

We thank the two reviewers for their careful reading of our manuscript and their constructive comments.

We revised our paper in order to account for their remarks and we provide below detailed answers to the various issues raised by the reviewers and corrections in the manuscript are highlighted in blue.

Answer to Referee #1

Comment 1). *The model, as well shown in figure 1, takes into account the weak layer heterogeneity: if a strong zone in the basal layer is met, the tensile failure of the slab is likely to occur. However I expect from the model that the energy release increases as the basal crack increases (i.e. as the crack propagates far from the original spot - red star in figure 1). In other words, I expect that, as the basal crack propagates, it is more and more difficult to arrest. Dynamical effects should also strengthen this observation. Please clarify: in figure 1 it seems that the distance from the position of the super weak zone has no influence on the position of the secondary tensile slab crack.*

Answer to comment 1). As the reviewer suggests, the position of the tensile fracture in the slab, if existent, also depends on the position of the initial failure in the weak layer. This is due to the fact that the stress concentration at the crack tip (or the fracture energy, equivalently) increases naturally with the crack size. Moreover, dynamic effects are likely to strengthen this effect. However, nothing in the paper is in contradiction with this observation. This effect is naturally taken into account in the numerical simulations and thus in the statistical distributions obtained.

In order to clearly mention this important point, a new paragraph will be added at the end of Section 3. It will also indicate that the shear stress difference $\Delta\tau$ and tensile stresses σ_{xx} do not depend only on the heterogeneity, but also on the size of the basal crack through the stress concentration at the crack tip.

In addition to this new paragraph, figure 1 will be clarified accordingly so that there is no misunderstanding.

Comment 2). *page 6038, line 7. When referring to the typical length scale of the system "lambda", we suggest citing [Chiaia BM, Cornetti P and Frigo B (2008) Triggering of dry snow slab avalanches: stress versus fracture mechanical approach. Cold Reg. Sci. Technol., 53(2), 170178]*

Answer to comment 2). We will add this reference.

Comment 3). *page 6040 line 14. Change "... with increasing tensile strength from 1 to 0." into " ...*

Answer to comment 3). This is not what we meant but we agree that the sentence was not clear. It will be modified as follow: "As expected, this probability systematically decreases from 1 to 0 with increasing tensile strength σ_t ."

Comment 4). *page 6040 line 23. "Stress" is written twice*

Answer to comment 4). This will be corrected.

Answer to Referee #2

Comment 1). *In the formulation of the model section, slab and weak layer properties (and their respective ranges) are presented. It may be helpful to connect those properties to actual snow properties through discussion or reference.*

Answer to comment 1). We will add in section 2 realistic ranges of the parameters used in the model for the slab (density ρ , Young's modulus E , Poisson's ratio ν , tensile strength σ_t) and for the weak layer (cohesion c and its standard deviation σ_c and correlation length ϵ) as well as the associated references in the form of a table.

Comment 2). *Section 2, lines 3-4: that sentence isnt clear*

Answer to comment 2). This sentence will be reformulated.

Comment 3). *Section 3: Maybe consider carefully offering results prematurely. In lines 13-14 a result is mentioned, maybe consider building the approach followed by results.*

Answer to comment 3). This part presents qualitatively the results which are obtained in the simulations (two release types) and represents a preliminary paragraph to the more detailed parametric analysis of section 4) Quantitative results. To clarify this point, Section 3 will be renamed “Preliminary description of the results: two release types”.

Comment 4). *While a proxy for tensile strength and youngs modulus with density is given in section 4.2, I believe discussion on the relevance of the snow properties used in the model would strengthen the paper. Some properties are given with no link or discussion on their realism. I found myself looking to other references to verify some values, adding it to the paper would help in realism and readability.*

Answer to comment 4). See answer to comment 1) above.

Comment 5). *Section 4.1.5: The results appear very sensitive to E in very common density ranges (≈ 200 - 300 kg/m³). The discussion addresses this, but it seems that the very stark transition with small changes in E with probability of tensile failure is very interesting and may be worth more discussion*

Answer to comment 5). We would like to recall that a sharp transition of the tensile failure probability is not only observed with Young’s modulus E , but with most of the tested parameters (slab thickness D , tensile strength σ_t and coefficient of variation CV). In addition, the transition with E is not as sharp as it may seem since E is represented in log-scale in Fig. 5. Furthermore, note that for a real snow cover, when E increases, σ_t increases as well but with opposite effects on the tensile failure probability leading to more complex interpretations (as already stated at the beginning of paragraph 4.2). For these reasons we argue that this point does not need further discussion as it might disrupt the reader.

Comment 6). *Section 4.2, line 22: Large E (stiff) doesnt not imply strong snow. They are not the same thing.*

Answer to comment 6). We agree with this comment. The word “thus” will be removed.

Comment 7). *Section 5, lines 9-10: awkward sentence*

Answer to comment 7). This sentence will be reworded: “These simulations explained why hard and thick snow slabs are more prone to wide-scale crack propagation than soft slabs.”

Comment 8). *Section 5, lines 16-17: Could this be discussed more? The physical interpretation seems evident, why not expand the interpretation for the reader?*

Answer to comment 8). A sentence will be added to expand the interpretation that thick slabs induce a smoothing of the WL heterogeneity and thus lead to wider release areas.

Comment 9). *Section 5: When discussing release area, maybe remind the reader what the coincident probability implications are too.*

Answer to comment 9). We are not completely sure what the referee means by “coincident probability implications”. We think that the referee might refer to the fact that the exceedence probability of the release width from natural and artificial releases are almost the same. The interpretation of this observation is that the release area is mostly influenced by terrain characteristics (possibly smoothed by the snow cover distribution) and slab properties (density and thickness). This interpretation was already clearly mentioned in this section (page 6046, lines 4-5). Nevertheless, we will add a new sentence in this part to clearly mention that the width of the release zone does not seem to be influenced by the triggering mode, as supported by Fig. 7.

