neighboring grains. The spheres at grain boundaries are linked by an elastic constitutive model that supports both tension and compression in the normal direction and shear in the tangential direction. Once the tensile strength of the bond is exceeded in response to the stresses that arise when the sample is loaded the cohesive bond is broken. Once broken the grain/spheres interact through a model that supports elastic compression and frictional sliding. The DEM model is simple and avoids the complexity of a full polyhedral treatment. While this method creates some artificial roughness due to the inherent bumpiness in grain-grain contacts, as the sphere radius  $> 0$  the bumpiness is diminished.

As an initial presentation of a new snow modeling technique this paper strikes a good balance between laying out the model details, exploring the sensitivity of the model to the grain-grain constitutive model, sphere diameter, sample size (RVE), and friction coefficient. Finally, results of large-strain uniaxial compression experiments on muCT derived samples having different densities and microstructures are presented. The comparison shows that sample density is a good proxy for a samples response to compression from initial elastic loading through failure. I am sure that the model can be used to study

other loading states as well.

**Comment 1** I have questions about the cohesive failure model. Does the failure occur in one time step? There appears to be no strain-softening? If not then the failure will send shock waves that radiate outward from the failure site. This is mitigated to some degree by the global damping that is imposed on the system. If this is the case then the problem is better addressed by the addition of strain-softening so that failure is attenuated over a number of time steps.

The failure of one sphere-sphere contact occurs in one time step. The failure of one grain-grain contact composed of multiple sphere-sphere contacts occurs generally in more than one time step. We effectively observed some elastic waves that radiate from the failure site. As explained 1.8 p.1434, we used a numerical global damping to dissipate energy in the system. This damping also dissipates some of the elastic energy released from a failure site. The sensitivity analysis revealed a low sensitivity of the macroscopic behavior to the global damping which strongly affects these elastic waves. We therefore believe that these waves do not lead to extra bond failure and so do not affect the macroscopic behavior. Considering more advanced microscopic contact laws (e.g. including sintering) is an ongoing work.

**Minor comments** P 1432 L 1: “The interactions between the members of two different clumps are frictional and cohesive, ...” Should mention elastic as well. **Changed as suggested.**

P 1436, L 10: “... and snow deforms ...” – “... and the snow sample deforms.”. **Changed as suggested.**

P 1436, L 25: “compacity” I learn something new every day. **Changed into “density”.**

## Reply to referee #3

The authors present a discrete element snow model based on 3D tomography images of the snow. The model is capable to describe the rapid and large deformation of the snow dominated by grain rearrangement. The voxels of an upscaled binary X-ray tomography image are replaced with elastic spheres. Snow grains are considered as rigid bodies, and they are represented in the model as clumps of bonded spheres. Bonds between the ice grains are represented by a cohesive contact law. Simulation results of the confined compression of snow samples of different snow types are presented and analyzed. Sensitivity analysis of different model parameters is also presented. Based on the simulations the authors conclude that density alone is a sufficient descriptor of the rapid compression behavior of snow, the microstructure only plays a secondary, practically negligible role.

The authors address an important point in snow mechanics. Namely, the effect of microstructure on the mechanical behavior of snow. X-ray tomography is a modern and popular tool to obtain detailed microstructural information of snow. The authors apply a sophisticated method to convert the tomography data into a discrete element (DE) model. As they correctly point it out, a DE approach is more suitable for the simulation of large deformations than a finite element one. In this respect it is correct to use a DE model to simulate snow compression which is dominated by the large displacement and rearrangement of the ice grains in the snow. On the other hand, a microstructure-based DE model consist of a large number of spheres resulting in a long computational time. The authors claim that a DE model is computationally more effective than a finite element simulation. While it is possible that a voxel based finite element model requires more computational time, an adaptive tetrahedral mesh can reduce the computational time considerably. The Youngs modulus and tensile strength of a 4 mm x 4 mm x 4 mm snow sample (similar size that is used by the authors in the present

paper) can be easily calculated on a common desktop computer with a single processor. I am not sure if this is the case with the simulations presented here. In fact, the authors never mention the computational time and hardware required to run their simulations. The work presented here is an important contribution to our understanding of the mechanical behavior of snow. It is a novel method, and with further development it can be a useful tool to study snow deformation on the microscopic level. The manuscript is clearly written with high quality figures. It is easy to understand, the relevant works are referenced with one notable exception (see below).

