

## ***Interactive comment on “Combining damage and fracture mechanics to model calving” by J. Krug et al.***

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### **General comments**

This paper introduces a new framework for modeling iceberg calving in a numerical ice sheet model by combining principles of viscous damage mechanics and elastic fracture mechanics. A viscous damage model is formulated for characterizing the weakening of the ice surface associated with surface crevassing. A scalar damage variable is then advected with the flow, such that the history of fracturing is carried along with the flow while simultaneously enhancing the flow locally. As damage is advected, it evolves depending on the state of stress in the ice according to a simple evolution equation. Once a threshold level of damage is reached, an analytical fracture model is then applied to an idealized brittle crack that penetrates from the ice surface down to

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the depth at which the computed damage is equal to the threshold or critical damage. If the fracture model predicts propagation of the crack down to a depth below sea level, then calving is prescribed. This new framework is applied in two-dimensional flowline simulations of the evolution of Helheim glacier, with the background ice flow treated using the Stokes equations with power-law (Glen-type) rheology.

I think this approach holds promise for modeling calving in ice sheet models. The work is well motivated and the manuscript is generally well written. The introduction of analytical elastic fracture calculations to a problem that is modeled over viscous timescales is a nice approach to accounting for a short-timescale process such as calving. I commend the authors for carrying out a range of sensitivity analyses to determine the influence of the newly-introduced model parameters on the results. I have only one potentially serious concern regarding the damage model formulation, though it may just be a misunderstanding on my part. The remainder of my comments are minor recommendations aimed at clarifying a few issues, placing some aspects of the work in broader context, and pointing out some issues that might be explored in future work if not addressed at present.

### **Specific comments**

My biggest potential concern surrounds the issue of damage and hydrostatic pressure in the ice. As such, I think that some explicit mention needs to be made of how the pressure term is being handled in the ice flow model. This is because the effective stress  $\tilde{\sigma} = \sigma / (1 - D)$  should be substituted for the Cauchy stress (not just the deviatoric stress) in the Stokes equations, with the result that the damage scalar should be mapped onto the pressure term as well. This can take different forms depending on whether an approximation to the Stokes equations is adopted, but damage should affect the pressure in addition to the viscosity, in the form of something like  $\tilde{p} = p / (1 - D)$ , as pointed out by *Pralong and Funk* (2005). It's not clear whether you accounted for this, or whether you only modified the viscosity term (Section 2.2.2). For that matter,

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it's unclear whether *Pralong and Funk* (2005) accounted for this dependence either (or whether it drops out somehow), as their Equation 26 appears to contain the original (unmodified) pressure rather than the effective (damage-dependent) pressure. Maybe I'm missing something here, but I'm concerned that perhaps the pressure is being treated incorrectly.

Another issue that relates more to the discussion and context of this work is the description of damage mechanics generally. It is incorrect (or at best misleading) to state that damage mechanics is only applicable up to the point where a macroscopic fracture first forms. Though the paper does not exactly say this, the reader might imply that damage mechanics only applies to the realm of slow, sub-critical crack growth. Damage mechanics is actually a much more general and versatile framework; it can be applied over viscous and elastic timescales, and is widely used to model both the initiation of a macroscopic fracture *and* the subsequent propagation of this fracture (for a review, see *Bažant and Jirásek*, 2002). Damage mechanics is especially appropriate for modeling fracture propagation in heterogeneous materials (such as glacier ice), for which the crack "tip" may be ill-defined due to an inherent zone of microcracking ahead of the traction-free crack. Damage as a notion of the "smeared" influence of fractures is just as appropriate for this type of fracture propagation as it is for the coalescence of a fracture to begin with. A great example of this in a glaciological context comes from the seismic data of *Bassis et al.* (2007), which show that a propagating rift tip is surrounded by a diffuse zone of fracturing. In a modeling context analogous to crevasse propagation, *Borstad and McClung* (2011) used damage mechanics to simulate elastic tensile fracture propagation in cohesive snow. My point here is to caution the authors against defining damage mechanics narrowly, especially since it is still a very new concept in the field of glaciology and might find applicability to a range of problems over a range of timescales.

Finally, how would the role of crack tip shielding influence the calving results? The stress intensity factor is calculated for the case of a lone fracture, or at least a fracture

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distant from any neighbors. Yet this stress intensity factor is calculated everywhere that the damage reaches the critical level, as if a field of crevasses existed (which is more likely to be the case). In a field of closely-spaced crevasses, the stress intensity factor at each crack tip is reduced due to the influence of stress shielding by neighboring cracks. This should make it more difficult for a single fracture in a field of crevasses to propagate down to sea level. There are a number of ways that this could be represented or accounted for in your model, and this would have implications for the appropriate values of the other model parameters necessary to pass the "sanity check" of your model. It might be worth at least discussing this issue in the text.