Influence of weak layer heterogeneity and slab properties on slab tensile failure propensity and avalanche release area

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Abstract. Dry-snow slab avalanches are generally caused by a sequence of fracture processes including failure initiation in a weak snow layer underlying a cohesive slab followed by crack propagation within the weak layer (WL) and tensile fracture through the slab. During past decades, theoretical and experimental work has gradually increased our knowledge of the fracture process in snow. However, our limited understanding of crack propagation and fracture arrest propensity prevents the evaluation of avalanche release sizes and thus impedes hazard assessment. To address this issue, slab tensile failure propensity is examined using a mechanically-based statistical model of the slab-WL system based on the finite element method. This model accounts for WL heterogeneity, stress redistribution by elasticity of the slab and the slab possible tensile failure. Two types of avalanche release are distinguished in the simulations: (1) full-slope release if the heterogeneity is not sufficient to stop crack propagation and to trigger a tensile failure within the slab; (2) partial-slope release if fracture arrest and slab tensile failure occurs due to the WL heterogeneity. The probability of these two release types is presented as a function of the characteristics of WL heterogeneity and of the slab. One of the main outcomes is that, for realistic values of the parameters, the tensile failure propensity is mainly influenced by slab properties. Hard and thick snow slabs are more prone to wide-scale crack propagation and thus lead to larger avalanches (full-slope release). In this case, the avalanche size is mainly influenced by topographical and morphological features such as rocks, trees, slope curvature and the spatial variability of the snow depth as it is often claimed in the literature.

1 Introduction

Dry-snow slab avalanches are generally caused by a sequence of fracture processes including: (1) failure initiation in a weak snow layer underlying a cohesive slab, (2) crack propagation within the weak layer (WL) and (3) tensile fracture through the slab which leads to its detachment (McClung, 1979; Schweizer et al., 2003). During the past decades, theoretical and experimental studies have gradually enhanced the knowledge of the fracture process in snow allowing a better estimation of snowpack stability (McClung, 1979; Jamieson and Johnston, 1990; Föhn et al., 1998; Jamieson and Johnston, 1998, 2001; Schweizer et al., 2006; Sigrist and Schweizer, 2007). Despite these advances, the limited understanding of crack propagation and fracture arrest propensity still limits the evaluation of the avalanche size and thus impedes avalanche forecasting and hazard mapping. This limitation is *inter alia* due to the multi-scale spatial variability of the snowpack and to the complex microstructure of snow.

Avalanche hazard mapping procedures have recently seen growing popularity of coupled statistical–deterministic models in order to evaluate the run-out distance distribution and the probability of exceedence of a threshold pressure at any location of the run-out zone (Barbolini et al., 2000; Naaim et al., 2003; Ancy et al., 2004; Eckert et al., 2007, 2008, 2010). These coupled models require the evaluation of the release volume, combination between the release depth and area. For the evaluation of the release depth, empirical techniques already exist (Swiss guidelines, Salm et al., 1990) and more recently, a coupled statistical–mechanical model was proposed by Gaume et al. (2012) and Gaume et al. (2013) taking into account both mechanical and meteorological factors in a probabilistic framework. On the other hand, the precise position of the release area and the evaluation of its spatial extent have been less investigated. Maggioni et al. (2002) and Maggioni and Gruber (2003) analyzed a database of well-documented avalanche events with respect to several topographic characteristics and showed that the mean slope angle, the curvature and the distance to the ridge are the most important parameters influencing avalanche release area distribution. Failletaz et al. (2006); Fyffe and Zaiser (2004, 2007) used cellular-automata approaches to compute avalanche release area distributions. These models include a source of stochastic variability such as the heterogeneity of weak layer mechanical properties. Interestingly, these models are able, under certain conditions, to reproduce the power-law area distributions observed from field measurements (McClung, 2003; Failletaz et al., 2004). Finally, Simenhois and Birkeland (2014) recently suggested different mechanisms that may control fracture arrest propensity.

In this paper, we extend a mechanically-based probabilistic model developed in a previous study (Gaume et al., 2012, 2013) to analyze which snowpack parameters are influencing slab tensile failure propensity and, hence, the extent of the release area. In a first section, we recall the main characteristics of the model and present the changes made compared to its previous version. Then, in the second section, two release types are distinguished and presented. In the third section, we quantify the influence of the characteristics of weak layer heterogeneity and of slab properties on the position

of slab tensile failure. Finally, in the last section, the results are applied to dry-snow slab avalanche release.

60 2 Formulation of the model

In this paper, the mechanical model proposed by Gaume et al. (2012) and described in detail in Gaume et al. (2013) is used and extended. We recall here its main characteristics.

The simulated system is a uniform slope composed of a slab and a weak layer (WL) of length $L = 50$ m. The numerical simulations are carried out using the finite element code Cast3m in 2D (plane stress condition). Gravity is the only applied external force and the system is loaded by progressively
65 increasing the slope angle θ until failure. The main change compared to the model of Gaume et al. (2013) concerns the constitutive law of the slab. We use here an elastic–brittle law, characterized by the existence of a tensile strength σ_t , in order to take into account the possible tensile failure of the slab. The density of the slab is denoted ρ . The elasticity of the slab is characterized by its Young’s
70 modulus E and its Poisson ratio ν . The weak layer is modeled as a quasi-brittle (strain-softening) interface with a Mohr-Coulomb failure criterion characterized by a cohesion c and a friction coefficient μ . Spatial heterogeneity of the weak layer is accounted for by a stochastic Gaussian distribution of the cohesion c with a spherical covariance function of correlation length ϵ . The average cohesion is denoted $\langle c \rangle$, its standard deviation σ_c and the coefficient of variation $CV = \sigma_c / \langle c \rangle$. The range of
75 parameter values used in the model, compared to realistic values for snow coming from field and laboratory experiments, are summarized in Tab. 1.