**Comment 1** The most comprehensive discrete element snow model is presented in M. Michael: A Discrete Approach to Describe the Kinematics between snow and a Tire Tread, PhD thesis, University of Luxembourg, 2014. This must be mentioned and referenced.

Changed as suggested.

**Comment 2** The bottleneck of microstructure-based snow simulations is the huge computational power required. The authors take steps to reduce the computational time of their simulations by using unrealistic material properties (elastic modulus, density), but they never mention the time and hardware their simulations require. Do they run on a desktop machine with a single processor or they require a supercomputer with 100s of processors?

The simulations presented in this paper run on a desktop computer with a single processor (2.7 GHz) and 16 Gb RAM. The typical computing time of the simulation is on the order of one day to one week. More precisely, the test shown in section 3.1 runs in 15 h with, in average, 8 time-step iterations per second. It requires 2.5 Gb RAM. The sample s-RG0 of rounded grains ( $4^3 \text{ mm}^3$ ) is composed of 800 grains, and described with about  $10^5$  spheres ( $1.5 \times 10^5$ , if the spheres in the inside of the grains are not removed). The mean number of sphere-sphere contacts per cohesive inter-granular bond is 14.

To answer this point, the computing costs are now described in section 2.3.2.

**Comment 3** It would be very useful to include the 3D picture of representative samples of the different snow types used in this study (for example s-DF, s-FCDH and S-RG0). Similar to figure 1.

As suggested, a 3D picture of sample s-DF, s-FCDH and s-RG0 (see Figure 2) was added to the paper.

**Comment 4** In this model the bonds between ice spheres are represented by several sphere-sphere contacts. As with every discretization, there is a minimum number of spheres that can properly represent a continuous contact. Mixed deformation modes like bending (the most common deformation mode of the necks in snow compression) require a fine discretization i.e. a large number of spheres at the contacts. This should be studied by comparison with finite element simulations of bond deformation or at least mentioned in the paper. At a very minimum, the number of spheres at the narrowest necks in the different snow models should be given.

We agree that bending forces can be affected by the number of spheres used to describe one inter-granular contact. However, the sensitivity analysis showed that the simulated mechanical behavior is relatively insensitive to the number (if reasonably large) of spheres used to describe the microstructure. The comparison with finite-element would be interesting, but very time-consuming and limited to the elastic phase (which is not the objective of this work). Moreover, the finite-element method is also related to a certain discretization of the microstructure and the deformation of the grains themselves modifies the stress distribution in the bonds, which limits the comparison with the DEM approach.

