The Cryosphere Discuss., 9, 1315–1343, 2015 www.the-cryosphere-discuss.net/9/1315/2015/ doi:10.5194/tcd-9-1315-2015 © Author(s) 2015. CC Attribution 3.0 License.



This discussion paper is/has been under review for the journal The Cryosphere (TC). Please refer to the corresponding final paper in TC if available.

Quantifying the resolution level where the GRACE satellites can separate Greenland's glacial mass balance from surface mass balance

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Received: 20 January 2015 - Accepted: 31 January 2015 - Published: 26 February 2015

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Published by Copernicus Publications on behalf of the European Geosciences Union.





Abstract

Mass change over Greenland can be caused by either changes in the glacial mass balance (GMB) or the precipitation-based surface mass balance (SMB). The GRACE satellite gravity mission cannot directly separate the two physical causes because

- it measures the sum of the entire mass column with limited spatial resolution. We demonstrate one theoretical way to indirectly separate SMB from GMB with GRACE, using a least squares inversion technique with knowledge of the location of the glacier. However, we find that the limited 60 × 60 spherical harmonic representation of current GRACE data does not provide sufficient resolution to adequately accomplish the task.
- ¹⁰ We determine that at a maximum degree/order of 90 × 90 or above, a noise-free gravity measurement could theoretically separate the SMB from GMB signals. However, current GRACE satellite errors are too large at present to separate the signals. A noise reduction of a factor of 9 at a resolution of 90 × 90 would provide the accuracy needed for the interannual SMB and GMB to be accurately separated.

15 **1** Introduction

Mass change occurring over the ice sheets can be divided into two parts: changes due to dynamical responses of glaciers (thinning and calving), and changes due to large-scale patterns in surface melting, runoff, sublimation, and precipitation. The dynamical response is known as glacial mass balance (GMB), while the atmospherically forced
²⁰ signal is the surface mass balance (SMB). These two types of mass change are typically modeled or measured separately. One exception to this rule is when the ice sheet mass balance is measured by satellite gravity, such as the Gravity Recovery And Climate Experiment (GRACE); these measurements are sensitive to the sum of all mass changes, without the direct ability to separate one cause from another. In this paper, we demonstrate one theoretical way to separate SMB from GMB using GRACE, based on a priori knowledge of glacier locations on the ice sheet. Using simulations, we de-





termine the GRACE spatial resolution needed to separate GMB and SMB around large glaciers within acceptable error limits.

In recent years, inverse least squares estimation techniques have been used to localize the smoothed signal observed by GRACE into more precise, geophysically-relevant ⁵ regions (Schrama and Wouters, 2011; Jacob et al., 2012; Sasgen et al., 2012; Bonin and Chambers, 2013; Luthcke et al., 2013; Wouters et al., 2013). Most often, these techniques have focused on the mass change over all of Greenland, or else within 8–16 large drainage basins covering the island. Hypothetically, this technique could be expanded to include regions designed to contain the mass signal of the largest of ¹⁰ Greenland's glaciers. From a purely mathematical perspective, the least squares approach should be able to separate a localized GMB signal from a wider-spread SMB

signal, provided one knows the location and approximate area covered by a glacier. However, Bonin and Chambers (2013) found out via simulation that estimating mass change via an inversion modeling method, even over relatively large SMB basins, can
result in trend errors of ~ 20 % of the long-term trend signal in basins losing the most mass and approaching 100 % of the trend signal in more stable basins. All else equal, the smaller the area, the greater the uncertainty in the inversion results. Because of the relatively small spatial scale of even the largest glaciers, the GMB has not previously been computed using this technique.