### Line-by-line Comments

- p. 1632, line 21 (and elsewhere): Throughout the manuscript the word "important" is used in places where I think you are referring to a quantitative magnitude, or something being "large" rather than qualitatively important in the sense of being meaningful or significant. You might want to look at where you are using this word to make sure the reader is not confused or misled.
- p. 1633, line 28: the van der Veen references are formally about the propagation of single surface or basal crevasses, and are not strictly about calving events
- p. 1637, lines 18-20: See specific comment above about the broader applicability of damage mechanics that can also include macroscopic fracture propagation. In other words, damage does not have to be limited to long-timescale viscous deformation, nor does it cease to become applicable once a macroscopic crack forms. I have no problem with the way you use damage mechanics in this study, but I think that readers should know that there is a broader context in which damage mechanics can also be applied.
- p. 1638, lines 5-7: "Damage" is not mentioned anywhere in the work of *Rist*

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*et al.* (1999), moreover I don't see how your assertion here is supported by this reference.

- p. 1638, line 17: does not the effective stress enter the Stokes equations in general, and not just the rheological law of Eq. 3?
- p. 1638, line 26: I think that the use of the variable “B” for the damage enhancement factor is a bit unfortunate, since some ice flow models use “B” to represent the ice rigidity (related to the rate factor), especially since damage and ice rigidity are often written next to each other (e.g.  $(1 - D)B$  in *Borstad et al.*, 2013, also in this journal). Is there another variable that could be used here?
- p. 1639, line 19: do you mean “splaying” crevasses?
- p. 1639, line 25: “envelope”
- p. 1639: for a flowline model, a maximum principal stress criterion seems like a good choice. However, a multiaxial criterion, such as von Mises, could still be used to represent the scalar level of stress for which fractures first appear (indeed the von Mises criterion reduces to the maximum principal stress criterion for a state of uniaxial tension). *Vaughan* (1993) found that a von Mises criterion corresponded well with the pattern of surface crevasse occurrence on many glaciers. As *Rist et al.* (1999) points out, crevasses indeed tend to *open* normal to the direction of maximum tensile stress even if the actual state of stress is multiaxial (you seem to be implying on line 18 that you wish to model crevasses opening under uniaxial tensile stress). Moreover, the fact that von Mises, or any other criterion, is often used for plasticity is irrelevant for whether it is physically applicable in another context.
- p. 1641, line 10: similar comment, the ice does not need to undergo pure (uniaxial) tension for damage to evolve, it just needs the maximum principal stress (in a multiaxial stress state) to exceed the threshold stress.

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- p. 1641, lines 17-18: a reference is needed here to support the claim about calving timescales approaching the speed of sound.
- p. 1643, lines 2-5: is it correct that the weight function method was calculated using vertical coordinates corresponding to the finite element mesh? If your mesh had a vertical resolution of 5 m near the surface, and if the typical depth of the critical damage contour was 5–15 m, then does this mean that only 1–3 vertices was used to calculate the weight function? This seems a bit coarse, and I would doubt that the results would be mesh-independent.
- p. 1643, line 15: “notched”
- p. 1645, line 9: you might substitute “becomes filled with water” for “is filled by water” as the latter makes it sound like the crevasse is already filled with water before it propagates down to sea level.
- p. 1646, line 21: why not specify a vertical temperature profile? Wouldn't that be more physically realistic, since the temperature at the base should be warmer than the temperature at the surface?
- Equation 17: can you be a bit more specific about how and where this friction coefficient is applied in the model?
- What is the time step size in the model? Do the results (calving event size or frequency) have any dependence on the step size?
- p. 1648, lines 23-24: the stress threshold seems like it should be physically related to (if not equivalent to) the tensile strength of snow, firn, or ice, depending on the nature of the glacier surface. *Pralong and Funk* (2005) considered hanging alpine glaciers, for which the tensile strength of the snow or firn at the surface, which is likely to be in the range of 10–50 kPa (*Borstad*, 2011), would be appropriate for the stress threshold. *Vaughan* (1993) inferred tensile strength

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values in the range 100-400 kPa from analyzing strain rate data in the vicinity of crevasses. Since an approximate diagonal in Figure 7 defines a set of acceptable model parameters (threshold stress and damage enhancement factor), it might be possible to further constrain these parameters from a knowledge of the properties of the firn layer where the fractures first originate. Do you have any information about the depth and density of the firn layer for Helheim? Even seasonal snow can have tensile strength reaching 0.1 MPa, so it might be possible to further constrain your considered range of threshold stresses to something like 0.1–0.2 MPa.

- p. 1649, lines 5-6: I'm confused by this statement, can you clarify? Are you trying to keep  $B \times \chi$  within some range?
- p. 1650, line 20: The "steady" advance...
- p. 1651, line 20: is it really appropriate to report all of these parameters to 3 significant digits?
- Figure 9: it's clear that the calving event sizes do not fit a gaussian pdf, so why plot them as such? It would seem better to first determine what kind of distribution (e.g. log-normal, Poisson, etc.) best fits the results, and then plot the appropriate cdf. This could facilitate comparison with observational data in the future. For example, if observations indicate that calving event sizes follow a log-normal distribution, then you would want a model that also produced calving event sizes that follow such a distribution.

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