Dear Scientific Editor,

We would like to thank the three reviewers for their critical, but constructive comments on the Discussion version of our manuscript.

In the supplement we outline in detail how we implemented most of the suggestions in the next version of the manuscript to make it more attractive for the community. These changes are highlighted in blue in the new version which is attached as supplement as well. Abstract, Introduction, but also parts of the other Sections have been completely rewritten to clarify the focus and conclusions of our work.

During the revision process an error in the implementation of the calculation of the friction heat in Elmer/Ice was discovered. We are grateful for the additional time that was allocated to us for the revision and have adjusted all figures and results to the updated, corrected implementation. Hence our conclusions differ partly from the previous version. Changes related to this issue are highlighted in red in the new version. The importance of friction heat is now less compared to the earlier version and we are able to better explain the observed acceleration of Franklinbreen and ongoing fast flow of the other outlets by relating this fast flow to the contribution of firn heating to the heat budget of the ice cap.

Sincerely yours,
Martina Schäfer (and all Co-Authors)
1. General comments

This paper presents numerical experiments on temperature and flow speed conditions of a relatively large ice cap on Svalbard. Full stokes thermo-mechanically coupled model is applied to reproduce observed surface velocity field. A special feature of this modeling work is the inversion of the drag coefficient of the basal sliding law. Spatial distribution of this coefficient was computed by minimizing the cost function. A series of numerical experiments was performed to reproduce ice temperature measured in a borehole drilled interior of the ice cap. The sudden speed up observed in Franklinbreen was simulated with special attention on the effects of different heat sources (strain heat, friction heat, and firn heating) on the fast flow. This is a continuation of the authors’ previous work (Schäfer et al., 2012), in which similar model was applied to the same region. The improvement from the previous work is the inclusion of thermodynamics in the model and prognostic experiments. Basal conditions are important in the studied ice cap because of the fast flowing outlet glaciers. Thus, the combination of inverse modeling and full stokes 3D thermomechanically coupled modeling is valuable to better understand the dynamic and thermodynamic structure of the ice cap. The validity of the model framework and numerical schemes are supported by the previous work (Schäfer et al., 2012). Nevertheless, the paper is not well focused on the subject appeared in the paper title. For example, a large part of the text is dedicated to the borehole temperature profile, which is not related to the fast flowing feature. I also found the experimental results are not well presented. For example, not much explanation is given to Figures 5 and 6, and the reason why those two locations are highlighted in Figure 6 is unclear. The aim of the prognostic simulations is also not clear. Accordingly, no conclusive interpretation is given for the role of each heat source in the fast flow condition. The authors state in Conclusion that firn heating is important, but advection of heat from the surface firn layer to the bed takes too long to explain the speed up in Franklinbreen. I also find the manuscript is not well balanced, i.e. experimental results (4.2 and 4.3) are very briefly mentioned as compared to the lengthy description on the model setup (3 and 4.1).

I list below my major concerns followed by more specific and technical comments. I hope the manuscript will be improved to more focused, better organized and concise paper, which meets the standard of publication.

We thank the reviewer for this critical, but constructive assessment. We strive to come after the demanded changes and give more details below.

1. Major concerns

(1) Main focus of the paper

The model is well prepared and a number of numerical experiments were performed. However, questions to be solved by the modeling are not clear. I expected the triggering mechanism of the speed up at Franklinbreen would be discussed in the context of englacial and subglacial temperature regime. In the experiments, however, ice temperature field is driven by basal flow conditions estimated by the inversion of surface speed, rather than taking ice temperature as the control of fast flow. The role of the strain heat, friction heat and firn heating in the fast ice flow is discussed only in a qualitative manner, and it does not reach clear conclusions. The experiments to reproduce the borehole temperature provide an insight into the influence of temperature and surface melt on englacial temperature field. It might be interesting if the paper is focused on the effect of firn heating on thermal regime of a polythermal glacier.

All reviewers agree that the paper was not well focused. We rearranged therefore completely Abstract and Introduction to improve its previous diffuse structure. Regarding your more specific modelling question, the issue is that we discuss two distinct thermodynamic phenomena which are however related:

1) The influence of mechanical heat production (strain and friction heat), which are efficient locally in the fast flowing areas, on the basal condition in these fast outlet regions, in particular Franklinbreen.

2) The influence of firn heating on a measured profile in the interior of the ice cap, which affects at a first view the thermal regime of the centre of the ice cap but through advection (or meltwater flow) also the outlets.

The first heat-sources are simply output of the model, while firn heating requires and additional parameterisation which needs to be calibrated. Hence they are presented differently and the calibration of
the firn heat formulation is discussed in detail. Both phenomena are related because both have impacts on the onset or maintain of fast flow in the outlets. We drastically worked over structure as well as text in order to clearer formulate this out.

(2) Sliding under freezing basal condition

It appears that the model allows basal sliding even when the bed temperature is below freezing temperature. I assume that the drag coefficient in the freezing zone is sufficiently large, but is this controlled in the model? It is not realistic if the model predicts fast sliding over a region well below freezing temperature. Terms "cold based sliding" (page 5121, line 17 and 19) and "submelt sliding" (page 5121, line 1; page 5124, line 27) are used, which give an impression that the model allow significant sliding over a frozen bed. Hindmarsh and Meur (2001) and Seddik et al. (2012) allow sliding at subfreezing temperature, but its magnitude decreases exponentially as temperature decreases below melting temperature. This point should be addressed properly.

Of course, the reviewer is right in that sense, that there is a slight discrepancy between the observed sliding parameter distribution and the areas of temperate base. But we would like to emphasize that this phenomenon occurs only over areas close to the terminus of two of the southern outlets and is not a general observed issue. This discrepancy mainly might be caused by the method-inherent fact that the inversion method of the mechanical sliding parameters is decoupled from the computation of the temperature field, that itself contains a certainly wrong assumption of steady state. This issue, though, cannot be resolved, as it would need a prognostic computation with all climatic as well as basal friction parameters being determined over a period of several thousand years. Unnecessary to mention that a spin-up run with a full Stokes model (even if just deployed over a smallish ice-cap) is beyond computing capabilities for the time being. On the other hand, the results of the inversion are certainly also not perfect and lead to wrong or unphysical sliding parameters for example over areas with imprecise bedrock data. The correlation between areas with poor bedrock data (see Fig. L2 at the end of this letter), the ice thickness distribution (the 20m contour has been added to what is now Fig. 9) and the areas of “cold based” sliding (now Fig. 9) strongly points to a problem in the bedrock data.

On the other hand the thin ice-thickness in these areas could also be a consequence of a recent local thinning. Then ice temperatures would certainly not yet be at steady-state and in reality be much warmer than calculated.

We interpret the critics of the reviewer as a motivation to better elaborate those shortcomings in the text and tried to come after this demand (see new Section 5.3.1).

(3) Surface and bed elevation

DEM reported for 1990 is used to reproduce the surface velocity field in 1995, 2008 and 2011 (page 5113, line 4). It is argued that surface elevation change is small, but ICE- Sat data are usually sparse in space. If the focus of this paper is fast flowing glaciers, accurate surface elevation is crucial. Using the same DEM limits the detailed discussion especially on the rapidly changing Franklinbreen. Sparseness of bed elevation data is also problematic (page 5114, line 14-18; page 5122, line 19-24; page 5123, line 11-13). Bed geometry is clearly important in fast flowing glaciers, and it is directly related to the basal conditions estimated in this study. I suggest the authors to present the surface and bed elevation data used in this study and discuss their influence on the results more in detail.