A significant reason inversion techniques give weak results in very small areas is due to the innate limited spatial resolution of the GRACE Release-05 (RL05) data. With a maximum degree/order of only 60, a strong spatially-localized signal is effectively indistinguishable from a weaker, more spread-out signal. However, at higher maximum degrees, such signals become distinct (Fig. 1) – and thus, should be separable by the

²⁵ least squares inversion process. We thus aim to answer two questions. First, how high of a maximum degree/order of gravity coefficients is needed to separate the localized, large-magnitude GMB from the larger-scale, smaller-magnitude SMB? Second, what reduction in satellite errors is required for current or future satellite gravity missions to separate the signals with reasonable uncertainty?





2 Description of inversion method

Throughout this paper, we use a modified version of the least squares inversion method described mathematically in Bonin and Chambers (2013). This technique uses a set of pre-defined "basin" shapes on a $1^{\circ} \times 1^{\circ}$ grid, including all of Greenland as well as the

- ⁵ surrounding land and ocean area. Each basin has a prescribed internal mass distribution assumed; using those weights, its smoothed appearance at a particular spherical harmonic resolution is computed. The goal is then to determine the appropriate multiplier for each basin, such that when converted to GRACE-like spherical harmonics, the set of multipliers times the shape and weight of the smoothed basins results in as close
 a match as possible to the GRACE observations. Our least squares inversion technique
- computes the set of basin multipliers optimally, using no additional constraints or regularization.

We choose to use 13 SMB basins covering Greenland, roughly based off the island's drainage (Fig. 2). To this we add 13 external basins: 4 local ocean basins and 9 nearby

¹⁵ Iand basins. The latter specifically include nearby Iceland, Ellesmere Island, and Baffin Island, all of which are known to have large ice mass imbalances themselves. Unlike in Bonin and Chambers (2013), we add to this a set of three GMB basins, which overlap the SMB territory. These define three of the most significant glaciers in Greenland: Kangerdlugssuaq, Helheim, and Jakobshavn. The former two glaciers lay entirely atop 20 SMB basin 4, while Jakobshavn is atop basin 7.

In Bonin and Chambers (2013), we assumed that mass was distributed evenly within each individual basin. However, that assumption was only accurate to first order, since the SMB is dominated by higher losses near the coast. Here, we instead weight the 8 external Greenland SMB basins (1–8), Ellesmere Island, and Baffin Island using the

RMS from the RACMO2 ice model (Ettema et al., 2009). This accentuates coastal mass change. The internal Greenland SMB basins and other external basins are still assumed to have uniform mass distribution. The GMB basins are each dominated by a single 1° × 1° grid cell, with 1–3 non-zero neighboring cells whose weights are defined





by modeled ice loss rates (see Fig. 5a) (Price et al., 2011). We do not assume that the actual modeled time series of glacial mass loss is correct, but merely use the model to determine the relative likely distribution of mass loss in neighboring grid cells, compared to loss within the central cell.

5 3 Definition of the simulation sets

We wish to quantify the accuracy of the inversion method. Uncertainty will be the sum of errors from three main places. First, errors from the inversion method itself will come from an imperfect fit of the SMB to the weighted basin collection. We have reduced these "misfit" errors by using coastally-dominated weights from RACMO2, but such weights may be imperfect and in any case do not take into account the changes in spatial variability of the signal over time. Similarly, while the central location of the glaciers is well known, the weights given to the secondary cells surrounding it are less definitive and may result in inaccuracies in the least squares fit. Last are the satellite errors from GRACE itself, particularly the north/south "stripes" which dominate unsmoothed and unconstrained GRACE data. No basin is designed to hold these stripes, as they are not part of the targeted signal, so instead they will collect in other basins and contaminate the fit there.

To parse out which problem causes which least squares fit error, we have created a GRACE-like simulation where the "truth" will be perfectly known. The simulation contains four parts: an external ocean and hydrology signal, a SMB signal, a GMB signal, and an estimate of GRACE stripe errors. The external signal occurs everywhere outside of Greenland itself, and is kept constant. The hydrology model used is the average of the GLDAS-Noah (Rodell et al., 2004) and WGHM (Döll et al., 2003) models. The ocean model is one run at the Jet Propulsion Laboratory (JPL) as a contribution to the Estimating the Circulation and Climate of the Ocean (ECCO), available at http://grace.jpl.nasa.gov. The JPL_ECCO model is based on the MIT general circulation model (Marshall et al., 2007). It is a baroclinic model forced by winds, pressure,



and heat and freshwater fluxes from the National Center for Environmental Prediction (NCEP) operational analyses products and also assimilates satellite altimetry and other in situ observations (Fukumori, 2002; Kim et al., 2007).