Besides the evaluation of avalanche release depth distributions, this model formerly enabled us to evidence a heterogeneity smoothing effect caused by stress redistributions due to slab elasticity. This elastic smoothing effect is characterized by a typical length scale of the system Λ (see Chiaia et al.,
80 2008; Gaume et al., 2013, 2014a, for more details).

3 Preliminary description of the results: two release types

Two types of avalanche release were obtained in the simulations: (1) full-slope release, if the entire simulated slope becomes unstable without tensile failure within the slab (Fig. 1a); (2) partial-slope release, if tensile failure occurs within the slab so that only a part of the slope is released (Fig. 1b).
85 Importantly, however, for both release types, the primary failure process observed is always the shear failure of the weak layer. Slab tensile failure, when existent, systematically constitutes a secondary process.

In the case of a full-slope release, the heterogeneity magnitude is not sufficient to trigger a tensile failure within the slab. The basal crack in the weak layer thus propagates until reaching the top
90 boundary condition which can be seen as an anchor point (Fig. 1a). In the context of a real avalanche

starting zone, this boundary condition can represent a strong geomorphological feature susceptible to trigger the tensile failure (ridges, rocks, trees, local convex zone, etc.).

In contrast, for partial-slope releases, the cohesion variability in the weak layer is sufficient to generate the tensile failure of the slab within the simulated system. Local strong zones can effectively
 95 stop the propagation of the crack and the excess of stress is redistributed in the slab and induces slab tensile opening.

For a constant slab failure strength and a constant average cohesion of the WL, the occurrence of full or partial-slope release is intimately linked to the heterogeneity of WL cohesion as well as smoothing effects due to the elasticity of the slab. These combined mechanisms lead to shear stress
 100 heterogeneities in the WL which modulate the shear stress differences $\Delta\tau$ existing between adjacent elements of the WL. This shear stress difference $\Delta\tau$ ultimately induces generation of normal stresses in the down-slope direction σ_{xx} (σ_{xx} is positive in tension and negative in compression). If σ_{xx} exceeds locally the tensile strength of the slab σ_t , then the brittle failure of the slab occurs. This process is illustrated in Fig. 1 for both release types. A full-slope release corresponds to $\sigma_{xx} < \sigma_t$
 105 everywhere in the slab whereas a partial-slope release means that σ_{xx} can be locally larger than σ_t .

Note that the position of the tensile failure in the slab, if existent, also depends on the position of the initial basal failure in the weak layer. This is due to the fact that the stress concentration at the crack tip increases naturally with the crack size. Hence, the shear stress difference $\Delta\tau$ and tensile stresses in the slab σ_{xx} depend not only on WL heterogeneity, but also on the size of the basal crack.

110 4 Quantitative results

For each set of the model parameters, 100 FE simulations were performed for different realizations of the WL heterogeneity with a constant average cohesion $\langle c \rangle$. As explained, each simulation led to either a full-slope or a partial-slope release. Note that due to the WL heterogeneity, the release occurs for different values of the slope angle θ (see Gaume et al., 2013, for more details). Besides, we only
 115 considered cases in which the load was sufficient to trigger the WL failure. Hence, the slab thickness D was chosen higher than the critical thickness $D_c = \langle c \rangle / (\rho g)$ to ensure this assumption. The results are presented in terms of partial-slope release probability also called tensile failure probability P_t . This probability represents the probability that σ_{xx} exceeds locally (i.e. at the crack tip) the tensile strength of the slab σ_t and is thus $P_t = P(\sigma_{xx} > \sigma_t)$. As the variability of the tensile stress σ_{xx} is
 120 due to the WL Gaussian heterogeneity, σ_{xx} also follows a Gaussian distribution of average $\langle \sigma_{xx} \rangle$ and standard deviation $\sigma_{\sigma_{xx}}$. Hence, the exceedence probability $P(\sigma_{xx} > \sigma_t)$ can be analytically expressed and is given by:

$$P_t = P(\sigma_{xx} > \sigma_t) = \int_{\sigma_t}^{\infty} p(\sigma_{xx}) d\sigma_{xx} = 1 - \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\sigma_t - \langle \sigma_{xx} \rangle}{\sqrt{2} \sigma_{\sigma_{xx}}} \right) \right] \quad (1)$$

with $p(\sigma_{xx})$ the probability density function.

125 In the first part of this section (parametric analysis), the influence of the characteristics of WL
heterogeneity (cohesion standard deviation σ_c and correlation length ϵ) and of the slab's properties
(slab thickness D , tensile strength σ_t and Young's modulus E) on $P_t = P(\sigma_{xx} > \sigma_t)$, $\langle \sigma_{xx} \rangle$ and
 $\sigma_{\sigma_{xx}}$ is shown. The influence of each parameter is studied while the other parameters are kept con-
stant. Then in the second part of this section (application to slab avalanche release), the relations
130 existing between the different mechanical properties of snow are taken into account.

4.1 Parametric analysis

4.1.1 Tensile strength σ_t

Figs. 2, 3a, 4a represent the probability of tensile failure P_t within the system as a function of
the tensile strength σ_t for different values of the correlation length ϵ (Fig. 2), of the coefficient of
135 variation CV (Fig. 3a) and of slab thickness D (Fig. 4a). Tensile strength varies between 0 and 1 kPa.
As expected, this probability systematically decreases from 1 to 0 with increasing tensile strength σ_t .
The data points obtained with the FE simulations have been fitted using Eq. (1) allowing to compute
the average tensile stress $\langle \sigma_{xx} \rangle$ and its standard deviation $\sigma_{\sigma_{xx}}$ (which are the two parameters of
the fit). The good agreement between the modeled exceedence probability P_t [Eq. (1)] and the FE
140 simulation results confirm that σ_{xx} follows a Normal distribution resulting from the Gaussian WL
heterogeneity.