The reviewer is correct in claiming that satellite data is sparse and the bedrock data certainly has inconsistencies. We do relaxation runs to partly at least identify possible issues. Nevertheless, there is no way out of the eternal dilemma in theoretical glaciology to get too little input from the field. All we can offer is to clearer state those possible sources of errors and will include a few lines along the following ideas even though we do not want to overload the paper with this topic:

With respect to the bedrock data (see Pettersson, 2011 for details) we admit that it’s certainly sparse, but all what we have. In our opinion the DEM captures the first order variability in the bed (i.e. major valleys etc.) with some exceptions. Of course second order variations (bumps etc.) will influence the details in the ice flow and so on, but will imprint on the first order features. Our focus is on the first order scale (i.e. fast-flow or not) and the second order might not be that important.

With respect to the surface data, a comparison (in preparation for publication, Pettersson et al.) between the NPI DEM (1990) used in the current study and the SPOT-Spirit DEM (2007) has been done. A difference of around 10m over Franklinbreen between the two DEMs is observed and we are thus rather confident that we can use the same DEM for 1995, 2008 and 2011.
We modified the data presentation in Section 2 according to this.

(4) Presentation of the results

The output of the 3D thermo-mechanically coupled model is complex. To discuss such results for a certain objective, the data set has to be processed and presented specifically for each purpose. Unfortunately, most of the plots just present computed values over the ice cap (Figures 2, 4, 6 and 8), and no further processing was performed. Moreover, the focus of the paper is fast flowing features, but only Figure 9 is prepared for this purpose. The readers would expect more of this kind of plot, and more direct and quantitative evidence of the authors’ argument. For example, I am interested in englacial temperature structures of Franklinbreen before and after the acceleration, as well as in the fraction of basal sliding and internal deformation.

We partly disagree with this assessment. Also Fig. 2 (distribution of basal sliding coefficients) showed a zoom over Franklinbreen and Fig. 5 (changes in basal temperatures linked to different processes, mainly driven by high velocities/shear-rates) was linked to the wider topic of fast-flow features. A figure showing the difference of surface and bedrock velocities, illustrating the importance of sliding compared to internal deformation is now included.

We did not include any vertical cross sections since we do not see in which way such information would contribute to the understanding of our simulations and results. The focus of our study lies on the basal temperature as imperative precondition for the existence of basal motion and the temperature distribution within the ice-body is (besides the calibration of the firn model) to a lesser extend of importance. Fig. 6 and Fig. 7 (vertical temperature profiles) summarize in our opinion the essential information about vertical temperature variations. We would however be happy to follow concrete suggestions what kind of figure and/or vertical cross section the reviewer judges beneficial for the presentation and understanding of our results.

3. Specific comments and Technical corrections

Abstract: Abstract should be more focused on your work and achievements. The first paragraph describes the back ground of the study, and it occupies half of the abstract. Results are given in the last paragraph, but it does not convey the significance of the study. It should be restructured to be concise and efficient.

As suggested by all reviewers, the abstract has been rewritten taking into account the various suggestions of all reviewers to improve it.

page 5098, line 28: "purely temperature dependent sliding law" is not accurate and sounds odd.
We have avoided this formulation in the new version.

page 5099, line 13: Citing Iken’s work (1981) here is not appropriate.
We have replaced the reference to Iken and added other references for surging glaciers.

page 5099, line 19-20: "... even close to pressure melting point, i.e. that undergoes some form of basal sliding." » This is not clear to me.
This idea has been reformulated. We meant that we use the term fast flow to describe ice flow occurring significantly faster than it would be possible by internal deformation – even at ice temperatures close to pressure melting point, when internal deformation is highest. Most of the ice flow occurs hence through some form of basal sliding/motion.

page 5101, line 14: "... this approach allows neither ..." » allows investigations on ?
We avoided the formulation “neither ... nor”, but find “allows investigations on” does not reflect what we want to say. What we really want to emphasize is that we only obtain a set of numbers, corresponding to a certain snapshot, but no direct understanding about the physics behind.
The sentence has been replaced by: "It [This approach] does not, however, give a direct relationship between temperature and in-situ friction parameter. No predictions for the basal friction parameters at other times than the inversion are possible.”

Page 5102, Research area and observational data: A set of figures explaining the geographical setting of the study site is needed. A part of Figure 1 gives the location of the ice cap, but no information is given for surface and bed elevations.

Description of the surface and bed data is given in separate subsection 2.1. We have modified Figure 1 so that the explanation of the general geographical setting is improved and included some information about surface and bed elevations. For more detailed figures we point the reader to the previous publication Schäfer et al. 2012.
From 2008 to 2011 no large changes occurred. There is a change at the southwestern corner. Is this due to a larger data gap? Yes, there are large changes in velocity visible in Figure 1 between 2008 and 2011. However, this change is an artefact due to limitations in the satellite data that create larger uncertainty in the results. We commented on this higher up in the text (page 5103, lines 17-20), but have clarified this further in the revised manuscript (see also comment to Figure 1 below).

"climate mass balance" Why not surface mass balance? We adopt the conventions from: IACS Glossary of Glacier Mass Balance and Related Terms, http://unesdoc.unesco.org/images/0019/001925/192525E.pdf. Climatic mass balance is the sum of the surface mass balance and the internal mass balance. It excludes mass changes through ice-flow, but comprises melting and refreezing inside the ice-body.

"n and t are normal and . . ." n should be defined right after it appeared in line 14. Correct, has been moved.

"beta" is named as the "basal friction parameter" here, but as the "basal friction coefficient" in the same paragraph (line 20), and later called as "basal drag coefficient". Please be consistent. Good point, is homogenized to clarify.

We of course mean heat conductivity of ice. The text has been modified to clarify this.

Equation 9 (now number 4) has been corrected.


"verified in (Schäfer et al., 2012)" verified (Schäfer et al., 2012) Right, is corrected.

"In the distribution . . ." This sentence is not clear. Has been rewritten, see answer to Reviewer 2.

I find this paragraph is lengthy. I understand that steady state temperature solution was assumed because of lack of data for spin-up experiment. This paragraph has been rewritten.

"small temperature dependency" of what?

... of the result of the inversion.

It is not clear what is guaranteed by the short time step. As shown in Zwinger and Moore (2009), inconsistencies in the geometric representation of either the surface or the bedrock manifest in locally strong variations of – in particular – the vertical components of the velocity field (simply induced by the incompressibility condition). This can lead to unnaturally fast transient behaviour of the free surface that usually damps out quickly, but consequently needs a high temporal resolution in order not to introduce instability. We have added the reference to the text.

"does not lead to visible improvements" Can you show a statistic value? Observational DEM tend to produce unrealistic velocity variations, thus smooth surface may be required to estimate more accurate spatial pattern of the basal friction. As discussed in the previous point, we in particular smooth out the errors in the DEM by diffusing them in short, highly temporally resolved runs. We include a figure about the vertical velocity-component and changes in surface adjustment as a criterion for the smoothness of the solution in the next version of the paper.

To judge the improvements by running this complex scenario, the final temperature and beta fields have been compared visually. Locally temperature differences of a few degree and changes in the logarithm of beta of a factor 2-3 can be observed. However we judge that these improvements do not justify the huge computational effort necessary. Prognostic simulations on a century scale have for example shown that such changes in beta do not significantly influence the evolution of the ice cap.

"(close to) equilibrium", "quasi equilibrium" Do you need to stress the solution is not in an equilibrium state?
This paragraph has been completely rewritten since it was not clearly formulated (now in 4.3 “Calibration of the firn heating formulation”).

Page 5116, line 13-17: This sentence is too long and unclear.

It has been reformulated.