We use an ensemble of semi-randomized simulations for the SMB and GMB sim-⁵ ulations, in hopes of determining the effect that mismatches between basin weights and the signal have on the fit. Similarly, we use an ensemble of randomized collections of stripe-like correlated harmonics (Bonin and Chambers, 2013) to estimate the likely impact of the GRACE errors on our fit. The methods used to create the latter three ensembles of "truth" simulations are described in the following paragraphs.

10 3.1 SMB simulation definition

An excellent model choice for SMB over Greenland and the surrounding areas would have been the RACMO2 ice model. But we have already used RACMO2 to compute the SMB basin weights, and using the same data to both fit with and fit to would significantly underestimate the actual errors caused by a least squares solution with real data.

¹⁵ We thus choose to simulate plausible SMB signals, using RACMO2 as a baseline. Long-term trends and the monthly climatology together make up 83 % of the RACMO2 variability across Greenland, including over 95 % of the coastal signal, making them the dominant terms in need of careful reproduction. As such, we separated the actual 2002–2012 RACMO2 signal into a long-term trend, a 12 month climatology, and the remaining residual. Using these pieces, we created six semi-randomized simulation SMB "truth" simulations.

To accomplish this, we first developed a technique to create randomized but locallycorrelated maps of the Greenland area, Iceland, and the ice-covered parts of Ellesmere and Baffin Islands. The local correlation is critical, since SMB tends to be caused by fairly long-wavelength weather phenomena like precipitation and temperature anomalies. A "truth" series which varied randomly from one 1° × 1° grid cell to the next would not only poorly represent reality, but would also fit differently into the larger SMB basins than a broader signal would. So we began with purely random values in each 1° × 1°





land grid cell, but then averaged over all grid cells within $\pm 3^{\circ}$. The smoothed grid was then normalized to a mean of 0 and SD of 1 across all grid cells (Fig. 3a).

We used such random, locally-correlated grids to vary the SMB signals away from RACMO2 in a physically-meaningful manner. The trend maps (x^{SIM}) for each of the six simulations were created by multiplying the actual RACMO2 trend map (x^{RACMO2}) by a 3° -smoothed random map ($r^{3°}$), which is randomized afresh for each realization. We used a weighting term, α , to determine the relative amount of disturbance to perturb the RACMO data by, as defined by

$$x^{\text{SIM}}(\text{lat, lon}) = \left[1 + \alpha \cdot r^{3^{\circ}}(\text{lat, lon})\right] \cdot x^{\text{RACMO2}}(\text{lat, lon}).$$
(1)

We chose α = 0.5, or a variability of 50 % away from the original signal in any 1° × 1° bin. Figure 3b shows an example of this technique on the trends, after subtracting off the original RACMO2 trends for visibility's sake. Critically, this technique means the high-signal coastal areas contain most of the variation, while the quieter interior of Greenland is adjusted with smaller variations away from the expected trends. Also, the signals are spatially correlated, as would be expected from physical processes such as changes in regional temperature and melting, or in precipitation. The process is repeated to create a set of alternative maps of each month's typical climatology, based on the RACMO2 climatology.

For both trends and climatology, we are probably creating a conservative estimate, since RACMO2 has been determined to have much less than 50 % error (Ettema et al., 2009). However, error estimates in such studies have focused on the errors in the total mass change over all of Greenland, not the mass change in a far smaller area like a single grid cell. Since positive and negative errors will tend to average out over large areas, we presume that local 1° × 1° RACMO2 errors will be significantly larger than global ones. Since we have no information on how much larger the local errors really are, we choose to err on the side of caution and create differences away from our basin weights larger than what we are likely to encounter in reality.



