Response to Referees of tc-2014-25

Terence J. Hughes, in behalf of all authors of "Quantifying the Jakobshavn Effect: Jakobshavn Isbrae, Greenland, compared to Byrd Glacier, Antarctica", 28 October 2014.

Response to Referee #1.

Referee #1 states, "My main concern regarding the methods centered around Figure 5 (which is too cramped to be easily readable) and the relative lack of explanation of the geometric approach. While the resulting equations are explored in depth, there is no clear example of how a geometric force balance is supposed to work. (Perhaps a simpler case, like an unbuttressed ice front, would be helpful.)"

I propose to address this concern by requesting an enlarged version of Figure 5 in the revised version published in *The Cryosphere* and adding an appendix that derives the geometrical force balance for the two simple end members, linear sheet flow and linear shelf flow, each using only two triangles.

Referee #1 states, "My main concern regarding the presentation of results is that the discussion remains largely in terms of the parameters associated with the geometrical approach," concluding with, "(W)hat can the geometric approach give me that I can't get from continuum mechanics? I'd be very interested to read a revised version that was more clear on this point."

The geometric approach provides a visual understanding of the force balance that is elusive when using the purely mathematical approach based on integrating the Navier-Stokes equations for the force (momentum) balance. For example, the force driving gravitational spreading for linear longitudinal flow can be subdivided using simple triangles and a rectangle into components that are resisted by a downstream compressive force, an upstream tensile force, a basal drag force, and a side drag force, and the deviator stresses that are associated with these resisting forces. The components of the gravitational force are linked to reductions of ice-bed coupling, due to an increasing thawed bed fraction for sheet flow, an increasing floating ice fraction for stream flow, and an increasing unbuttressed ice fraction for shelf flow. These distinctions are not as readily extracted from continuum mechanics.

Referee #1 lists several specific comments. Here are my replies, linked to page numbers.

Page 2044. I will include ice-sheet vulnerability in the Amundsen Sea Embayment.

Page 2046. I will state all these conditions are treated by ISSM.

Page 2048. Our values of thawed fraction f are calculated directly from Equation (3) using values of τ_O and τ_S obtained from Figure 2, for a frozen bed and a thawed bed, respectively, for which the flow characteristics are internal shear through the whole ice column and basal sliding. We assume $f = 0.5$ separates ice motion dominated by internal shear from ice motion dominated by basal sliding. It's a "broad brush" assumption.

Page 2048. I can locate Byrd Glacier in Figure 1, but it would clutter the figure so I would prefer not to. Most readers will know where Byrd Glacier is located.

Page 2049. Yes, flow is in the negative x direction.

Page 2049. No. Equation (2) is differentiated for constant τ_0 in this case.

Page 2050. Thank you. I'll try to eliminate the confusion. Yes, ice thickness and elevation are observed quantities in this paper.

Page 2051. I'll improve the explanation for two yield criteria.

Page 2053. I should have written "the effect of the actual reduction of A near the bed…"

Page 2054. Equation (8) was derived by Paterson (1994). His derivation also appears in Appendix O of *Holistic Ice Sheet Modeling* (Hughes, 2012).

Page 2056. "Tension" causes converging flow and "compression" causes diverging flow in continuum mechanics. For linear flow, tension thins ice and compression thickens ice in our treatment.

Page 2058. I'll ask to have Figure 5 enlarged.

Page 2060. I'll rework this paragraph. The basic idea is the ability of basal water to "float" overlying ice doesn't stop at the ice-shelf grounding line.

Page 2061. I mean replacing these gradients over distance Δx with abrupt finite changes at x. This assumes the gradients are small enough to be ignored compared with h_B and ϕ. I don't know if that assumption is justified.

Page 2062. I assume the shallow ice approximation applies to ice elevations where ice is grounded. The actual ice elevation must take into account places where ice is floating, which would require a lower ice elevation. The actual ice elevation is a balance of these two, depending on the floating fraction of ice. This is an assumption, and I will so state.

Page 2067. I'll add the Walker et al. (2008) citation if Reviewer #1 provides it for me.

Page 2071. Buttressing fraction f_B, like floating fraction ϕ, applies along x up ice streams, whereas ϕ_0 applies at ice-shelf grounding lines. I need to make that clear.

Page 2072. "Buttressing" is correct, see Equation (53).

Page 2073. An ice shelf can be grounded along its sides in an embayment and locally where there are ice rumples and ice rises. The more of these, the less "floating" the ice shelf is.

Page 2075. Both the Thomas and Zwally effects allow $\phi_B = 0$ independently, which removes all constraints imposed in the Jakobshavn Effect, see Table 1. They are old friends. This just my way of taking a final bow to them, since it is probably my last paper.