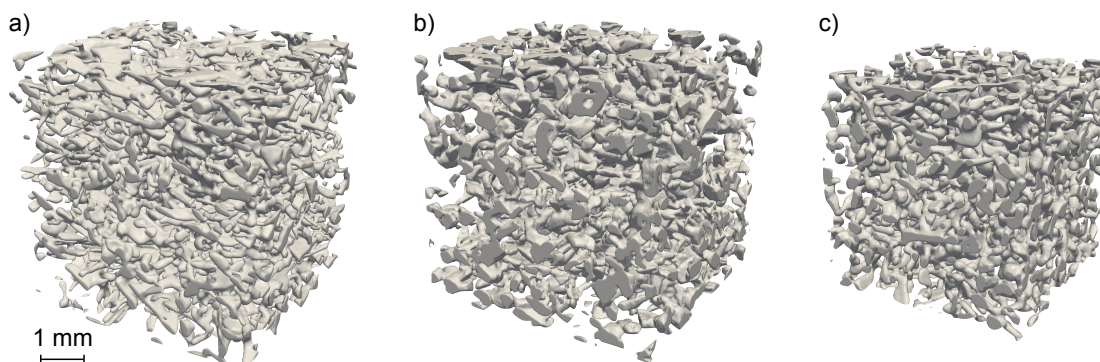


Figure 2: Example of different snow microstructural patterns used in this study: (a) sample s-DF, (b) sample s-FCDH and (c) sample s-RG0. The size scale is the same for the three images.

Sample name	Sphere radius ( $\mu\text{m}$ )	Grain number	Total sphere number	Mean sphere number per contact
s-DF	17.18	2900	546 773	16.2
s-DFRG	19.64	2703	263 495	9.0
s-FCDF	24.13	1261	212 066	17.9
s-FC	20.93	2011	371 824	21.2
s-FCDH	24.13	1809	247 618	15.1
s-RG0	19.64	1967	273 069	13.6
s-RG1	24.55	1261	212 066	17.8

Table 1: Details on the discretization of the microstructure of the different samples into clumps of spheres.

To clarify this point, the mean number of spheres used to describe one bond in the sample s-RG1 used to illustrate the method (sections 3.1 and 3.2) is now indicated in section 2.3.2. Further information on the discretization of the grains into clumps of spheres is also indicated in Table 1.

**Comment 5** The authors write on page 9 line 8: “However, for the parameters considered in this study, the mean effective intergranular friction coefficient remains close to the microscopic friction coefficient defined at the sphere-sphere interaction.” It is hard to believe (and it contradicts to my experience with discrete element modeling) that surfaces consisting of spheres with different size will show similar friction coefficients. What parameters do you refer to?

As shown in Figure 3 in the paper, the bumpiness of the grain contact creates a non-constant shear force resisting to sliding for a constant normal force. This force depends on the relative position of the two contact planes composed of spheres. Since the description of the planes with spheres is periodic, the relation between normal force and shear force for a given sliding direction is also periodic (to the extent that boundary effects are limited). This period is proportional to the sphere radius. But the scatter and the mean value of the friction coefficient (ratio between the mean shear force and the normal force) are not sensitive to the sphere size, for relatively large contacts. Moreover, the sensitivity analysis showed that the simulated mechanical behavior is relatively insensible to the size

of the spheres used to describe the microstructure.

**Comment 6** By using a physically unrealistic value for the Young's modulus,  $E$ , the contact behavior becomes completely unrealistic. Therefore, a direct comparison with real snow measurements becomes impossible. This should be emphasized in the paper. It should be also mentioned that changing  $E$  will change the point of bond failure.

As pointed by the reviewer, the strain at inter-granular failure is over-estimated in our approach since the chosen contact stiffness is artificially low to reduce the computing costs. Indeed, the Young's modulus of ice is estimated to be around  $10^{10}$  Pa (Petrovic, 2003) and we use a microscopic contact stiffness of  $10^7$  Pa (Table 2). First of all, the aim of this paper is to propose and present a new methodology to model large deformations of snow directly based on the microstructure captured by tomography. The evaluation with experiments and the subsequent adjustment of the microscopic contact law ( $E, \sigma_{ice}, \tan \varphi$ ) is beyond the scope of the paper. Second, in the model, grains (or clumps) are rigid. Therefore, there is no explicit link between the Young's modulus of ice and the elastic modulus to be used in the microscopic contact law. Third, in case of non-cohesive discrete element models (without clump), it has been shown (e.g. Cleary, 2010; Lommen et al., 2014) that the overall mechanical behavior (not the elastic phase) is not sensitive to the contact elastic modulus when the interpenetration remains limited ( $< 0.5\%$  of grain radius). As shown in Figure 4c of the paper (now Fig. 6c), the inter-penetration is on the order of a few percents of the sphere radius. In our model, the simulated behavior might be thus sensitive to the elastic modulus, but is not completely unrealistic (see answer to comment 8). In conclusion, a direct comparison with real snow measurements is possible but might require an adjustment of the microscopic contact law ( $E, \sigma_{ice}, \tan \varphi$ ) and, in particular, of the elastic contact modulus. This is now mentioned at the end of the paper.

