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Interactive comment on "Quantifying the Jakobshavn Effect: Jakobshavn Isbrae, Greenland, compared to Byrd Glacier, Antarctica" *by* T. Hughes et al.

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1 General Appreciation

This paper addresses a fundamental issue of glacier mechanics, seeking to understand how the mechanical behavior of grounded ice is affected by the basal resistance. This is a problem in need of insight, and the authors adopt a different tack from most of their contemporaries.

The basic idea is to express the mechanics in terms of a parameter ϕ, which in some way is correlated with the amount of water at the bed, and thereby affects the basal resistance. This in itself is no different from many other works, who express the quantity and effect of basal water in terms of quantities such as the mean water thickness, or the effective pressure.

However, Hughes et al. go somewhat further by also wishing to relate ϕ to the horizontal stresses. In order to explain this, I need to abandon the authors notation, which I don't think is helpful, and so far as I can see, has led to an error.

A fundamental distinction in all glacier mechanics lies between the Cauchy stress and the deviator stress. The Cauchy stress enters the momentum balance equations, while at least for incompressible materials such as ice, the deviator stress is directly related to the shear stress. Generally these are notated differently, and there is value in this, because special rules apply to adding and differencing Cauchy and deviator stresses. Adding two Cauchy stresses produces another Cauchy stress; substracting them can produce another Cauchy stress or a deviator stress, depending upon the details. Adding or differencing a deviator stress to a Cauchy stress, if it produces a well-defined quantity, produces a Caucy stress, while adding or differencing deviator stresses will not in general produce a Cauchy stress. All this is well-known to glaciologists who follow these rules automatically, but it is worth pointing out that the manipulation of these quantities requires some care.

Hughes et al. (HEA) discuss horizontally acting stresses in terms of a compressive stress σ_C and a tensile stress σ_T. This confused me for some time, until I concluded, I hope not mistakenly, that the compressive stress was a Cauchy stress and the tensile stress a deviatoric stress. The horizontal components of these stresses are in general of this nature. (HEA make things very difficult for readers by not adopting a consistent sign convention for stresses, e.g. compressive negative, tensile positive).

In glacier mechanics, excluding solutions obtained by solving the Stokes equations, the mean value of the normal Cauchy stress σ_{zz}, which is cryostatic, is given by $\sigma_{zz} = -\frac{1}{2}\rho_i g h_I$,where h_I is the depth.

Not very far in from the calving front, above a frictionless bed, the horizontally acting

Cauchy stress is $\sigma_{xx} = -\frac{1}{2}\frac{\rho_i^2}{\rho_w} g h_I$,and then since by definition, in plane flow, the deviatoric stress τ_{xx} is given by $2\tau_{xx} = \sigma_{xx} - \sigma_{zz}$,the mean value is given by the well-known formula due to Weertman (1957) $2\tau_{xx} = \frac{1}{2}\left(1 - \frac{\rho_i}{\rho_w}\right)\rho_i g h_I$.We can see that Hughes's et al. $\sigma_T = 2\tau_{xx}$ at the calving front

HEA then make the daring assumption that, upstream of the grounding line, where frictional resistance has affected the horizontal stress, that

$$\sigma_{xx} = -\frac{\phi^2}{2}\frac{\rho_i^2}{\rho_w} g h_I - \frac{1}{2}\left(1 - \phi^2\right)\rho_i g h_I, \tag{1}$$

where one might call the first term on the right-hand side the 'water term' and the second term the 'ice term'. It seems that HEA are saying that even under grounded ice, the presence of water can be accounted for in the water term, and that this exerts a horizontal Cauchy stress. Certainly, this formula is widely accepted at the calving front ($\phi = 1$) and above frozen bed ($\phi = 0$).

This rearranges to $\sigma_{xx} = -\frac{1}{2}\left(\phi^2\frac{\rho_i}{\rho_w} + \left(1 - \phi^2\right)\right)\rho_i g h_I$,and then, corresponding to Hughes et al. (12) $2\tau_{xx} = \frac{\phi^2}{2}\left(1 - \frac{\rho_i}{\rho_w}\right)\rho_i g h_I$,(at least this is the only way I can derive equation 12). I call this an assumption because I don't see any justification for it - which isn't to say it isn't right or useful. All the triangles are sketches of the way the Cauchy stresses vary with depth, except that I don't see why σ_T or equally τ_{xx} should vary with depth in this linear fashion.