Page 5117, line 5: What are the mass balance and surface temperature conditions during the 30 year simulation?

This is a temperature simulation without evolution of the upper surface, hence no mass balance is needed. Surface temperature is kept constant as stated in the paper.

Page 5117, line 14: Do you assume the mass balance is constant from 1995 to 2008? What about temperature?

Both are kept constant. It is in fact the mean present day mass balance.

Page 5118, line 6: “largely unaffected by temperature distribution” » No data are given to justify this statement.

This has been shown the earlier paper Schäfer et al. (2012) as cited in Section 4.1.1. We have replaced the reference to 4.1.1 by the reference to this paper. We also have added an explication of these observations, see answers to questions of other reviewers.

Page 5118, line 10-12: This sentence is not clear.

See Reviewer 2: “are connected”.

Page 5118, line 18-24: I wonder how the authors can judge whether signals are artifacts or not. Uncertainties in the surface and bed elevation also influence the results.

It is correct that from the information presented in the manuscript it was not possible to judge whether these artefacts or not. Since slight changes elsewhere than Franklinbreen are of minor importance, we decided not discuss them in the new version of the manuscript. The text has been modified accordingly.

Page 5119, line 3: “ice core” » "borehole"

Right, has been corrected.

Page 5119, line 12: "1920ies" » ?

We would kindly ask the reviewer to clarify the point of his statement, as by this comment we simply do not know what action we are supposed to take.

Page 5119, line 15-17: "the uncertainty in the basal drag coefficient strongly impacts . . ." » Are you sure about this? The temperature pattern does not influence on the pattern of the basal drag coefficient (page 5113, line 20-22). Any sensitivity test?

The temperature pattern does not strongly influence the pattern of basal friction parameters (see also comment to Reviewer 2) since it only affects very little ice-velocities through the viscosity. However, the basal friction parameter field has impacts on the velocity and hence on the temperature field through advection. Sensitivity test have been conducted, but since the paper is already rather long we limited ourselves to present the more important results. Some tests have been presented in Schäfer et al, 2012.

Page 5120, line 6-8: It is difficult to read this sentence without the definition of Pmax.

Pmax equations have been added.

Page 5120, line 17-22: This part is not clear.

We agree that the second part of the last paragraph of this section was unclear and limited the text to the first part.

Page 5121, line 4: "importance" of what?

We meant the importance of friction heating, this sentences has however completely be changed.

Page 5121, line 9: "contribution" to what?

... impact of strain-heating on the temperature field.

Page 5121, line 11-12: Please refer Figure 5 to justify your argument.

A reference to this figure has been added in the paragraph.

Page 5121, line 19: "cold based sliding" » Do you mean the glacier is sliding over a frozen bed? Or do you attribute this unrealistic result to the sparse bed elevation data?

As stated above (Major concern 2), we agree that there is a discrepancy between the observed sliding parameter distribution and the areas of calculated temperate base. This discrepancy is certainly due to the method-inherent fact that the inversion method of the mechanical sliding parameters is decoupled from the computation of the temperature field and that both (result of inversion as well as temperature field) have errors.

Page 5121, line 25: Isn’t it obvious to get strain heating where ice is sliding?

For friction heating it is not a surprise, yet, for strain heating in the definition as we understand it, we do...
not see a direct link.

We modified the sentence on page 512, line 10-13 to clarify:

“Nuth et al. (2010) observe a balanced or slightly positive volume change over Franklinbreen (average of 0.06±0.12 km3 yr⁻¹) for the period 1990-2005. Even though their result is subject to a large error, it seems unlikely that the observed recent reduction of the acceleration between 2008 and 2011 is driven by a mechanism involving thinning.”

Please see also comment above, by reduction of acceleration we mean that the glacier was still accelerating, but less than before.

We wanted to say: “... firn heating influences the englacial temperature distribution and in turn affects the englacial hydrology.” In the new version this sentence has however been replaced by “... since the englacial temperature distribution affects the englacial hydrology and vice-versa.”

We added some sentences in 4.3 (now 4.4) and 5.4 (now 5.3.2) to better motivate our simulations. Indeed, we are aiming in discussing the speed up of Franklinbreen.

By no means we claim that sliding laws used by other works are wrong. We understand that we have to be more careful with our conclusions. What we wanted to state is, that there is evidence in our inverted basal sliding coefficient distribution that points towards the fact that a direct coupling of increased sliding with temperature in places does not seem to apply in our approach. The text has been changed accordingly.

For future simulations a basal drag parameter constant over time cannot be assumed, since the basal drag parameter obtained with the 1995 and 2008 inversion differ significantly. The paragraph has been reformulated:

“The observed drastic change of inversely determined bed friction coefficient between these snapshots (Sect. 5.1) renders prognostic simulations using a single in time constant set of these parameters highly inaccurate.”

We have and reformulated the sentence.

This sentence does not exist anymore in the new version.

Fig. 2: Here and other plots, please provide (a), (b), (c) . . . for each of the sub plots.

Fig. 3, legend: "initialisation run with ideal T profile" » "initialization", What do you mean by "ideal T profile"?

That is indeed wrong, “ideal T profile” should be “depth dependent T profile”.

Fig. 4, right: Is this the correction relative to the initial ice thickness in (%)?
This figure displays the ratio between the correction (final surface elevation – initial surface elevation) and the initial thickness.

Fig. 5: The unit in the plot is no correct. "°C" » "degree C"

This has been changed on all figures.

Fig. 5, caption: "The last figure remains unchanged whether . . ." » What do you mean?

As discussed in 4.2 we investigated a steady-state (in time constant) as well as an in time changing component driven by changing climate. The point was, that the last sub-picture in Fig. 5 shows the same distribution no matter whether the temporal change in firn heating is accounted for, or not. The simulation names have been added.

Fig. 6: The stake number above the plot should be removed.

The titles have been modified. The choice of these two locations was motivated by the fact that they are locations of some of the best surveyed measurement sites, even though these measurements have finally not been used in this manuscript.

Fig. 8, caption: "Difference in change of surface elevation" » "Change in surface elevation"?

Correct.

Fig. 9: Please provide a scale. It is hard to distinguish the lines delineating the sliding area when the paper is printed.

Both subfigures have the same scale for the texture (temperature distribution) which is shown on the left side of the figure. The colours for the lines are given in the caption and in the figure.
In this manuscript Schaefer et al. use a finite element implementation of the Stokes equations to invert measured surface velocities for a basal friction coefficient. They analyze the observed speed-up of Franklinbreen, and show how it is connected with a decrease in the friction coefficient. They then analyze the influence of various heat sources. They conclude that several of the heat sources, most importantly firm heating and basal friction, greatly impact the basal temperature distribution, but they are unable to conclude on the role of these heat sources on the speed-up of Franklinbreen.

This is an interesting manuscript, it is mostly well written and reaches substantial conclusions. I recommend that it be published in The Cryosphere after some issues are addressed. These issues mostly concern the writing and presentation of the material, and many suggestions are made below. I would place this somewhere between minor and major revisions. I would be willing to review a revision, but do not feel that I need to.

Thanks for this positive feedback and the concrete suggestions for improvements below.

The authors might also find a recently published manuscript (Habermann et al., 2013, TC) interesting, where inversions for basal friction were made in the context of the retreat and acceleration of Jakobshavn Isbrae.