While the trends and climatology describe the strongest parts of the RACMO2 SMB estimate, 17 % of its variance is driven at other frequencies, including significant interannual variability. To simulate both higher- and lower-frequency variability in the simulated data, we used a random walk process. We first created a series of the random, locally-correlated maps described previously, one for each desired month of simulated data. The final non-trend, non-climatology simulation at month *i*+1, x_{i+1}^{SIM} , is a weighted combination of that month's random map (r^{4°) and the final map of the previous month (x_i^{SIM}):

$$x_{i+1}^{\mathsf{SIM}} = \beta \cdot x_i^{\mathsf{SIM}} + [1 - \beta] \cdot r^{4^\circ}.$$

After comparing with the actual RACMO2 residuals in various places (for example, Fig. 4), we chose a value of $\beta = 0.85$, thus constraining each new month's signal fairly tightly to the previous one. Once the entire randomized time series was created, we removed the mean and normalized each grid cell to have the actual RACMO2 SD in that cell. The latter gives the coastlines more variability, as they have in reality, while retaining spatial correlations with the nearby grid cells and temporal correlations with neighboring months.

Each final SMB simulation series is made from the summation of trend, climatology, and random-walk pieces, for each month. We created 6 simulations of 10 years of SMB simulation, designed to represent the GRACE years 2002–2011. We transformed these into spherical harmonic representations of maximum degree/order 60, 75, 90, 120, and 180 for use in the least squares inversion process.

3.2 GMB simulation definition

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In comparison, the simulated GMB signal is artificially simple. We considered using a random walk process, similar to that used in the residual SMB simulation, but decided to avoid such unnecessary complexity. Firstly, we did not have access to good, monthly

to avoid such unnecessary complexity. Firstly, we did not have access to good, monthly measurements of the mass signal in any of the three glaciers we were looking at, so we

(2)

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had no clear estimate of the expected variability, particularly at sub-annual frequencies. Secondly, the GMB basins are only 2–4 grid cells in size, and are each dominated by a single central grid cell, so there is minimal concern about signal overlap from nearby glacial bins with vastly different temporal signals. Instead, we kept things simple and ⁵ manufactured a piecewise linear "truth" signal for each glacial basin (Fig. 5c). This will help us to determine how well different changes in slope can be seen using the least squares inversion technique. The expected signal is of roughly comparable magnitude

- to modeled estimates (Howat et al., 2011) and is thus much larger than the SMB signal is, though across a far smaller area.
- We expected misfit errors from the GMB to arise from the imperfect basin weightings 10 we gave to the non-central glacier cells. To test how large an effect that has, we created an ensemble of 6 simulated GMB series, each to maximum degree/order 60, 90, 120, and 180. Each run has the same total GMB signal per glacier, but we altered the spatial distribution of that signal slightly each time (Fig. 5b vs. a).
- To do so, we first computed the average weight originally given to the non-central grid 15 cells (W_{sides}). We then altered the glacier's original grid cells (including the central cell) by adding half of W_{sides} times a random value (computed with a mean of 0 and variance of 1). Those neighboring grid cells which originally had zero weights were shifted away from zero by randomized weights a tenth as large as W_{sides} . The new weights around
- the glacier were then summed and normalized by the original sum of weights, so the 20 total signal strength was the same as in the ideal case. The same GMB time series was divided among 6 different randomized distributions of weights, in addition to the ideal weights case used in the basin definitions. The difference between the inverted results gives us an estimate of the sensitivity of the least squares process to imperfect glacier basin weights. 25

3.3 **GRACE** stripe simulation definition

We used GRACE RL05 solutions from the Center for Space Research (CSR) for harmonic cases 60×60 , 96×96 , and 120×120 to create stripe-only simulations. The



first two series are freely available on the Physical Oceanography Distributed Active Archive Center (PODAAC) website (ftp://podaac.jpl.nasa.gov/allData/grace/L2), while the latter is an experimental case run in the same manner and kindly provided by Himanshu Save at CSR. Despite the slight mismatch in maximum degree, we represent the errors for the 90 × 90 simulation cases with the 96 × 96 stripes.