I can at least accept the equations in principle until equation (20), noting the defintion of σ_F on line 2059:11. Here it is clear that a deviatoric stress σ_T is being combined with a Cauchy-type stress σ_W. Equation (20) looks like the plane-flow version of the stress equations used by Muszynski and Birchfield and by Van der Veen in the1980s., but here we have a Cauchy stress gradient term $\Delta\left(\sigma_F h_i\right)/\Delta x$ and no deviator stress gradient term as obtained by the authors just mentioned.

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In view of what I have said above, I believe that somewhere, probably in the subtraction/addition of the force triangles, an error has been made. There is nothing in the 'calculus' of triangular force balance that prevents it from being written down algebraically. I can't really deal with anything below (20) in view of my conclusion, though I have contributed some minor queries.

This paper is close to being unreviewable. It is apallingly difficult to break it down into its constituent parts, which I suppose that this is an inevitable consequence of its claim to be "holistic". The equation layout is not good, for which I tend to blame the typesetting software of the journal. If the authors believe that they are correct, I suggest that they rederive (20) carefully, using algebra, and explain how a stress with Cauchy magnitudes (which are huge compared with deviator stresses) enters into this equation.

2 Minor Points

1. A major irritation in reviewing this paper was the swapping between differential notation e.g. $\mathrm{d}h/\mathrm{d}x$ and difference notation e.g. $\Delta h/\Delta x$.What is the point of this? There are even examples of the limit process being shown.2045:10 "literally pull" - true, but needs to be modified with a length scale. Does it pull significantly all the way to the divide? How far?
2. 2046:9 Paragraph break would be good here.
3. 2046:10 "show signs of the Jakobshavn effect". The Jakobshavn Effect is described above as "a group of positive feedback mechanisms". If ice-streams are only showing part of it, i.e. individual mechanism that have been discussed before, you cannot say they are "showing signs" of the JE.
4. 2047:1 Para break would be good here.

5. 2047:1 “Our alternative treatment provided here is holistic in the sense it provides continuity from sheet flow to stream flow to shelf flow.”. But so do full Stokes, Blatter, and some vertically integrated. Are you drawing a distinction between your approach and these approaches?
6. 2047:4 "Our approach uses ice-bed coupling as the major contributor to ice thickness, which we measure directly by radar sounding". Appears confused - thickness is related to mass, while ice-bed coupling is related to momentum. Requires clarification.
7. 2047:5 "avoids using partial differential equations" . I see lots of ordinary differential equations in the paper, what is the point of this statement?
8. 2048:7. "frozen bed....." reference required.
9. 2048: 13-15 You should state whether these thawed bed fractions are an arbitrary classification, or based on data or modelling?
10. 2049: "boot-strapping" - "iterative". ’Boot-strapping’ has a more specific statistical meaning.
11. 2050: 10-13. How do these authors reach these conclusions?
12. 2050: 5. I don’t get this - are you saying that the shear stress is the same in the thawed region no matter what the effective pressure is, and, that the water pressure/effective pressure only affects the horizontal Cauchy stress σ_{xx}? Does ϕ vary in the thawed patches? Is this a convenient assumption, or do you have a suggestion as to why this might happen?
13. 2051: 14-21. What is the point of the elaborate geometrical constructs in Figure 2. How do they prove what you want to prove o p. 2052?
14. 2052: 12-21 I don’t understand the point of this paragraph.

15. 2052: 24. change "move up" to "expand" etc. The patches are not migrating.
16. 2053: 3-4 "This linkage....". I disagree, it is simply the thawed fraction which reduces basal traction, as shown by your eqn. (3).
17. 2053. 23-26. You should state upfront that changing n is a mathematical convenience to represent enhanced shear near the bed.
18. 2054 11-21. Pa s is a more commonly used unit for viscosity.
19. 2055 14-22 Haven't you previously (in this paper) argued against the ice being unfrozen on top of hills/riegels.
20. 2055 25. "supersaturates" - not a term usually applied to till. Do you mean "dilated"?
21. 2056. 6 "thickness of ice that floats in water". Which water?
22. 2056; 10. "This condition" - which sort of condition, a logical condition, or an environmental condition. As far as I can see, h_F is a notional quantity, so how can this statement not be true - all you've done so far it seems is define h$_F$.
23. 2056 18-20. I take exception to this statement. It is clear from definitions below that σ_C is a Cauchy stress while the remainder are deviatoric stresses (or resistive stresses in the Whillans/Van der Veen mechanical description). Until a reader has worked this out, the whole scheme is incomprehensible.
24. 2057. 15. I would disagree with the sentence "P$^*_W$ is an effective basal water pressure .... caused by h_W". The reason is that I think h_W is a notional quantity - how is it measured? - so how can anything be caused by it?
25. 2058. I get nothing from the development on 2058. I gave up on it, because I think that h_w is a notional quantity. As I say at the top (equation 1), I think that