Parts of this paper are concerned with the thermal hypothesis for surge initiation, which, I believe, originally comes from Clarke et al (1984, J. Can. Earth Sc.). I wonder whether this is a bit of a distraction. The paper is not really able to contribute all that much to or against this hypothesis, so maybe too much emphasis is placed in the Introduction. Reading the Introduction the first time frankly left me a bit confused as to the goals of the paper. One of the issues is that there are glacier accelerations with quite different causes and symptoms: tidewater glacier retreats and surges. Tidewater glacier retreats are initiated at the glacier front and show a pattern of thinning, acceleration and retreat that propagates inland. Surges, on the other hand, are initiated farther upglacier, they propagate down with a wave of thickening ice, leaving thinner ice upstream. Franklinbreen is a marine terminating outlet glacier, so one could be left with the initial impression that its acceleration is like those of tidewater glaciers in Alaska or Greenland. But a look at the patterns of elevation change etc leaves one with the impression that it is behaving like a surge. The paper would benefit if this distinction was clearly made in the beginning of the paper.

As suggested by all referees the Introduction has been rewritten. We have strived to link our manuscript to the existing literature pointed out by you.

With respect to the different fast flow modes, see our answer below to comment on p5099, l16/17.

More detailed comments:

p.5099 l.13: It’s not clear to me what ‘natural variability’ refers to here. Are you referring to the ‘tidewater glacier cycle’? In that case you should reference Meier and Post (1987, JGR). The Iken (1981) reference seems irrelevant in this context. The Introduction has been rewritten following the different suggestions. We avoid the term “natural variability”, since it is misleading and changed the references.

l.16/17: the triggering of fast flow in tidewater glacier retreat and in surges is clearly different, at the very least in its spatial pattern. So you do limit yourself to some degree. It is not a bad idea to pursue general patterns, but here it leads to more confusion than insight.

Thanks for pointing this out. We agree that the way we had presented our approach was a bit confusing and state now that we simply focus on the outlets of Vestfonna without assigning them a fast flow mode.

l.20: I recommend using ‘basal motion’, precisely to avoid associating a mechanism such as sliding or sediment deformation.

Good idea, however in the context of models where basal motion is implemented as basal sliding, we stick to the term basal sliding.

p.5100, l.10: I think an Alaskan reference would make more sense here, such as Kamb et al., (1984, Science), since that’s where the hydrologically controlled surges were originally described

The reference has been replaced (we assume that you meant Kamb85).

p.5101, l.28: You might better motivate the paper by stating here the reasons for assessing the evolution of temperature.

Thanks again for a suggestion how to improve our Introduction.

p.5102, l.6: IPY was the International Polar Year (not Geophysical)
There is indeed a confusion, in line 6 it should be IPY (Polar), but in line 7 IGY (Geophysical).

p.5103 l.12: 2 cm is not a velocity error, it’s a displacement error.
Right, has been corrected.

p.5103 l.21-24: Why do you need to interpolate the data sets first? Can’t your misfit functional just quantify the misfit to measurements in areas where you have them? Interpolating and then fitting removes the inversion one step from the original data that seems unnecessary.

We agree that it would be more appropriate to evaluate the misfit exactly at the location of the measurements. However, as the Robin inverse method requires to impose the observations as Dirichlet conditions, it is necessary to interpolate the observations at the mesh nodes which usually do not correspond to the measurement locations. Hence some interpolation will always be necessary.
Also, in 2008 and 2011 data was completed with data from 1995 and interpolation/smoothing again was necessary during this process. Additionally, for prognostic simulations we need to know the friction parameter over the whole ice cap. Since this parameter spans many orders of magnitude and includes high local variations it seems to be more encouraging to interpolate the simpler velocity distribution rather than the complex friction parameter distribution obtained from the inverse method.

p.5105, l.14: Appendix A does not serve a real purpose. These equations are stated in many textbooks and papers, and the only parameters given there are those of gravity and density.
Appendix A has been taken out, the equations have been rearranged (less about the mechanical part of the model, but more details about the thermal part).

p.5106, Eqn(4): You probably don’t want to redo calculations, but Cuffey and Paterson offer a more detailed discussion of A(T) that probably results in more accurate values.
We assume you refer to equation (3.35) in Cuffey and Patterson with just a single value $A_{\mu}$. Indeed, we would be hesitant to redo all our simulations in order to introduce a slightly different formulation that does not change viscosity in a significant way to expect vastly deviating results.

p.5108 l.15: What is $\mu$?
Mu is the effective ice viscosity, its definition has been added.

l.20-23: How is this heat source [firn heating] applied? Is it a uniform source in the uppermost one, two, ? layers?
As can be deduced from Eqs. (12) – (14) it is a depth-dependent distribution, hence not a uniform heat source. It is given by Equation (14) and applied in all layers where $Q(d)$ is not zero, i.e. in all layers from surface to the penetration depth $d_{\text{pen}}$, where $\Delta T$ tends to zero. At the location of the recorded temperature profile, this heat source is applied for example in the uppermost 6 layers.

p.5109,l.9: Maxwell et al. (2008, J.Glac) might be an appropriate reference here, since they introduced this iterative method.
Ok, has been added.

p.5111 l.4-6: This sentence is difficult to understand. Please simplify.
*In the distribution of the basal sliding coefficient regulating the velocity field is determined before a purely mechanical spin-up is conducted (surface relaxation, Sect. 4.1.2).*

->
First, the distribution of the basal friction parameter regulating the velocity field is determined. Then, a purely mechanical spin-up is conducted followed by the calculation of a temperature field making certain assumptions.

l.13-15: Same. Nested sentences, while I much like them, are best avoided in scientific writing, as they, after multiple readings, often still lead to confusion.
*The inherent problem if starting from a DEM purely on observed data, is that an instantaneous solution for both the mechanical and the thermo-dynamical problems is needed as a starting point.*

->
Starting from a DEM purely based on observed data rises an inherent problem since a consistent and instantaneous initial condition for the thermo-mechanically coupled system is required.

l.24: Aren’t you referring still to a spin-up from deglaciated conditions? So surely, there wouldn’t be air temperature and precipitation records for that long?
To our information that kind of long time simulations have been made on a larger scale, for example in “Simulation of the Eurasian ice sheet dynamics during the last glaciation” (P.-L. Forström and R. Greve, Global and Planetary Change 42 (1-4), 59-81 (2004)). Despite the fact that small ice caps like VSF might deserve a more elaborate approach for climate forcing, the lack of knowledge of the basal sliding parameters and existence as well as extents of fast flow areas is unavoidable.
The Seroussi reference deals with Greenland, and it is not clear that it would be applicable to surge-type glaciers.
We agree that a priori we cannot know if results from Greenland are applicable, however it's another publication dealing with inversion and influence of the thermal regime.
I.25: delete 'it was'
Correct, will be done.

It would be interesting how your mismatch between modeled and observed velocities compares to the mismatch that is expected from the velocity errors (see e.g. Truffer (2004, J.Glac.); or better, the book by Parker (Geophysical Inverse Theory)). The interesting thing to know would be whether you are able to fit velocities to within the observational error (avoiding overfitting, see e.g. Habermann et al., 2012) or whether the method does not find a solution for basal stickiness that fits the velocity data well. This would indicate other unresolved errors, such as those of the model (temperature, geometry, ...). The binning in the figure in Schaefer et al. (2012) is too coarse to assess this. You should also show on Fig. 3, where you choose your final solution.

Modelled and measured velocities and errors have been compared and discussed in Schäfer et al. 2012. For the sake of clarity, we prefer not to present a similar or even more complex analysis again in this paper since the results are because of the small changes in the inverse method not very different. A reference to the earlier paper has been added.