In each case, we began with the full GRACE series, then removed as much of the geophysical signal as possible, to end with what we hoped was mostly errors in GRACE. To do this, we first removed the JPL_ECCO ocean model and the hydrology model used previously (the average of GLDAS and WGHM), as well as the RACMO2

- ¹⁰ model over Greenland and a far rougher linear estimate of the mass change over Antarctica. None of these models are perfect, so we fit a mean, trend, annual, and semiannual signal to what remained. We know that much of the remaining trend and annual signal is important geophysical signal, but some stripes also fall into those categories. To further separate that, we pulled aside only the trend/annual components
- ¹⁵ of the harmonics which explained at least 50 % of that harmonic's full variability. That fraction is added to the "model" and removed from the "residual". The result is a set of "model" maps that do not visibly show stripes, and a set of "residual" maps that are heavily dominated by stripes (Fig. 6a and b).

We then applied the stripe simulation technique described in Bonin and Chambers

- (2013). The technique is based on the observation by Swenson and Wahr (2006) that due to the north–south stripes, same-order odd-degree harmonics tend to correlate, as do same-order even-degree harmonics. We used the actual variance and harmonic cross-correlations from the residual GRACE series to make randomized sets of north– south stripes with approximately the same spatial distribution as what is actually seen in
- ²⁵ GRACE (Fig. 6c). We created 10 randomized variations of the stripes for each GRACE series (degrees 60, 96, and 120). The stripe simulation technique begins to break down at high degrees/orders, overweighting the stripe amplitude within ~ 5° of the poles at maximum degree 96 and ~ 10° of the poles at maximum degree 120. To reduce this false effect, we were forced to apply a latitude-based normalization scheme for the





 96×96 and especially the 120×120 simulated stripes. This reduced the simulations' bin-based RMS to levels matching the original stripe RMS for each maximum degree.

We chose to create simulated stripes, rather than directly use the residual signal as the GRACE errors because a close look at the residuals reveals that some probably-

real interannual signal remains in several of the coastal Greenland basins, even after the trend/annual fit and removal. This is caused by an imperfect SMB/glacial model and the fact that not all remaining signal is perfectly linear or annual. In terms of the simulated stripes, it implies that our stripe estimates will tend to somewhat overstate the true north–south stripes, since the variance of the remaining interannual signal signal will go into simulated stripes. This makes our stripe simulation a slightly conservative estimate of the expected GRACE errors.

4 Analysis and discussion

We computed the basin amplitudes from each of the above simulations via the least squares inversion process, resulting in time series for each basin which can be directly
computed to the average "truth" signal in each place. In Sect. 4.1, we compare each SMB and GMB "truth" input to its inverted response, to determine the "misfit" errors caused by using imperfect basins in the least squares method. In Sect. 4.2, we compare the maximum level of stripe errors which are permitted for a given signal-to-noise ratio to the actual stripe errors estimated from GRACE, to determine if either the current GRACE or a future probable satellite gravity mission might be able to accurately separate the glacier signal from the SMB signal.

4.1 Method errors due to imperfect SMB and GMB basins

Figure 7 shows the average RMS basin error from the six GMB-misfit-only simulation cases, for each of the 13 SMB basins and the 3 GMB basins. Specifically, this is the ensemble average of the RMS of the difference between the "truth" basin amplitude



and the inverted basin amplitude. The effect of spatial resolution is seen clearly: with decreasing maximum degree/order, the errors increase. Basin 7 (which overlaps with Jakobshavn Glacier) and the interior basins (9–13) most drastically show the degradation, particularly between maximum degrees 75 and 60.