We add subsection 3.2.3, details on lines 188 and 194-196 to clarify the sensitivity to  $E$ .

**Comment 7** Do you expect your model to describe real snow behavior? Would it be possible to fit the model to real snow measurements? What are the fitting parameters (if there are any) in the model? These points should be discussed in the paper.

See answer to comments 6 and 8.

**Comment 8** On page 9 line 22 you write: "the elasticity of the contacts is expected to have little influence on the macroscopic response of the sample." Why do you expect this? In the initial, elastic phase, as well as in the final, dense compaction phase  $E$  should have a strong effect. A sensitivity analysis must be done to prove this.

In the limit of rigid grains (inter-penetration small compare to the sphere radius), the elasticity of the contacts is generally shown to have little influence on the macroscopic response (except in the macroscopic elastic phase) of the sample (e.g., Cleary, 2010; Lommen et al., 2014). This is why we expected the little influence of  $E$  on the overall mechanical behavior in our domain of interest (the brittle frictional phase). We add this precision 1.188, 1194-196.

We evaluated the sensitivity of the computed stress-strain curves to the microscopic Young's modulus (see Figure 3). Indeed,  $E$  has an effect on the compute stress-strain curve, especially in the two phases mentioned by the reviewer, namely the elastic and dense compaction phases. We already explained in the paper that the elastic phase is for sure not well represented by the model, as written 1.3 p.1442 ("we would not expect the present DEM model to provide a correct macroscopic Young's modulus"), 1.6 p.1441 ("the strain interval for the elastic phase is overestimated"). For the dense compaction phase and high macroscopic strains, we mentioned 1.3-5 p.1443 that "the assumption of

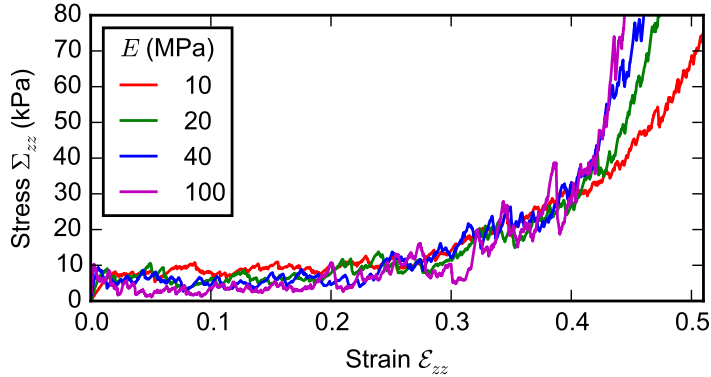


Figure 3: Computed stress-strain curves for different Young's moduli.

rigid grains is no longer valid and the simulated behavior becomes very sensitive to the value chosen for the Young's modulus”.

Actually, the granular interpenetration encountered during the brittle/frictional phase (see Figure 5c in the paper) are too large (around a few percents) to ensure completely the assumption of rigid grains ( $< 0.5\%$ ). That is why there is some sensitivity to  $E$  in this phase. For a future comparison to real experiments, this sensitivity cannot be neglected and the value of  $E$  might be adjusted (increased probably). In the context of the paper, this sensitivity remains limited and do not change the feasibility of the approach and the conclusion that the main effect of the microstructure on the mechanical behavior under compression is expressed through its density.