4.1.2 Correlation length ϵ of WL cohesion

The influence of correlation length ϵ is shown in Fig. 2a. The tensile failure probability P_t decreases
in general with increasing correlation length ϵ . The average tensile stress $\langle \sigma_{xx} \rangle$ decreases with ϵ
145 while its standard deviation $\sigma_{\sigma_{xx}}$ increases. The influence of ϵ is thus more pronounced for small
values of the tensile strength σ_t . Hence, the higher the correlation length, the farther a crack propa-
gates leading to more full-slope releases and thus potentially larger avalanches.

The correlation length ϵ characterizes the spatial structure of the cohesion heterogeneity. A nil
correlation length corresponds to a completely random signal whereas a large correlation length
150 indicates the existence of spatial structures in the signal with similar values (see Gaume et al., 2013,
for some examples). Hence, an increase of the correlation length smooths the WL heterogeneity and
reduces the shear stress difference $\Delta\tau$ between two elements, resulting in a lower fracture arrest
propensity. On the contrary, a very low value of the correlation length implies a WL heterogeneity
without spatial structure and thus large local variations that can stop the propagation of the crack.
155 This observation qualitatively explains the observed trend of the results.

4.1.3 Standard deviation σ_c of WL cohesion

The influence of the standard deviation σ_c is then investigated and represented in Fig. 3. For a constant slab thickness $D = 1$ m and correlation length $\epsilon = 0.5$ m, Fig. 3a shows, as would be expected, the increase of the tensile failure probability with increasing coefficient of variation $CV = \sigma_c / \langle c \rangle$.

160 This dependence is even clearer in Fig. 3b for which P_t is represented as a function of CV for $\sigma_t = 150$ Pa and different slab thickness values. Inset of Fig. 3a also shows that the average tensile stress $\langle \sigma_{xx} \rangle$ as well as its standard deviation $\sigma_{\sigma_{xx}}$ linearly increase with the coefficient of variation explaining why the curves in Fig. 3a are progressively shifted to the right with a lower rate of decrease with σ_t as the coefficient of variation increases.

165 Indeed, a large value of the coefficient of variation induces large local variations of the WL shear stress resulting in high tensile stresses within the slab and ultimately favors fracture arrest. As a consequence, the tensile failure probability increases with increasing variability.

4.1.4 Slab thickness D

As shown in Fig. 4a, the tensile failure probability P_t in general decreases with increasing slab thickness D . Furthermore, the higher is D , the faster the probability decreases with σ_t . The values of σ_t for $P_t = 1$ are almost unaffected by the slab thickness D while the value for $P_t = 0$ is decreasing with increasing D . These characteristics correspond to the relations of $\langle \sigma_{xx} \rangle$ and $\sigma_{\sigma_{xx}}$ with the slab thickness D , which both decrease with increasing thickness. Note, that as mentioned before, simulations were carried out for $D > D_c = c / (\rho g) = 0.41$ m in this case, to ensure that a WL shear failure can be initiated. Indeed, if no failure is initiated in the WL, no slab tensile failure can occur.

In more detail, Fig. 4b reports the tensile failure probability P_t directly as a function of D for different tensile strength values. For $\sigma_t < 150$ Pa, P_t is approximately equal to 1, whereas it is approximately equal to 0 for $\sigma_t > 900$ Pa. For intermediate values of σ_t , the tensile failure probability generally decreases from $D = 0.5$ m to $D = 2$ m.

180 It has previously been shown (Gaume et al., 2013, 2014a) that the slab thickness D , together with the slab elastic modulus E , has an important smoothing effect on WL spatial variability due to stress redistribution. Consequently, a large value of the slab thickness D smooths the cohesion heterogeneity by reducing the apparent standard deviation of the WL heterogeneity, in contrast to the case of a completely rigid slab in which the stresses would follow exactly the heterogeneity variations. As shown before, a reduction of the standard deviation leads to a decrease of the tensile failure probability, which explains the results described above. This result is also clearly illustrated in Fig. 3b. The rate of increase of the tensile failure probability P_t decreases with the slab thickness. Hence, as the slab thickness increases, the WL heterogeneity is smoothed and thus a larger value of CV is required to obtain $P_t = 1$, i.e. only partial-slope releases. As shown in the inset, the smoothing

190 effect by the slab thickness can be accounted for simply through a scaling of the standard deviation with a linear function of the slab thickness $\rho g f_c(D)$ with $f_c(D) = 0.64D - 0.17$ (in meters).

4.1.5 Slab Young's modulus E

Fig. 5 represents the influence of slab Young's modulus on the tensile failure probability P_t for a correlation length $\epsilon = 2$ m, a slab thickness $D = 1$ m and a slab tensile strength $\sigma_t = 600$ Pa. The tensile failure probability increases with the slab Young's modulus E . Indeed, the slab being brittle-elastic, the tensile stress σ_{xx} increases linearly with the deformation in the down-slope direction ϵ_{xx} according to $\sigma_{xx} = E' \epsilon_{xx}$ until σ_{xx} reaches the tensile strength σ_t (with $E' = E/(1 - \nu^2)$). Hence, for a constant deformation ϵ_{xx} which is imposed by the displacement gradient in the WL interface and thus by the WL stresses variations, the tensile stress is lower in the case of a low Young's modulus than for a high one (see insets in Fig. 5). As a consequence the slab failure criterion, namely $\sigma_{xx} = \sigma_t$ is met easily in the limiting case of a rigid slab.

Moreover, as shown in Gaume et al. (2013, 2014a), slab elasticity also induces an important smoothing effect on WL heterogeneity. This effect leads to a reduction of the apparent standard deviation which, together with the previous explanation, explains the decrease of the tensile failure propensity with increasing elasticity (decreasing Young's modulus E).