- ⁵ The specific case of basin 7 is explained by the effect of Jakobshavn Glacier: the two overlapping basins have large and consistently anti-correlated error time series, particularly in the 60 × 60 case. We take this to mean that, at small maximum degrees, the inversion technique cannot appropriately separate the spatial maps of basins 7 and Jakobshavn, and it tends to place some of the signal that belongs in one basin
- into the other. We hypothesize that the reason basin 7 sees so much stronger areas than the other basins (including basin 4, where the other two glaciers reside) is first because both the Jakobshavn and basin 7 mass loss signals are very strong, and second because basin 7 is the smallest of the SMB basins and a significant percentage of its high-signal coastline is also covered by the Jakobshavn basin. The strength of
- this mismatch is highly sensitive to the spatial resolution used, however. The basin 7 and Jakobshavn SMB misfit errors are cut to a third merely by increasing the spatial resolution from 60×60 to 75×75 , and drop to approximately the same error level as the other external basins by maximum degree/order 120.

While the SMB-basin-mismatch errors decreased with increasing spatial resolution,

- oddly, the same pattern did not hold true for the GMB-basin-mismatch test. Figure 8 shows the average basin errors caused by varying the pattern of glacier signal weights. It is true that in most basins, increasing the maximum degree/order from 60 to 90 (or above) reduces the errors. However, in the critical basins 4 and 7, which overlap the glaciers themselves, the situation is less clear. Basin 4 shows effectively identical errors
- for the 60 × 60 and 90 × 90 cases, as does the overlapping Kangerdlugssuaq Glacier (though Helheim follows the more expected pattern). In basin 7, the errors are inverted to what we had expected, with larger errors occurring at higher spatial resolution. We have no explanation for this. We do note that the size of these errors is still a third or





less the size of those from the SMB misfit test, making this puzzling result a secondary impact, at least.

To visualize the relative size of the above misfit errors compared to the "truth" geophysical signals, we have plotted the inverted glacial signals from all of the 36 combina-

- tions of SMB and GMB simulations in Fig. 9. In the dark solid lines, we show the "truth" signal from each glacier basin. Particularly at low maximum degrees/orders, the simulation realizations tend to clump in groups of six, demonstrating the relative strength of the SMB-misfit errors over the GMB-misfit errors. Figure 9 clearly demonstrates that the method-only errors do not cause an insurmountable hurdle to our ability to sepa-
- rate the SMB from GMB signal. Although the standard RL05 60 × 60 solutions do not provide sufficient spatial resolution to do the job, a 90 × 90 solution, which is certainly a plausible achievement either with improved releases of GRACE or with a later satellite gravity mission would, hypothetically, do the task well. By maximum degree/order 90, and with improving skill at higher degrees/orders, a perfect and error-free GRACE solution could be inverted to easily discern relatively small changes in inflection in the
- glacial (and non-glacial) Greenland signals, even given expected imperfections in basin definitions.

4.2 Allowable vs. actual GRACE stripe errors

Unfortunately, GRACE observations are decidedly not perfect and error-free. Indeed, the north–south stripe errors dominate any individual map made from unconstrained, unsmoothed GRACE data. Numerous techniques exist to reduce these stripe errors, including a variety of spatial smoothings, correlation-based destriping methods, and spatial and temporal constraints; however each necessarily impacts the signal along with the error. More critical to our interest here, they effectively reduce the spatial reso-

²⁵ lution of the GRACE data, by damping both error and signal at higher degrees/orders. To use any such post-processing method would undo the high-resolution benefits the previous section found, making it difficult to reach the needed resolution to even theoretically separate SMB from GMB signals. As such, we choose to use no such method.