Page 2076. I’ll add reference Smith, B.E., Fricker, H.A., Joughin, I.A., and Tulaczyk, S. An inventory of active subglacial lakes in Antarctica detected by ICESat (2003-2008), Journal of Glaciology, 55(192), 573-595, 2009.

Page 2089. Great surface roughness not sufficiently smoothed by a running mean is the main case for $\phi > 1$. Aitbala Sargent applied a dissection method to suppress those values, as noted in the text.

Referee #1 lists some technical comments, also by page.

Page 2049. Replacing I with i would require retyping too many equations. May we keep the equations as they are?

Page 2053. We put A in the denominator, following the pioneering work of John Nye.

Page 2056. The arrow means “approaches”. Isn’t that a common usage?

Response to Referee #2.

In his General Appreciation, Referee #2 writes, referring to basal water thickness or effective pressure in treating stream flow using continuum mechanics, "Hughes et al. go somewhat further by also wishing to relate ϕ to the horizontal stresses. In order to explain this, I need to abandon the authors' notation, which I don't think is helpful, and so far as I can see, has led to an error." Our floating fraction of ice in stream flow is ϕ, taken as a measure of ice-bed coupling. Referee #2 then proceeds with his demonstration. Here I must assume his "Cauchy stress" is the spherical hydrostatic or lithostatic stress, with applied stresses being the sum of spherical stresses and deviator stresses that "deviate" from the spherical state of stress and are therefore directional.

Here is my response. Spherical stresses in ice (lithostatic pressure) and water (hydrostatic pressure) are exerted in all directions, and therefore in the direction of ice flow in which gravitational flow is resisted by deviator stresses. For stream flow, where ϕ is used, gravitational stresses in horizontal direction x of ice flow are represented geometrically in Figure 5, using triangles and a rectangle. The gravitational driving force along x is the *area* of the largest triangles in Figure 5. These triangles consist of smaller triangles and a rectangle in which the gravitational force at x is resisted by an upstream tensile "pulling" force, a downstream compressive "pushing" force, a basal shear force, and a side shear force, which are respectively linked to deviator stresses σ_T, σ_C, τ_0, and τ_S. The compressive force is much larger than the tensile force, as referee #2 notes, and this is shown in the middle panel of Figure 5, where the area inside the bold lines is linked to σ_C and the much smaller remaining area is linked to σ_T. Within the bold lines, the force linked to σ_C is shown to be composed of forces linked to τ_0, τ_S, and σ_W. In addition, the combined forces linked to σ_T and σ_C balance the total gravitational force given by the triangular area linked to basal ice pressure P_I. Then $\sigma_T + \sigma_C = P_I$, as seen in Equations (12) and (13), with $\sigma_C >> \sigma_T$, as shown by the areas in Figure 5. In addition, flotation stress $\sigma_F = \sigma_T + \sigma_W$, see Equation (15), where σ_W is a water stress linked to the basal water pressure. None of this conflicts with the continuum force balance along x presented by referee #2. Referring to Equations (12) through (18), referee #2 states, "I can at least accept these equations in principle."

Referee #2 was confused regarding compressive stress σ_C, finally concluding it is a Cauchy stress (what I would call a spherical stress). It isn't. It represents all the downstream resistance to ice flow. This resistance includes side drag and basal drag along the downstream part of the ice stream, plus buttressing by an ice shelf at its grounding line with the ice stream when the ice shelf is confined in an embayment and/or locally pinned to the sea floor to produce ice rises and ice rumples on the ice surface. In the absence of an ice shelf, buttressing is provided by the vertical wall of water at the calving front where ice would otherwise become afloat. Equation (43) relates the compressive force to all these downstream resisting forces, all of which produce deviator stresses that resist ice flow.

Referee #2 seems to be confused on one point, however. He states, "All the triangles are sketches of the way the Cauchy stresses vary with depth." He must think the sloping sides of the right-triangles represent variations with depth. No. Only the *areas* of these triangles and the rectangle are important, and represent components of the longitudinal gravitational force. Only ice thickness h_I appears in Equations (12) through (18), with no variations along z.

Referee #2 thinks Equation (20) is in error, so he "can't really deal with anything below (20)". However, Equation (20) is merely Equation (19) divided by the vertically averaged ice pressure (after + replaces – for the last term on the upper line). Equation (19) is the standard equation for the linear force balance along x, with Equations (16) – (18) substituted for basal shear stress τ_O, side shear stress τ_S, and longitudinal force gradient $\partial(\sigma_F h_I)/\partial x$ (referee #2 calls this last term a "stress gradient" but it is a force gradient). There is no error in Equation (20), so Referee #2 could have continued. Equation (20) accounts for all the components of the longitudinal gravitational force in Figure 5. It's perfectly valid.