4.2 Application to slab avalanche release

The results of the previous parametric analysis should be interpreted with care and one should keep in mind that, for snow, several of the previous parameters are linked which may lead to more complex interactions. For instance, the result about the influence of Young's modulus on the tensile failure probability might seem contradictory to avalanche observations. Indeed, taken as it is, this result would imply that it is easier to trigger a tensile failure in **stiff and strong** snow than in softer snow. If this line of reasoning is pursued, hard snow slabs would result in smaller avalanche size than soft slabs which is clearly in contradiction with avalanche observations. Hence, even if the result behind Fig. 5 is consistent, from a mechanical point of view, it cannot be directly applied to dry-snow slab avalanche release. To do so, one needs to take into account the relation between the density of the slab ρ , its Young's modulus E and its tensile strength σ_t .

New simulations were therefore performed, for which the dependence between ρ , E and σ_t was taken into account using the relation proposed by Sigrist (2006):

$$E(\rho) = 9.68 \times 10^8 \left(\frac{\rho}{\rho_{ice}} \right)^{2.94}, \quad (2)$$

220 and

$$\sigma_t(\rho) = 2.4 \times 10^5 \left(\frac{\rho}{\rho_{ice}} \right)^{2.44}, \quad (3)$$

with $\rho_{ice} = 917 \text{ kg/m}^3$. The density of the slab was varied between 80 and 250 kg/m^3 leading to values of the Young's modulus between 0.7 and 20 MPa and tensile strength between 0.5 and 10 kPa. Two types of simulations were performed: (1) simulations with a constant slab thickness and therefore a different slab mass for the different densities; (2) simulations with a slab thickness which was varied in order to keep a constant load on the WL for the different densities.

In both cases, the slab tensile failure probability P_t decreases as a function of slab density if considering the above mentioned inter-dependencies (Fig. 6). Besides, it appears that for densities larger than about 150 kg/m^3 , the tensile failure probability becomes very small, meaning that all the releases are full-slope. Thus, for large enough densities, the WL layer heterogeneity has no influence on the position and the extent of the avalanche release area. In this case, this result would suggest that topographical and geomorphological features control the size of the release area. This result is illustrated below the graphic with two cases of avalanches: (left) a soft and shallow slab for which the release area is quite small compared to the maximum potential extent of this starting zone; (right) a hard and thick slab for which the release area is very extensive and controlled by terrain features. In the latter case, the tensile failure generally occurs at the transition between slope angles larger and smaller than 30° due to the basal friction (the crack face friction angle of snow being around 30° , van Herwijnen and Heierli, 2009).

5 Discussion

The proposed approach allows us to compute the slab tensile failure probability from WL spatial variability characteristics and slab properties using the finite element method. First, a parametric analysis showed the influence of each model parameter on the tensile failure probability. Then, more realistic simulations were performed taking into account the link between the mechanical properties of the slab. **These simulations explained why hard and thick snow slabs are more prone to wide-scale crack propagation than soft slabs.** However, one might also argue that the density is generally linked to the thickness, the higher the thickness: the higher the density due to settlement. Nevertheless, even if this link was taken into account, the main finding of Fig. 6, namely that the WL heterogeneity influences fracture arrest propensity only for soft slabs, would still remain relevant since the tensile failure probability would be even lower. Indeed, as shown in Fig. 4, an increase in slab thickness decreases the tensile failure probability, **and thick slabs thus lead to larger release areas by smoothing the WL heterogeneity.** Furthermore, since the WL was modeled as an interface, the bending of the slab observed in field propagation saw tests (van Herwijnen et al., 2010), which can increase the tensile stress (Gaume et al., 2014b; Schweizer et al., 2014), was not taken into account. This bending effect would probably induce a higher transition density, since more partial-slope releases would occur.

From the presented approach, a rough estimate of the avalanche release area can also be proposed. For a tensile failure probability equal to zero, the avalanche release area would be equal to the maximum area A_{max} allowed by the terrain or the snow cover distribution. On the contrary, for a tensile failure probability equal to 1, the avalanche release area would be close to zero. Hence, the release area A can be approximated in a first approach by:

$$A = (1 - P(\sigma_{xx} > \sigma_t)) A_{max}. \quad (4)$$

The avalanche release area would thus be different from A_{max} only for very soft slabs in which case it would also be a function of the characteristics of WL heterogeneity and slab properties. The form of the proposed estimate of the release area is in good quantitative agreement with the result of Fyffe and Zaiser (2007) who studied also the influence of WL heterogeneity parameters on the size of the release zone using a cellular automaton model. These authors showed that the release area increases with the tensile strength of the slab and then tends to a limit given by the maximum size of the system for a tensile strength $\sigma_t \approx 4000$ Pa (calculated using the dimensionless analysis proposed by Fyffe and Zaiser, 2007, and the parameters of our model) corresponding to a density $\rho \approx 170$ kg/m³.

Fig. 7 reports the exceedence probability of the width L of 369 natural and 5323 artificially-triggered slab avalanches that were observed during the winters 1998 to 2010 in La Plagne (France). These data were presented in detail in Gaume et al. (2012), in which the focus was on the avalanche release thickness rather than the width. The inset shows that the avalanche width L is not very well correlated with the release thickness D despite a relatively slight apparent increasing trend which is statistically not significant ($R^2 = 0.15$ for a linear regression). Our numerical results suggest that L should not be correlated with D at all, except for very low densities and thus small avalanches. However, as mentioned before, bending effects induced by WL failure (which were not accounted for) would tend to increase the tensile failure probability and consequently the dependency of L with D . Furthermore, the roughness of the terrain is progressively smoothed during the season as the slab thickness increases. Hence, the avalanche maximum extent A_{max} also potentially increases with increasing slab thickness D (Veitinger et al., 2014). Besides, we remark that, the exceedence probability $P(\geq L)$ does not appear to depend on the avalanche triggering mode, natural or artificial. Hence, the width of the release area is essentially uninfluenced by the triggering mode. On the contrary, Gaume et al. (2012) showed an important difference between natural and artificial triggering on the release depth distribution, natural avalanches being influenced not only by terrain and mechanical aspects, but also by recent snowfall distributions. This similarity between the width distributions of naturally and artificially triggered avalanches confirms that the release area is mainly influenced by terrain characteristics (possibly smoothed by the snow cover distribution, Veitinger et al., 2014) and slab properties (density and thickness).