However, we do choose one simple technique to reduce the errors at no spatial cost: applying a year-long temporal moving window to the data. While this will remove or dampen any high frequency "truth" signal, it is the longer-period Greenland ice mass signal we are most interested in for climate change, which means there is only a limited
⁵ cost to removing some stripes in this way. On the other hand, a majority of the stripe RMS occurs at periods of less than one year. For example, in the 120 × 120 case, removing the high-frequency temporal signal reduces the bin-by-bin stripe RMS to 15% or less their original size around the globe. Due to the way basin analysis averages through stripes, this results in yearly-averaged stripe basin RMS values of only about one-third the size of the full stripe basin RMS.

Figure 10a shows the quadrature-summed SMB and GMB basin misfit errors from the previous section, now using yearly-smoothed data. If GRACE had no satellite errors, Fig. 10b would show the signal-to-noise ratio (SNR) of the inversion technique, computed by dividing the basin RMS of the ideal "truth" signal by the summed misfit RMS errors. In this idealized case, the SNR increases everywhere as spatial resolution

- RMS errors. In this idealized case, the SNR increases everywhere as spatial resolution improves. The SNR is below 1.0 (errors are larger than the signal) for the interior basins (9–13) at a maximum degree of 60, but improve at higher resolutions (except interior basin 9, which has a very small "truth" signal size). Basin 7 has the lowest SNR of the coastal basins, regardless of maximum degree/order, and basin 4 among the next low-
- est, which is rather concerning since those are the basins nearest to the glaciers and in most need of accurate separation. However, using at least 90 × 90 (noiseless) data would result in a SNR greater than 5 for all coastal basins, which ought to be sufficient for the separation task.

To potentially separate the SMB from GMB, we assume a minimum desired stripeinclusive SNR of 2.0 – that is, the signal RMS must be at least twice the total error RMS of the stripes and basin misfit errors combined. In Fig. 10c, we show the maximum stripe basin RMS possible to meet this SNR > 2 goal, given the known basin misfit





errors and "truth" signals, using

$$SNR = \frac{RMS_{truth}}{\left(RMS_{SMB}^{2} + RMS_{GMB}^{2}\right) + RMS_{stripes}^{2}} \ge 2$$

and

$$\text{RMS}_{\text{stripes}} \leq \sqrt{0.25 \text{RMS}_{\text{truth}}^2 - (\text{RMS}_{\text{SMB}}^2 + \text{RMS}_{\text{GMB}}^2)}.$$

⁵ The maximum level of allowable stripes is largely independent of maximum degree/order. Because the "truth" RMS (which does not change with maximum degree/order) is significantly larger than the misfit errors, equation 4 is dominated by the first squared term under the radical, and only slightly altered by the second. As such, while the maximum allowable stripe RMS values are slightly lower for degree/order 60 in all basins, except for in basin 7 and the internal basins, the difference is trivial (and nearly invisible in the figure).

In comparison, the actual yearly-windowed inverted errors from the stripe-only simulations are large and grow larger quickly with increasing maximum degree/order (Fig. 11). While the non-glacier-overlapping SMB coastal basins of the 60 × 60 case are within the acceptable SNR > 2 ranges, by 120 × 120, the actual errors in all basins are much larger than needed to reach that target. In the critical glacier-overlapping basins, 4 and 7, the 90 × 90 errors are up to 9 times larger than the maximum allowable. The trouble is two-fold: first, the GRACE errors increase rapidly with degree, and second, the inversion technique preferentially dumps narrow signals, like stripes, into small basins, like the GMB basins, while "averaging through" more of the stripes over

larger basins.

(3)

(4)



5 Conclusions

A basin-based least squares inversion technique can theoretically be used to separate the SMB signal from the GMB signal, assuming sufficient spatial resolution of the input data. We found that a maximum degree of 60×60 is insufficient for this task, particularly

- ⁵ near Jakobshavn Glacier, but that a maximum degree of 90 × 90 can accomplish it with expected signal-to-noise ratios greater than 5 in all coastal SMB basins. Internal basins have smaller SNRs and may need to be combined into broader basins, if their far smaller mass distribution is to be exactly measured. The expected basin misfit errors are small enough to clearly discern fairly small interannual changes in the glacial signal.
- ¹⁰ A 90 × 90 spatial resolution is plausibly achievable for a future release of GRACE or a future satellite gravity mission.