Referee #2 concludes by writing, "This paper is close to being unreviewable." He thinks this might be because we attempt to make it "holistic" by combining sheet, stream, and shelf flow. We do this by using a thawed bed fraction f for sheet flow, a floating ice fraction ϕ for stream flow, and an unbuttressed fraction ϕ_O at the ice-shelf grounding line for shelf flow, all of which are used to quantify reduced ice-bed coupling. My hope was to make these processes clear, using geometry that can be visualized with triangles and a rectangle, and thereby avoid integrating the Navier-Stokes equations for the force balance. Deriving and using these equations can puzzle readers unfamiliar with calculus, and there is no visual representation to show how this force balance is done using continuum mechanics.

In his Minor Points, referee #2 gives 47 examples. I will respond in that order.

1. Upstream ice is pulled all the way to the ice divide if $\phi > 0$ all the way to the ice divide.

2. I agree.

3. I'll replace "signs" with "parts".

4. I agree.

5. Both approaches are holistic.

6. Ice elevation and thickness are reduced when ice-bed coupling is reduced.

7. Replace "This" with "Our geometrical approach to the force balance".

8. Reference *Ice Sheets* (Hughes, 1998, Chapter 3).

9. The thawed fraction is interconnected for $f > 0.5$ and disconnected for $f < 0.5$, based on a change in surface ice velocity.

10. I'll replace "boot-strapping" with "iterative".

11. I'll add "Ice is thinner under domes than over basins, so basal heat is conducted faster to the surface. Frictional heat in ice converging on ice streams should thaw the bed."

12. I'll add "Ice thins when a frozen bed thaws, as measured by thawed bed fraction f." Floating ice fraction ϕ doesn't enter into sheet flow.

13. These constructs show how ice-bed coupling is weakened when basal shear stress τ_O has two values, higher for strong coupling without sliding and lower for weaker coupling with sliding.

14. I'll begin this paragraph with "Reduced ice-bed coupling when a frozen bed thaws lowers ice elevations because τ_S replaces τ_F in Equation (3). These two stresses can be linked to separate yield stresses, as shown in Figure 2."

15. Replace "move up" and "move into" with "increasingly include".

16. I'll replace "This linkage provides the means" with "Bed topography provides locations".

17. I'll begin with your sentence.

18. I'll include units of Pa s for viscosity.

19. That was for sheet flow. This is stream flow.

20. I'll add "and therefore dilates".

21. I'll replace "water" with "basal water".

22. I'll replace "condition" with "flotation condition".

23. All stresses here, including σ_C, are resisting stresses because they are all directional stresses that resist gravitational flow in the x direction.

24. P_W^* is directional because it varies with floating ice fraction ϕ, which varies along x.

25. There is a physical basis for h_W. It is linked to P_W^* and ϕ in Equation (10), both of which are real.

26. Figure 5 shows how h_W is measured.

27. How do we measure them directly? See sections 5 and 6 to see how ϕ is measured and Equations (12)-(17) to see how these stresses are then calculated.

28. "Expressions (1)" aren't even mentioned here. Apart from that, I stand behind what I wrote in lines 15-27.

29. All stresses are resisting stresses varying along x, and therefore deviatoric in nature.

30. Water pressure applies a horizontal stress *normal* to a flat bed.

31. I can't put it better than I did.

32. The triangles show components of the gravitational driving stress that acts in the x direction. Triangles linked to τ_O and τ_S "point" in opposite directions, see bottom panel of Figure 5.

33. Before the last term on the upper line, the – should be +. That was a typing mistake. Correct it and Equation (20) is the standard equation for linear flow everyone uses.

34. I'll rewrite this sentence.

35. It allows me to ignore basal stresses linked to bed roughness if the bed isn't too rough, see Hughes (2012, Appendices E and P).

36. These two thickness become one ice elevation at the ice surface because the differences in ice-bed coupling at the base blend to become a single ice elevation at the surface.

37. Don't you mean Equation (30) on line 5?

38. Equation (31) is correct. See Hughes (2012, Appendix A, Equation (33) when $ij = xx$).

39. Equation (20) is correct.

40. If I could sit down with you, I could convince you.

41. An ice shelf is coupled to the bed along the sides of its confining embayment, around its surface ice rises, and under its surface ice rumples.

42. What hypothesis?

43. This analysis is best applied to a broad ice shelf of the kind shown in Figure 7. Since we did a flowband/flowline analysis for sheet, stream, and shelf flow to Byrd Glacier and Jakobshavn Isbrae, the equations here become greatly simplified.

44. Not much data for Byrd Glacier and Jakobshavn Isbrae to compare with other data.

45. Values of $\phi > 1$ are rare in our analysis, and can be linked to surface roughness we can't easily smooth.

46. By "almost fully buttressed" we mean ϕ_O we measure is very small.

47. The inset map for $n = 3$ shows where $(\varepsilon/\varepsilon_O) = 1$ is located.