We take the last iteration of Fig. 3 (the converged solution) as final beta field, this has been added explicitly to the text.
I.9-11: If I interpret the Schaefer et al (2012) sensitivity experiments correctly, than they address a uniform thickness change. I think it is also important to ask whether slope changes can make a difference. The thinning patterns shown in Moholdt et al. (2010) are quite uneven, and one could imagine that resulting slope changes make an impact. Can you address this?
No, we haven’t conducted such tests for the Vestfonna ice cap. In our opinion such tests should - because of their complexity - rather be conducted on a synthetic glacier. Also we think that setting up such a test suite for sensitivity tests are out of the scope of this paper where we focus on the heat sources and not on technical details of the inversion. To a certain extend we simply want to use the best friction parameter distribution available.

However, we have added a reference to Joughin et al. 2004, who found indeed an increased sensitivity to surface-slope errors compared to fairly robust results towards errors in surface velocity and bedrock topography data.

Why zero mass balance and not something close to observed? Zero mass balance will lead to thickening in the ablation area and thinning in the accumulation area that can quickly reach several meters and that would be compensated by surface balance in reality.

The surface relaxation is an artificial way to diffuse away inconsistencies in the prescribed geometries. Hence, they are very short runs where changes are triggered by local imbalances of the hydrostatic pressure gradient that quickly damp out. As those adjustments by far exceed changes by the usual accumulation/ablation patterns, it really doesn’t make a difference, if one is applied or not. In last consequence, not plugging this balance in at the time we did the relaxation for TCD version is linked to the fact, that at this point we did not have it ready to be deployed. Now, when updating our simulations for the corrected calculation of the friction heat, we updated to the mean present day mass balance.
In order to prove the (un-)importance of the precise choice of the mass balance, we attach a figure (Fig. L1 at the end of this document) showing the result of the relaxation run with present day mean mass balance (the same as used in the paper for the 1995-2008 simulation) and with zero mass balance. The difference is negligible and reflects the integrated mass balance over the short simulation time.
I.16: You could use continuity arguments to make a quick qualitative assessment. If the glacier gets faster, it has to be thinner by continuity. If the bedrock data shows thinning and the velocity data shows slowing down, then the bedrock data has a problem.
We have deleted the statement about the existence of mountain ridges and valleys in the bedrock since it is not essential for the manuscript. We do not really understand what the reviewer means with “continuity arguments”, however from Fig.4 in Petterson et al. 2011 (see Fig. L2) it is clearly visible that the bedrock of Idunbreen/parts of Bodleybreen and Rijpbreen are not resolved in a satisfactory way. Results of the surface relaxation and the calculated "cold based" sliding in exactly these regions also confirms that there is a problem with the bedrock data.
Yes, we do, it has been added.

p.5118: l.5: This is not shown in Sec. 4.1.1, it is shown in a previous paper. Correct.

l.10-13: Sentence is missing a verb: are connected? Correct.

p.5119: l.5: It is not known whether there was an earlier equilibrium, is it? No, it is not known whether there was an earlier equilibrium, however the shape of the measured profile suggests that the temperature distribution was at least similar to a steady-state before the recent changes in surface conditions.

l.15: ‘explained by various facts’ or ‘can occur for various reasons’ ok

l.21: ‘to’ -> ‘than’ ok

p.5120: l.4-6: I would interpret this exactly the other way around: To make an assessment whether a change in this heat source leads to changing basal conditions, you show that one would need to include several centuries of temperature data. We agree that this heat source is not negligible for prognostic runs over several hundreds of years. The term “century scale” was not correctly used in the original text since we had prognostic runs for periods up to 100 years, but not longer, in mind. We have corrected the text to clarify this.

l.8: what is Pmax? Pmax equation has been added in the modified model part.

l.24: Such a comparison is not shown, but might be interesting to include. You only show that change in beta, but the reader does not know how v and tau change. A figure showing the difference between surface and bedrock velocities has been added. We assume that the reviewer makes with the question “how tau changes” reference to a possible change of the basal drag between 1995, 2008 and 2011. We prefer not to include such a figure since it does not reveal important insights connected to our main findings striving not to lengthen the manuscript even more. Also, we have added new figures of the friction heat $q_f=\tau_b \cdot v_b=\beta v_b v_b$ which in a way illustrate the product of beta and $v_b$ compared to beta.

l.26: I know this was shown in a different paper, but I’m somewhat baffled that the temperature distribution doesn’t matter more. It affects viscosity greatly. If more insight can be offered here, I would greatly appreciate it.

It is not a big surprise, if the inversion is forced by the same velocity field to obtain not vastly different values for the velocities and stresses, since the temperature differences are not introducing viscosity changes in orders of magnitudes. Also, the ice velocities of VSF are dominated by sliding, which is in our approach independent of temperature. This idea has been clearer formulated in the paper.

P5118 l.13: What do you mean by ‘internal structure’?
Will be replaced by: “When comparing the obtained basal patterns from 1995, 2008 and 2011, the fine structure of the basal drag coefficient in some of the outlet glaciers differs slightly.”

p.5122, l.23/24: Can you really say that? Without an assessment of 100+ years of temperature variation, you have no basis for this statement. The glacier could react now to a change in firn heating that happened at the end of the LIA. This statement has been taken out.

p.5123, l.9/10: Such a thickness change is also observed, perhaps not to that degree though. Yes, we agree, the observed thickness change is certainly a combination of real thickness change and the effect of fixing the geometry horizontally.

p.5125, l.17: Do you think the sliding law is the problem, or the other unkowns that enter sliding relationships, such as the evolution of effective pressure?

We would like to emphasize first that in this work no sliding law based on physical processes is used, our inversion only delivers us a set of numbers. With respect to the statement in the manuscript that “a sliding law incorporating mechanisms governing acceleration and deceleration” is needed, we agree that the term “sliding law” was misleading, especially since we used in the old version the term “sliding” for basal motion as well as for sliding of ice over the base. With our current understanding of basal motion we do not know well enough which processes play which role for basal motion to judge if it is possible to describe them as sliding (i.e. sliding of ice over the base) or if sediment and hydrology models will be necessary. We have reformulated the paragraph avoiding
the term “sliding law”.

Figure 1: The binning is strange with strange intervals and it is not clear at the upper end. White seems to indicate fast flow, but also missing data. What is the issue for 2011? The lower left shows very speckled fast flow and a separation from slow flow that looks too linear to be real.

The figure has been changed to avoid any confusion in the colour scale and binning.

With respect to the lower left corner, this has been detailed in the text, however we suggest replacing the current sentence by the following to make it clearer:

“In the 2011 ERS-2 dataset, dual-azimuth offset-tracking was considered in the northern part of Vestfonna and here the matching error is estimated to be about 35myr⁻¹; in the southern part SAR data of only one orbit is available and the error of range-azimuth offset tracking is larger, on the order of 130myr⁻¹.”

Figure 2: What do the dots signify? Is it a reflection of mesh size?

Each dot represents one grid point of the inhomogeneous mesh, see Figure 5 in Schäfer et al., 2012.

Martin Truffer
Anonymous Referee #3

General statement

The manuscript entitled “Assessment of heat sources on the control of fast flow of Vestfonna Ice Cap, Svalbard” by Schäfer and colleagues investigates the influence of the ice thermal regime on the initialization and sustainability of fast flow regimes of several outlet glaciers of the Vestfonna Ice Cap. They use the Elmer/Ice software with a thermo-mechanical coupling based on the Full-Stokes equations to reproduce the glacier configuration and initialize their model using data assimilation of surface velocities to infer basal friction for different years. They first assess the importance of the different heat sources on the ice thermal regime and compare their results with measurements along a deep borehole. They then run simulations for about 13 years under different scenarios and compare the evolution of the velocity with observations. They conclude that basal friction is not solely driven by temperature changes but that other parameters such as hydrology or sediments also play a significant role.