Unfortunately, this is true in theory only. Realistically, when current GRACE noise estimates are included, a SNR > 2 is achievable for all coastal SMB basins except those where the simulated glaciers are located. In the glacier-overlapping basins, the

- ¹⁵ actual 90 × 90 GRACE RL05 errors are up to 9 times larger than those which would permit a SNR of 2. Since GRACE errors increase far faster with degree than the inversion method errors decline, this problem becomes worse as the maximum degree of GRACE increases, such that in the 120 × 120 case, the actual GRACE noise in all basins is too large to allow a SNR > 2. At higher desired SNR levels, the GRACE erman used to be brought down a similiar the further as the under and an the interm.
- ²⁰ rors would need to be brought down significantly further, as they depend on the inverse square of the target SNR.

Significant stripe reduction could potentially allow for SMB and GMB to be separated using the least squares inversion method. To do so, the GRACE noises would need to be reduced by approximately a factor of 9 at 90×90 or 30 at 120×120 . This noise

reduction would need to be accomplished without altering or removing the high-spatialresolution signal. We suspect that no post-processing scheme alone can currently accomplish this task, so the separation of the GMB from SMB using this method must await a new GRACE release or a future mission with smaller stripe errors.





Acknowledgements. The authors want to express great thanks to Himanshu Save at the Center for Space Research in Austin, TX, for the use of his 120×120 solutions and error estimates, as well as his helpful commentary on the research.

Support for this research was funded by the NASA GRACE Science Team program and by the NASA New (Early Career) Investigator Program in Earth Science.

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Figure 1. Impact of spatial resolution on the apparent shape and amplitude of a 1 cm signal over Helheim Glacier, given the apriori weight distribution in (e). Maximum degrees/orders are limited to (a) 60×60 , (b) 90×90 , (c) 120×120 , (d) 180×180 , and (e) the original 1×1 grid cells.







Figure 2. SMB and GMB basins for Greenland. GMB basin **(J)**akobshavn overlaps with SMB basin 7, while **(H)**elheim and **(K)**angerdlugssuaq overlap basin 4. White grid cells show the central GMB cell, while black are lesser-weight GMB cells.



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Figure 3. An example of the random part of the trend signal made using spatially-correlated randomization with 3° smoothing, before (a) and after (b) applying the RMS-based amplitude weighting.











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Figure 7. Average RMS difference from "truth" per basin for the SMB-only simulations, for data of increasing maximum degree, (a) in the SMB basins and (b) in the glacial basins.





Figure 8. Average RMS difference from truth per basin for the GMB-only simulations, for data of increasing maximum degree, (a) in SMB basins and (b) in the glacial basins.





Figure 9. Visualization of the spread caused by the SMB and GMB estimated basin misfit errors, at the three glaciers, for maximum degrees (a) 60×60 , (b) 90×90 , (c) 120×120 , and (d) 180×180 . Solid black lines denote the "truth" simulated signal per basin.







Figure 10. (a) Estimated errors caused by misfits between the SMB and GMB input data and the defined basin weights, **(b)** Signal-to-Noise Ratio using only the misfit errors, and **(c)** the maximum stripe level allowable to result in a SNR > 2 when stripes are included. All figures use yearly-windowed data.





Figure 11. Comparison of maximum allowed stripes (green boxes) based on SNR > 2, and the actual estimated stripes per basin (red lines) for the **(a)** 60×60 , **(b)** 90×90 , and **(c)** 120×120 cases. For **(b)**, the actual stripe signal is from the 96×96 GRACE runs. The orange dashed lines denote the actual stripes reduced by the factors of 3.5, 9, and 30, as needed to fall within the allowed values. All figures use yearly-windowed data.