the expression for σ_{xx} is a hypothesis, and there is no need to go through the elaborate argument on 2058. It's just manipulation of notional quantities as far as I can see.

26. 2058 21-25. Pretty much the same argument applies to the triangles. Most glaciologists are familiar with similar derivations of the driving stress relations for glaciers and icebergs (Nye and Weertman respectively). The difference between those derivations and the present case is that they deal with measurable quantities. h_W and its offspring are not measureable.
27. 2059. 12 "These are real stresses". How do you measure them directly?
28. 2059 15-27. This reads as an argument that because the expressions (1) is true at the calving front it must be true elsewhere, with the notional h_W.
29. 2059 . Equation (15) looks as though you are adding a Cauchy stress to a deviator stress to define σ_F, which has to produce another Cauchy stress. You need to clarify what σ_F is.
30. 2060. whole page. Water pressure does not apply a horizontal stress across a flat bed.
31. 2060. Last few lines. I don't understand this. Why instead of asking the reader to visually subtract (or add) as the case might be eight triangles, don't you write the force balance down. I don't see how you can have eight triangles anyhow?
32. 2060 last few lines It's also beyond confusing having the triangles point in the same direction, when presumably some of the forces are acting in opposite directions?
33. 2061 The upshot of this is that I cannot believe equation (20) is correct, having a Cauchy stress gradient in a relationship where every other glaciologist has

deviatoric or resistive stresses. I think that there must be a mistake in subtracting the triangles.

34. 2061 20 "Putting...". Not sure that this sentence is grammatical.

35. 2061. I see that you might want to approxiate the bed as a staircase, but this is going to increase stresses locally. It seems that this approximation allows you to introduce notional normal stresses that you do not permit to exceed the yield stress. Where has this got you?

36. 2061 18 I have no idea how you can have grounded and 'floating' thicknesses that are different. Can these two geometries be measured?

37. 2064 8 confirmation that σ_T is a deviator stress.

38. 2064 Eqn (31). This is wrong, it must have a cross-term $\dot{\varepsilon}_{xx}\dot{\varepsilon}_{yy}$ stemming from the inclusion of $\dot{\varepsilon}_{zz}$ in the invariant definition.

39. 2064 20. Since I don't understand (20) or rather don't believe it, I'm unconvinced by eqn. (32).

40. I am entirely bamboozled by pages 2065 - 2067.

41. 2067. 10 Why do we need a whole section on ice-bed uncoupling for shelf flow? There is none, surely? Presumably ϕ has a completely different meaning in this section - it looks as though it's a buttressing factor? I'm not sure this section helps at all with the fundamental hypothesis, and I suspect it's entirely possible to do the Hughes type analysis upstream of the grounding line by just varying this buttressing factor.

42. 2070. Equation on line 4. This looks as though it confirms my hypothesis regarding σ_{xx}in (1) .

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43. 2071-5. I haven't looked at this in detail. It looks like some kind of attempt to estimate the buttressing factor. I don't see any particular reason to believe it (no comparison with numerical solutions) and in any case it is not pertinent to the main point of the paper, which is the estimation of ϕ in the grounded part of the ice-sheet.

44. 2078. 16-24. There are a lot of data regarding grounding line location - how do these compare?

45. Figures 11 12, 16. Are all your values ϕ no greater than one? What does it mean if they are?

46. 2079 9 Be more precise about what "fully buttressed" means. How about using the Schoof flux formula (easily converted to a velocity formula) to estimate the buttressing parameter (I suspect that HEA and Schoof's quantities are the same at the grounding line)

47. 2104: Figure 2. x-axis requires some labelling, though I suppose it's obvious where '1' should be.

Interactive comment on The Cryosphere Discuss., 8, 2043, 2014.