The manuscript is supported by previous work based on the Elmer/Ice framework as well as work from the same author on the Vestfonna Ice Cap (Schäfer et al., 2012). The authors present here additional results on this ice cap and provide many different experiments and scenarios to assess the impact of the different heat sources. Simulations include deformation heat, basal friction heat or heat released during refreezing of melt water, some are based on steady-state assumptions and others are run as transient models. The focus of the paper is not always clear and the abstract does not really represent the content of the manuscript. Some very common equations are detailed in the manuscript while others that are not widely used are not provided. I also found myself sometimes confused between the experiments and the thermal model used in some simulations. All these aspects are detailed in the Specific comments section below. I therefore think that the text should be clarified, shortened and better focused in order to improve the quality of the manuscript and the pertinence of the conclusions.

Thanks for this critical, but constructive feedback. We answer below in more details to the suggestions for improvements. Abstract and Introduction have been rewritten, the model section about the thermal equations extended and a table summarizing all simulations introduced.

Specifics comments

The abstract does not really reflect the content of the text and the conclusions of the manuscript. The first part, which deals with general aspects and previous studies, is rather long and should be reduced as this kind of information should be provided in the introduction section. I think that it would be better to focus on the conclusions reached in this study. It is also not clear whether the authors investigate the influence of heat on basal friction or the influence of basal friction on ice thermal regime.

The abstract has been rewritten. The influence between friction and thermal regime by thermo-mechanical coupling in the rheology is a mutual one.

The model section is very detailed for some aspects that are fairly common and already presented in Schäfer et al. (2012) (e.g., Arrhenius law, evolution of free surface) but not so detailed when it comes to the latent heat induced by firn heating and Pmax model. A shorter description of the model with emphasizes on new or improved parts would help clarify this section.

The equation section has been rewritten with the focus less on the mechanical, but on the thermal model properties and equations.

The simulation section sometimes only provides simulation description and sometimes also includes discussion of the results. This section is not very clear and could be better organized. I would like for example to have a very short description of all the simulations performed in the introduction of the section and then go into the details as it is done in the manuscript. I think that adding name, similarly to what is done with 1995ss, that summarize the set-up for all experiments and a table with the description of all simulations would greatly clarify this section.

This is a good idea and will be done by introducing names for all simulations and a summary table.

Technical comments
p.5099 l.15: Add reference for surging glaciers
We assume that the reviewer makes reference to line 14 and have improved our references when rewriting the Introduction, see also comments to other reviewers to the same paragraph.

p.5101 l.22: Add reference for Elmer/Ice software
The Elmer/Ice reference cited later in the paper has been added here.

p.5102 l.10: situated → located

p.5102 l.18: What is the coverage of ground based and airborne radar? What is the distance between flight lines? How are these data combined to provide a two-dimensional map of bedrock elevation?
The coverage of the radar data is highly variable, but is generally less dense in the outlet glaciers. The outline of profiles and assimilation of all the data into one set is already published in Pettersson et. al 2011 and to avoid an even more lengthy manuscript we decided to exclude this from the present manuscript and use only the reference. We suggest to modify p.5102 l18 as in the new version, but if the reviewer feels it is essential to include more details, we could include a more exhaustive discussion in the revised manuscript.

p.5103 l.28 and followings: Can the authors quantify the acceleration?
A reference has been added and the text modified so that a quantitative idea of the acceleration is given:
“Between 1995 and 2008 a net speed-up of at least 100% (doubling of speed) in the Franklinbreen outlet can be seen (Pohjola, et al., 2011), which levelled in during 2008 until 2011. The southern branch continued to accelerate slightly, while the northern branch decelerated (Fig. 1).”

p.5104 l.2-5: This sentence is not very clear, it would help to rephrase it.
The speed up of Franklinbreen, the flow feature showing the biggest change since 1995 (reaching speeds comparable to other fast flowing outlet glaciers in 2008/2011), is modest compared to other Svalbard surging glaciers (Hagen et al., 1993).

->
Franklinbreen is the outlet glacier showing the biggest changes since 1995, reaching speeds comparable to other fast flowing outlet glaciers in 2008/2011, which, nevertheless, are modest compared to other Svalbard surging glaciers (Hagen....).

p.5104 l.14: How is the surface temperature changed? How often?
It’s not changed at all.

p.5104 l.18: How does the surface temperature coincide with measurements?
The stated sea-level surface temperature is estimated from weather stations located on the glacier. There is no other data available to compare with.

p.5105 l.10: What do the authors use for the surface accumulation? Do they include melt water run-off as they have a percolation and meltwater refreeze model?
Most likely the reviewer is referring to the kinematic boundary condition on the next page. The mass balanced used for the prognostic runs is discussed and presented in section 4.3 on page 5117 (now 4.4). For details about the mass balance model we refer to Möller et al., 2011.

p.5106 eq. 2 and 3: These equations are common knowledge and do not need to appear in the manuscript. Same for eq.5
Presentation of the model equations is redone, see other comments.

p.5106 l.13: What is the climate mass balance? Where does it come from?
See comment to Reviewer 1 about the definition of “climate mass balance”. At this part of the paper no mass balance is used. In other places we use output of a climatic mass balance model, see reference in the text.

p.5107 eq.7: no m needed here
Correct.

p.5107 eq.9: How is the temperature kept below melting point (T < Tpm)? What initial value is used for the temperature T ?
We use a consistent method that solves the variational inequality introduced by the heat-transfer equation under the constraint \( T < T_{pm} \), as it is explained in section 6.5. of Gagliardini at al., 2013. As initial value for the temperature field we use the depth dependent profile, which showed the best convergence properties.

p.5108 l.13: What is the link between \( u_b, \tau_b \) and \( vH, \tau_H \) on p.5106?
There was an inconsistency in the notations used in the manuscript, it is \( u_b=v| \) and \( \tau_B=\tau| \). We have homogenized this in the manuscript.

p.5108 l.8: Can the authors please provide the equation of the Pmax model?
Pmax equation has been added.
There are earlier citations for the regularization term. We have added Morlighem et al., 2010 as reference.

What kind of elements are used in the two-dimensional and three-dimensional mesh (triangles, rectangles, ...)? What type of finite elements are used (linear, quadratic, ...)? We use a hybrid mesh of triangles/quati-laters for surface meshes and hence – defined by vertical extrusion – wege type or hexahedral prisms. All elements have linear test functions and use either residual based schemes (Stabilized Elements) or additional element-internal functions (bubbles) for stabilization.

Rephrase this sentence

See answer to Reviewer 2.

I would actually start this section with a brief summary of all the simulations performed, with names and a table that lists them.

See above, very good idea, has been implemented, including the table and names.

It is not very clear what the authors do with the thermal part during the spin-up? Do they use the initial steady-state and keep it constant during the spin-up? Do they compute a new thermal steady-state at each time step (updated with ice thickness and velocity)?

As shown in Figure 3, there is an iteration between the calculation of steady-state temperature and inversion. Then, see Section 4.1.2, temperature and basal drag coefficient are kept constant during the mechanical part of the spin-up (surface relaxation), see also comment below to l18.

reasonable → reasonably

Correct.

this whole introduction of section 4.1 is not clear: it starts with a description of the relaxation but does not go into the details, so it is a little confusing.

It is difficult to anticipate what details the reviewer would expect, this part has been changed and we hope that it is clearer now.

This sentence is not very clear, should be rephrased.

The simulations to determine the best weight for the regularization term, $\lambda$ in Eq. (17) are performed for simplicity with temperature kept fixed to the depth dependent profile (Sect. 3.2) as done by Schäfer et al. (2012).