To clarify the points raised in comments 6, 7 and 8 about the choice of the contact law parameters, especially  $E$ , a section on the sensitivity to  $E$  was added (sect. 3.2.3) and the future work necessary to evaluate the model with experiments is now described at the end of the paper.

**Comment 9** What are the typical number of spheres in a model? This should be mentioned.

This is now mentioned in section 2.3.2.

**Comment 10** *Although it is not realistic, the calculated Youngs modulus should be mentioned in paragraph 3.1.1.*

We do not agree. This discrete element model is not suited and intended to compute Young's moduli, since the deformation is constrained to be inter-granular and the microscopic Young's modulus of ice is artificially low. Mentioning values of the macroscopic Young's modulus might mislead the reader.

**Comment 11** On page 14 line 16 you write: "It turns out that the value  $\tan(\phi)$  has little effect on the computed mechanical behavior for macroscopic strains in the range  $[0,0.2]$ ." This contradict to your conclusions in paragraph 3.1.2 where you conclude that in the brittle/frictional phase grain sliding (so the friction between grains) is the dominant mechanism. This requires clarification or further explanation.



The exact strains chosen to separate the three deformation regimes are slightly arbitrary. But here, we define the brittle/frictional regime for a macroscopic strain between 0.02 and 0.3. The friction coefficient has little effect on the computed mechanical behavior for macroscopic strains in the range [0,0.2]. In paragraph 3.1.2, it is explained that bond breaking, structural re-arrangement and grain sliding are the dominant mechanisms (see 1.14-18 p.1436). Friction plays a role in this regime (for macroscopic strain larger than 0.2) but is not THE dominant mechanism for the entire regime. This is never written in the paper. On the contrary, it is mentioned 1.18-20 p.1438 that “microscopic friction is not the dominant deformation mechanism in the so-called brittle/frictional phase”. There is no contradiction.

**Comment 12** Looking at figure 7a, it is interesting that  $\tan(\phi)$  has such a high effect in the final, dense compaction phase. This should be mentioned and explained in the text.

We added the following text (1.307-309) to emphasize this effect: “In the so-called dense compaction phase, the increase of non-cohesive intergranular contacts (see Fig. 5c) thus enhances the effect of the microscopic friction coefficient on the macroscopic mechanical behavior.”

**Comment 13** Figure 7b shows some surprising results. First, the slope of the curve in the initial, elastic phase should not depend on  $\sigma_{ice}$  since practically no bonds are broken yet. Second, in the final, dense compaction phase when most contacts are not cohesive any more,  $\sigma_{ice}$  should not have an effect on the compaction curve. How do you explain these?

As explained 1.2, p.1434, the Young’s modulus was set as  $E = 10 \cdot \sigma_{ice}$  to ensure that the interpenetration of grains remain limited. For the sensitivity analysis to  $\sigma_{ice}$ , we kept this relation to ensure that the microscopic strain at inter-granular failure remains constant. In Figure 8b (now), the slope of the curve in the initial elastic phase is proportional to the Young’s modulus which is here artificially proportional to the microscopic cohesion.

To clarify this point, we add this explanation in the legend of the corresponding figure.

We were also surprised by this dependance on  $\sigma_{ice}$  in the dense compaction phase, but the contradiction suggested by the reviewer (“most contacts are not cohesive anymore”) is not correct. Figure 5c shows that in the final compaction phase, the number of cohesive bonds is still important (more than the half of the initial bonds are still cohesive for a macroscopic strain of 0.5). That is why there can still be a high dependence on  $\sigma_{ice}$  in this phase. Note that the linear relation between macroscopic stress and microscopic cohesion is also enhanced by the fact that the microscopic strain at failure is kept constant in this analysis.

**Comment 14** It is a mistake to discretize the snow geometry using spheres with different diameters in the comparison of the simulated compression behavior of the different snow types. Especially, since the effective friction between grains can depend on the size of the spheres. 20 micrometers should have been used for all samples. It is in fact a bit suspicious that it is exactly those 3 samples that show slightly different behavior that have a sphere size of 20 micrometers instead of 25.