Finally, the results of the presented model suggest that the majority of the releases would be full-slope, i.e. not influenced by WL heterogeneity, especially for high densities. Hence, the potential extent of slab avalanche release areas will be controlled by topographical and geomorphological

features of the path such as rocks, trees, ridges or local curvatures induced by the terrain and the snow cover distribution. As a consequence, GIS methods based on terrain characteristics such as those developed by Maggioni and Gruber (2003) might be adequate to compute the potential extent of extreme avalanches. For more frequent avalanches, similar and recent methods taking also into account the spatial distribution of the snow depth and the induced terrain smoothing would be relevant (Veitinger et al., 2014).

6 Conclusions

We used a coupled mechanical-statistical approach to study the probability of occurrence of slab tensile failure of a slab-WL system using the finite element method. Two different release types were observed in the simulations: (1) Full-slope release if the WL heterogeneity is not sufficient to arrest crack propagation and trigger a tensile failure within the slab. Hence the crack propagates across the whole system. (2) Partial-slope release when the local variations of WL cohesion are substantial and can stop crack propagation and trigger the slab tensile failure. Importantly, for both release types, the primary failure process observed is always the basal shear failure of the weak layer. Hence slab fracture systematically constitutes a secondary process.

We have shown that the slab tensile failure propensity strongly depends on the model parameters such as the tensile strength σ_t , the slab thickness D , the correlation length ϵ , the standard deviation of the weak layer cohesion σ_c and probably other parameters that have not been varied in this study such as the average cohesion $\langle c \rangle$. In addition, we presented a simple statistical model capable of reproducing the tensile failure probability as a function of the model parameters. Two illustrations of this simple model are represented in Fig. 1. In the first case (Fig. 1a), the slab tensile stress σ_{xx} is always lower than the tensile strength σ_t . The basal failure thus propagates across the entire system until the top boundary condition which can be seen as a ridge, a rock, a tree or a local curvature. In the second case (Fig. 1b), a local zone of substantial tensile stress σ_{xx} due to strong variation of the WL cohesion generates a local tensile failure within the slab since $\sigma_{xx} > \sigma_t$.

For realistic values of the parameters and taking the link between the mechanical properties of the slab into account, the model results suggest that the releases are partial-slope only for low slab densities and rather full-slope for densities higher than about 150 kg/m^3 . Hence in most cases, one would expect that the extent of the release area is mainly controlled by the topography and the morphology of the path. this finding corroborates the results found by Maggioni and Gruber (2003) who analyzed the influence of morphological features of the path on the extent of the release area using a purely data-driven statistical approach, and brings some mechanical justification to the predominance of local terrain geometry in the position and extent of real avalanches. This result shows in particular that the release area will be extremely dependent on slope topography (local curvature, ridge...), on

the presence of rocks and trees for instance and also by the snow cover distribution which can induce terrain smoothing and thus help wide-scale crack propagation.

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	parameter	modeled values	typical values	references
Slab	ρ (kg m ⁻³)	80-250	100-300	Schweizer (1999)
	E (MPa)	0.1-20	0.1-50	Mellor (1975); Scapozza (2004); Sigrist (2006)
	ν	0.2	0.1-0.4	Ooizumi and Huzioka (1982); Schweizer (1999)
	σ_t (kPa)	0-10	0.1-30	McClung (1979); Conway and Abrahamson (1984) Jamieson and Johnston (1990); Sigrist (2006)
Weak layer	c (kPa)	0.5-1.5	0.1-4	Jamieson and Johnston (2001)
	CV (kPa)	0-0.5	0.2-0.6	Föhn (1989); Jamieson and Johnston (2001); Landry (2002) Kronholm (2004); Logan et al. (2007); Schweizer et al. (2008)
	ϵ (m)	0-5	< 10	Schweizer et al. (2008)
	μ	0.56	0.5-0.75	van Herwijnen and Heierli (2009)

Table 1: Parameters used in the model and typical values for snow from field and laboratory experiments. For more details about these parameters and a more complete review of the different related studies, see Section 3.3.2 in Gaume (2012).

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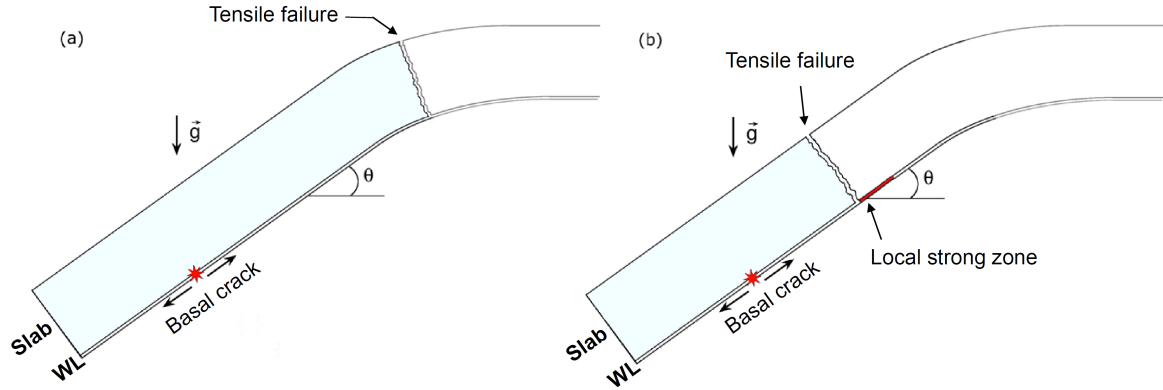


Figure 1: Schematic representing the two types of failure observed in the simulations. (a) full-slope release: the crack in the WL propagates across the whole slope. The position of the slab tensile failure would then depend on morphological features (rocks, trees, ridges, curvature...). (b) partial-slope release: the local heterogeneity is sufficient to stop the propagation of the crack in the WL and thus trigger the tensile failure within the slab. The red-coloured part of the weak layer represents a local zone of large shear strength.