The best value of the regularization parameter $\lambda$ in Eq. (13) is determined by L-Curve analysis (Hansen, 2001) from a plot displaying $J_{\text{reg}}$ (smoothness of the friction parameter) as a function of $J_0$ (match to observations). This analysis is done for simplicity only once using the 1995 velocity dataset and a fixed temperature distribution given by the depth dependent profile (Sect. 3.2) as done by Schäfer et al. (2012).”

How do they explain this better fit?

We assume that the reviewer makes “with better fit” reference to the fact that the costfunction is lower when iterating between inversion and temperature steady-state calculations.

This is in our opinion obvious, since without the iteration the temperature (and hence viscosity) remains fixed to the initial condition (depth dependent temperature profile) which is not very realistic. Even though the steady-state assumption for the temperature might be wrong, the obtained temperature (and viscosity) fields are at least somehow realistic and it is hence possible to better match the observed surface velocities.

Why do the authors use zero surface mass balance and not a more realistic value?

See answer to Reviewer 2.

Why do the authors evaluate the change on vertical velocity and not rate of thickness change (or surface elevation) change?

Errors in DEMs manifest in strong variations of vertical velocities (e.g., Zwinger and Moore, 2009), that in a prognostic runs implicitly adjust the free surface. These adjustments quickly damp out and hence vertical velocities (especially if they clearly exceed the expected emergence velocities) are a good indicator that there is still an adjustment going on. However in the present case surface elevation changes stabilize at the same time. A figure has been added, showing how surface elevation change and vertical ice velocities stabilize.

I do not understand why the mesh is changing? It should rather be the surface elevation here. Naturally, if the geometry of our computational domain changes we have to adjust our discrete representation of this domain, i.e. the mesh.

It is not clear from this paragraph if a new thermal steady-state was recomputed.
The paragraph has been rewritten.

p.5115 l.25: How do the authors deduce this from the temperature profile? See comment to Reviewer 1 on p.5115 l.15/17, we have modified the text leading to this conclusion.

p.5116 l.5: Consider adding a definition of ice lenses
We added such a definition: “Ice lenses are defined as discrete anomalies in density of the firn column observed by density measurements and by geophysical scanning (DEP) and by ocular inspection of the ice facies. The latter separate the ice facies due to difference in void space / air bubble content. These observations are standard when analysing ice cores.”

p.5117 l.16: Why do the authors start from the thermal steady-state? Why not use the simulation that best fits the measurements?
The simulation was run three times: neglecting firn-heating and starting from both of our firn-heating simulations (with and without the time-evolving part). Only in the third case it is possible to start from the simulation that fits best the measurements. The section has been reformulated.

p.5117 l.24: It is quite confusing that in the section, the authors sometimes just very quickly describe the experiment (e.g., subsection 4.3) and sometimes discuss the settings and some results with a lot of details (e.g., subsection 4.2). The authors should homogenize the different parts of this section. Section 4 has been reorganized following the different suggestions from all reviewers. In the precise case of 4.2 (now 4.3 Calibration of the firn heating formulation) discussion of the results and description of the setup of the simulation cannot be strictly separated, since the setup is a consequence of the results. We hope that the new version is clearer.

p.5118 l.7: I am quite confused by this statement (“The temperature evolution shows high sensitivity to such an inversion”). If the authors start from a thermal steady-state, the temperature should not evolve much.
The word “evolution” in “temperature evolution” was badly chosen. The temperature distribution over the ice-cap is very sensitive to the result of the inversion, see also answer to p. 5119, l. 15-17 (Reviewer 1).

p.5119 l.2: What year was the profile measured?
The profile was measured in 1995, this has been added.

p.5120 l.15: It is not clear what parameterization the authors are referring to here.
We meant the applied Pmax model, which is more or less only a set of parameters. We reformulate: “With the model formulation representing latent heat release ...”

See comment Reviewer 1.

p.5122 l.10: What kind of errors?
Velocity errors.

p.5122 l.20-23: Rephrase
The temperature signal of a recent increase in latent heat release cannot possibly have reached the bedrock to explain the recent acceleration (Fig. 7).

->
We wanted to say “the recent acceleration cannot be explained by the recent increase in release of latent heat (Fig.8), since it cannot possibly have reached the bedrock”, this idea is now however contained in a different way in Section 5.3.2.

p.5124 l.26-29: This paragraph is not very clear, should be rephrased.
This paragraph has disappeared when adjusting to our updated results.

Fig.1: What exactly is uxy, should be defined. What do the diamonds on the figure indicate? White color seems to indicate both very high velocities and no data. Southwest part in 2011 looks really weird.
Uxy stands for velocity in the xy plane or horizontal velocity. We removed this term as it is not used elsewhere in the manuscript and does not contribute to the understanding of the overall paper. We have modified the figure to avoid confusion about the colour scale (see also answer to Reviewer 2). The text describing the data problem in the south-western part in 2011 has also been modified (see answer to Reviewer 2).

Fig.4: What are the units? Are the authors referring to the end of initialization or relaxation?
The units are given in the scalebar, but have been added now also to the caption. We do not understand what the reviewer means by difference between initialization and relaxation. In the caption it reads now “surface relaxation” including the simulation name.

Fig.5: Is the temperature in this figure corrected for Tpm
All temperatures in the figures are given in deg.C relative to Tpm. This has been added to the figure
Legends.

Fig. 7: It would be good to have the 18m equilibrium line to compare with the 13m one. It had not been included since the 450yr graph is very close to the equilibrium, but it is added in the new version.

Fig. 8: Should be $\beta$ (not beta)

ok

Fig. 9: All the temperatures should either be in K or C. All colour scales are in deg.C (relative to Tpm). However we keep the definition of the sliding areas in Kelvin to avoid confusion since is not defined with respect to pressure melting point, but as absolute temperature.
Figures

Figure L1: Relaxation run with zero (left) and present day mass balance (middle) as well as the difference (right).

Figure L2: Figure about the bedrock data quality, Fig. 4b extracted from Pettersson et al. (2011)
Assessment of heat sources on the control of fast flow of Vestfonna Ice Cap, Svalbard

M. Schäfer¹,², F. Gillet-Chaulet³, R. Gladstone¹, R. Pettersson⁴, V. A. Pohjola⁴, T. Strozzi⁵, and T. Zwinger⁶

¹Arctic Centre, University of Lapland, Rovaniemi, Finland
²now at Finnish Meteorological Institut, Helsinki, Finland
³Laboratoire de Glaciologie et Géophysique de l’Environnement (LGGE), UMR5183, UJF-Grenoble 1, CNRS, Grenoble, France
⁴Department of Earth Sciences, Air, Water and Landscape science, Uppsala University, Uppsala, Sweden
⁵Gamma Remote Sensing and Consulting AG, Gümligen, Switzerland
⁶CSC – IT Center for Science Ltd., Espoo, Finland

Correspondence to: M. Schäfer (smartina.ac@gmx.de)
Abstract

Understanding the response of fast flowing ice streams or outlet glaciers to changing climate is crucial in order to make reliable projections of sea level change over the coming decades. Motion of fast outlet glaciers occurs largely through basal motion governed by physical processes at the glacier bed, which are not yet fully understood. Various subglacial mechanisms have been suggested for fast flow but common to most of the suggested processes is the requirement of presence of liquid water, and thus temperate conditions.