The size of the spheres has to be compared to “one size” representative of the microstructure geometry. Here, we use the equivalent spherical radius to define “one size” of the microstructure geometry. To be consistent, we chose smaller spheres for the samples with the smallest equivalent spherical radius. To avoid any suspicion, Figure 4 shows the stress-strain curve of the tree mentioned samples for a sphere size of about 20 and 25 micrometers. No significant effects of the sphere size on the stress-strain curve are observed. Moreover, as already shown in the sensitivity analysis, the sphere size has little effect on the simulated stress-strain curves. But to be consistent with the typical



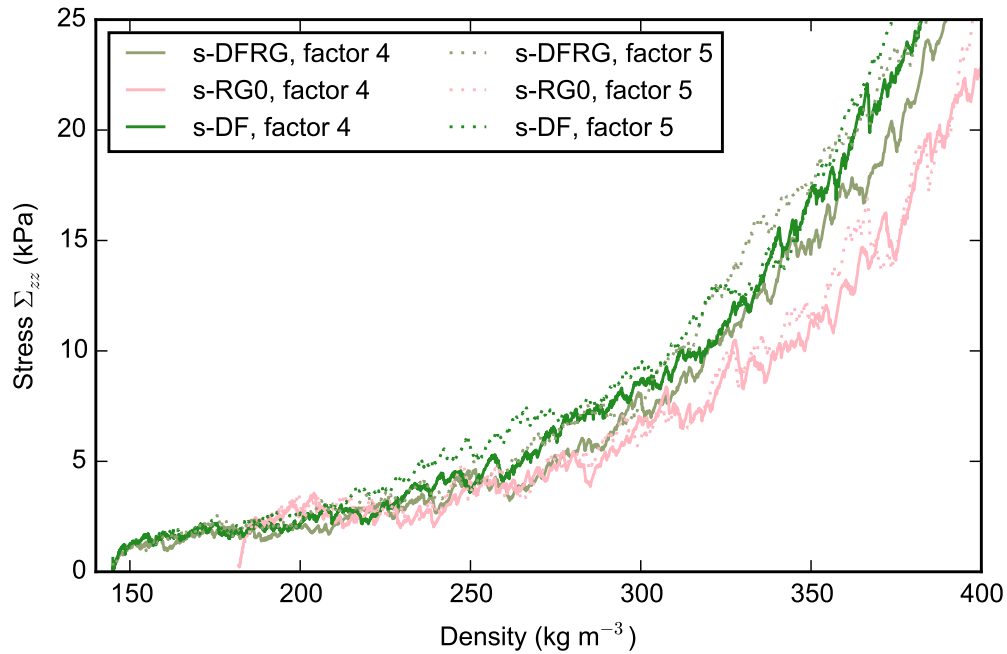


Figure 4: Simulated stress-density relation for the samples s-DF, s-DFRG and s-RG and two different factors (4 and 5) of resolution reduction.

size of the snow microstructure, we made the choice to distinguish these three samples from the rest of the data set. Note that we do not make any conclusion out of small differences observed in the stress-density curves since we cannot ensure that they are not due to modeling artefacts.

**Comment 15** Instead of "equi-temperature metamorphism" write "isothermal metamorphism". page 5 line 13 and page 17 line 4.

Changed as suggested.

## References

- Cleary, P. W.: DEM prediction of industrial and geophysical particle flows, *Particuology*, 8, 106–118, 2010.
- Lommen, S., Schott, D., and Lodewijks, G.: DEM speedup : Stiffness effects on behavior of bulk material, *Particuology*, 12, 107–112, doi:10.1016/j.partic.2013.03.006, 2014.
- Petrovic, J. J.: Mechanical properties of ice and snow, *J. Mat. Sci.*, 38, 1–6, 2003.