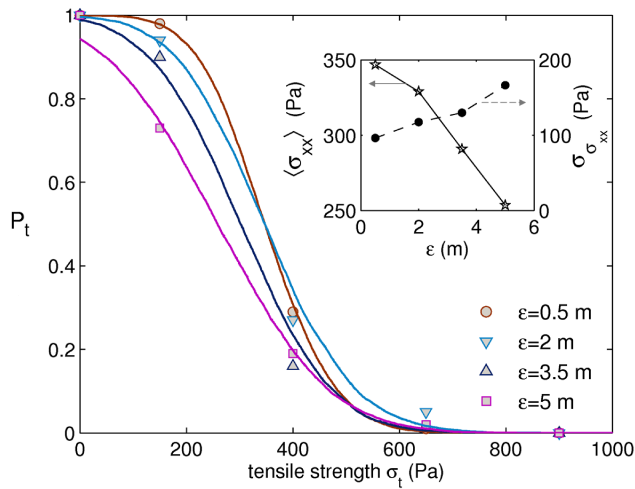


Figure 2: Probability of slab tensile failure P_t within the simulated system (partial-slope release) as a function of the tensile strength σ_t for different values of the correlation length ϵ , a coefficient of variation $CV = 0.3$, a constant slab thickness $D = 1$ m, slab density $\rho = 250$ kg/m³ and Young's modulus $E = 1$ MPa. In the inset, the average tensile stress $\langle \sigma_{xx} \rangle$ (solid line) and its standard deviation $\sigma_{\sigma_{xx}}$ (dashed line) are represented as a function of ϵ .

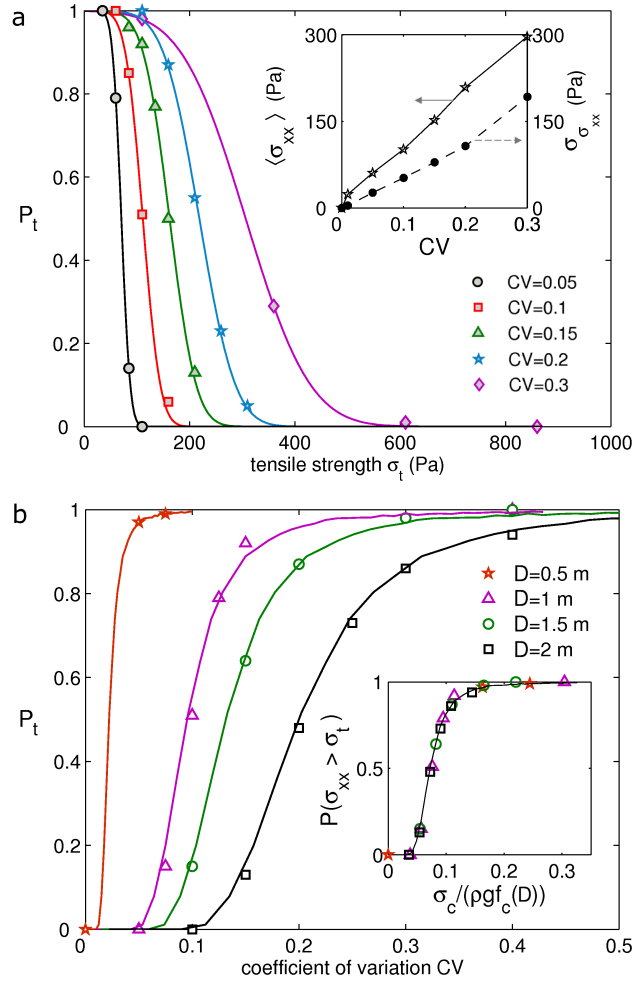


Figure 3: (a) Probability of slab tensile failure P_t within the simulated system (partial-slope release) as a function of the tensile strength σ_t for different values of the coefficient of variation CV and for a constant correlation length $\epsilon = 2$ m, a constant slab thickness $D = 1$ m, slab density $\rho = 250$ kg/m³ and a Young's modulus $E = 1$ MPa. In the inset, the average tensile stress $\langle \sigma_{xx} \rangle$ and its standard deviation $\sigma_{\sigma_{xx}}$ are represented as a function of CV. (b) P_t is represented as a function of CV for different slab thickness values, a tensile strength $\sigma_t = 100$ Pa and the same value of the other parameters as in (a). In the inset, the tensile failure probability P_t is represented versus the standard deviation scaled by $\rho g f_c(D)$ with $f_c(D) = 0.64D - 0.17$ (in meters).

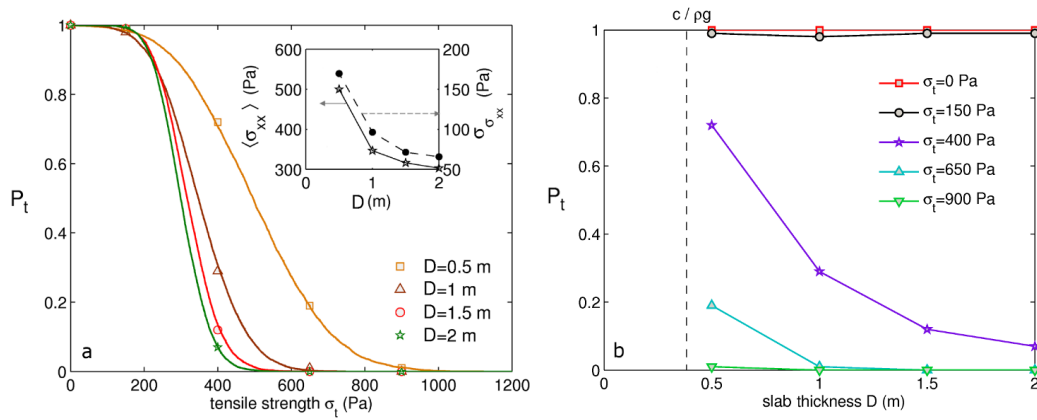


Figure 4: (a) Probability of slab tensile failure P_t within the simulated system (partial-slope release) as a function of the tensile strength σ_t for different slab thickness values and a constant correlation length $\epsilon = 0.5$ m, slab density $\rho = 250$ kg/m³ and Young's modulus $E = 1$ MPa. In the inset, the average tensile stress $\langle \sigma_{xx} \rangle$ and its standard deviation $\sigma_{\sigma_{xx}}$ are represented as a function of D . (b) P_t is represented as a function of D for different tensile strength values and the same value of the other parameters as in (a).

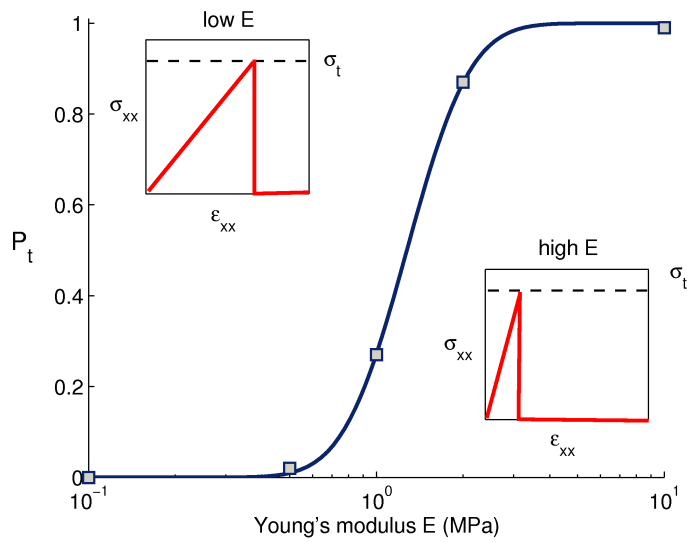


Figure 5: Slab tensile failure probability P_t versus Young's modulus of the slab for a correlation length $\epsilon = 2$ m, a slab thickness $D = 1$ m, a slab density $\rho = 250$ kg/m³ and tensile strength $\sigma_t = 400$ Pa. The insets show illustrations of the tensile stress σ_{xx} vs down-slope deformation ϵ_{xx} for a low (left) and a high Young's modulus (right).

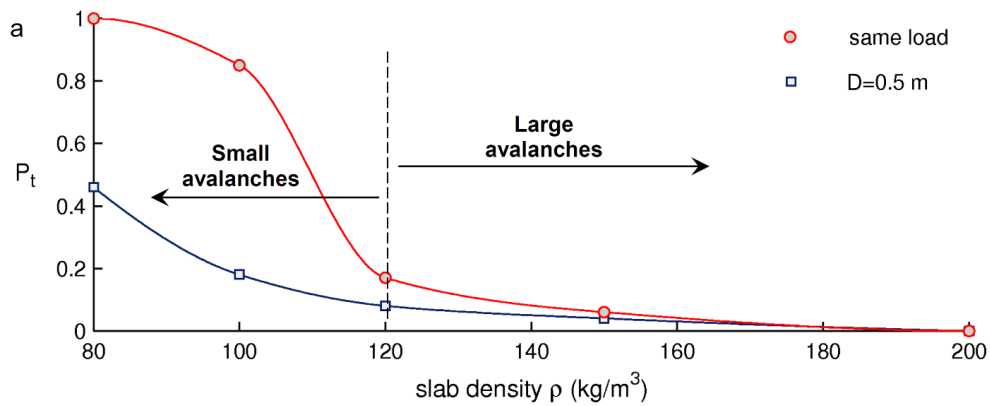


Figure 6: Slab tensile failure probability P_t versus slab density for $\epsilon = 2$ m and taking into account the relation between slab density ρ , its Young's modulus E and its tensile strength σ_t according to Eqs. (2) and (3). The red curve represents the results of simulations for a constant load and thus a slab thickness which decreases with slab density ($\rho D = 100$ kg/m²). The blue curve is for a constant slab thickness $D = 0.5$ m and thus a varying load. Below are two avalanche pictures illustrating the result: (left) soft and shallow slab of small size ©Anchorage Avalanche Center; (right) hard and thick slab with a very large extent ©Grant Gunderson.

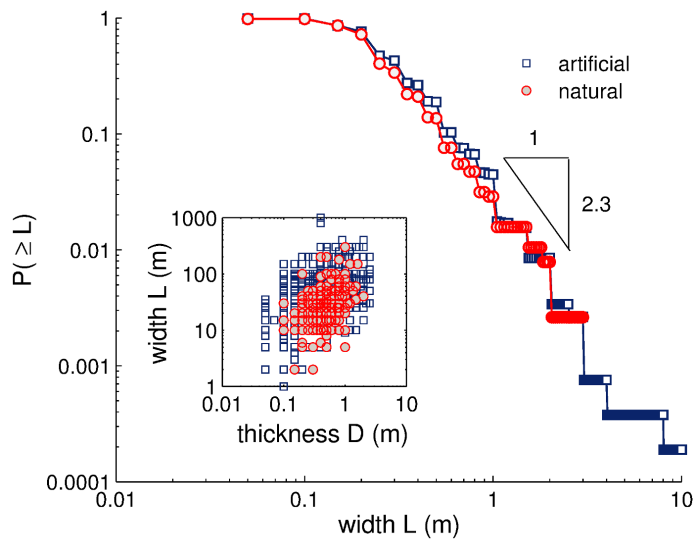


Figure 7: Cumulative exceedence probability of the width L of 369 natural and 5323 artificially-triggered slab avalanches observed during winters 1998 to 2010 in La Plagne (France). The inset shows the width L versus the release thickness D .