We use a combination of modelling, field, and remote observations in order to study links between different heat sources, the thermal regime and basal sliding in fast flowing areas on Vestfonna ice cap. A special emphasis lies on Franklinbreen, a fast flowing outlet glacier which has been observed to accelerate recently. We use the ice flow model Elmer/Ice including a Weertman type sliding law and a Robin inverse method to infer basal friction parameters from observed surface velocities. Firn heating, i.e. latent heat release through percolation of melt water, is included in our model; its parametrisation is calibrated with the temperature record of a deep borehole. We found that strain heating is negligible, friction heating is identified as one possible trigger for the onset of fast flow, and firn heating is a significant heat source in the central thick and slow flowing area of the ice cap and the essential driver behind the ongoing fast flow in all outlets.

Our findings depict a possible scenario of the onset and maintenance of fast flow on the Vestfonna ice cap based on thermal processes and emphasises the role of latent heat released through refreezing of percolating melt water for fast flow. However, these processes cannot yet be captured in a temporally evolving sliding law. In order to simulate correctly fast flowing outlet glaciers, ice flow models not only need to account fully for all heat sources, but also a sliding law based on the basal temperature as well as hydrology and/or sediment physics.
1 Introduction

Recent studies suggest that contributions to sea level change over the coming decades will be dominated by cryospheric mass loss in the form of discharge from fast flowing ice streams or outlet glaciers (Meier et al. 2007; Moon et al. 2012; Jacob et al. 2012; Tidewater Glacier Workshop Report 2013). Therefore, understanding the response of fast flowing features to changing climate is crucial in order to make reliable projections (Moore et al. 2011; Rignot et al. 2011; Shepherd et al. 2012; Dunse et al. 2012). Observations dating back several decades show multiple modes of fast ice flow behaviour including permanently fast flowing outlet glaciers or ice streams, networks of ice streams that switch between fast and slow flow (Boulton and Jones 1979), pulsing glaciers (Mayo 1978), short-term velocity variations of fast tidewater glaciers (Meier and Post 1987), and surging glaciers (Dowdeswell and Collin 1990; Howat et al. 2010; Dunse et al. 2012; Moon et al. 2012; Sund et al. 2009, 2011, 2014) showing occasional massive accelerations of a factor 10 or higher from a quiescent state. The underlying processes behind this range of behaviours are not yet fully understood and need to be addressed.

The flow in fast flowing glaciers occurs significantly faster than would be possible by only internal deformation – even at ice temperatures close to the pressure melting point, when internal deformation is highest. Fast glacier flow is therefore considered to be caused by basal motion through a combination of non-zero ice velocity at the bed, sliding over the bed and fast deformation of soft basal ice or subglacial sediments. Whereas ice deformation is relatively well explained today (Payne et al. 2000), the physical processes controlling basal motion remain to be better understood. Many processes and feedbacks have been suggested to influence basal motion, including sub-glacial hydrology (Kamb 1987; Vaughan et al. 2008; Bougamont et al. 2011; van der Wel et al. 2013), deformation of sub-glacial sediments (Truffer et al. 2000), heat production from sliding (i.e. friction heating, Fowler et al. 2001; Price et al. 2008), strain heating (Clarke et al. 1977; Pohjola and Hedfors 2003; Schoof 2004) or thermal instabilities (Murray et al. 2000). Common to most of the suggested processes is the idea that basal motion requires the presence of liquid water, and thus temperate conditions, at the base of the glacier.
Hence, in order to understand the mechanisms of fast flowing ice it is essential to study the processes maintaining and causing temperate basal conditions as well as more generally the mechanisms leading to changes in the thermal conditions in the ice. For this purpose we examine different heat sources and their impact on basal temperatures together with the redistribution of heat through advection over the ice cap.

Long time scale oscillatory behaviour of fast flowing ice streams can be solely explained by coupled flow and temperature evolution (Payne and Dongelmans, 1997; Hindmarsh, 2009; van Pelt and Oerlemans, 2012). However, shorter timescale oscillations, such as surges, require additional feedbacks or alternative mechanisms (Fowler et al., 2010). The mechanisms behind surges are poorly known, but have been suggested to be mainly hydrologically controlled on temperate (Alaskan type) glaciers (Kamb et al., 1985) or associated with thermal instabilities of polythermal (Svalbard type) glaciers (Payne and Dongelmans, 1997; Murray et al., 2000; Fowler et al., 2001, 2010). The assessment of the heat sources that contribute to the basal thermal regime is also essential for the understanding of such temporal and spatial oscillations.

Here, we use a combination of modelling, field, and remote observations in order to study links between the thermal regime, heat sources and basal sliding in fast flowing areas on Vestfonna Ice Cap. Vestfonna is one of the two major ice caps of Nordaustlandet, the second largest island of Svalbard. Compared to the neighbouring Austfonna ice cap, Vestfonna has a relatively small area of thick slow moving interior ice. It is instead dominated by fast flowing outlet glaciers that extend from the coast to close to the ice divide. All of the fast outlet glaciers on Vestfonna are thought to be topographically controlled (Pohjola et al., 2011) and several of them are believed to have surge-type behaviour (Dowdeswell and Collin, 1990). We will especially focus on the recent speed-up of Franklinbreen (Pohjola et al., 2011), one of the major outlet glaciers on Vestfonna, and simulate ice flow with the Elmer/Ice finite element Full Stokes ice dynamic model (Gagliardini et al., 2013). In the context of ice dynamic models, the term basal sliding is used rather than basal motion, because basal motion is typically modelled by sliding of the ice over the bed. The in-situ complexity is represented in the sliding law by a few free parameters (usually sliding or friction coefficients) - as long as specific sediment or hydrological models are not deployed.
While many large scale flow models use spatially uniform parameters for the friction law but only allow sliding when ice at the bed reaches the pressure melting point (Greve, 1997; Ritz et al., 2001; Quiquet et al., 2013), we solve an inverse problem to constrain spatially varying friction law parameters by determining the best match between model and observed surface velocities. This approach allows quantification of the basal sliding velocity which can help to constrain the in-situ processes (Morlighem et al., 2010; Pralong and Gudmundsson, 2011; Jay-Allemand et al., 2011; Habermann et al., 2012, 2013). It also improves the accuracy of the reproduction of spatial patterns in observed surface velocities. It does not, however, give a direct relationship between temperature and in-situ friction parameter. No predictions for the basal friction parameters at other times than the inversion are possible. From the model we extract information about basal frictional heating (due to sliding at the bed) and strain heating (due to internal ice deformation) as well as their possible evolution during the acceleration of Franklinbreen between 1995 and 2008. In addition we use the Wright $P_{\text{max}}$ formulation (Wright et al., 2007) for refreezing to assess the role of latent heat release due to refreezing of percolating surface melt water in the snow pack (firn heating) that is advected through the ice. This allows us to identify the heat sources responsible for a temperate bed in the fast flowing outlet glaciers. Comparing their time evolution provides insights in the driving mechanisms behind the observed recent acceleration of Franklinbreen and the conservation of fast flow in all outlets.

The research area and observational data have been presented in detail by Schäfer et al. (2012). Key features and additional data are described in Sect. 2. Sect. 3 describes the ice flow model. In Sect. 4 we outline our different simulations. Results with respect to the dominant heat sources, relationships and feedbacks between fast flow, acceleration and the thermal regime are discussed in Sect. 5 before we conclude in Sect. 6.

2 Research area and observational data

Vestfonna (VSF) is characterised by a varied surface topography with two main ridges and strongly pronounced fast outlet glaciers (Dowdeswell and Collin, 1990). In common with most
glaciers in Svalbard VSF is polythermal (Schytt, 1964; Palosuo, 1987). VSF was the target of a recent International Polar Year (IPY) project (Pohjola et al., 2011), and the observational record extends back to the International Geophysical Year (IGY) 1957–1958 (Schytt, 1964) when data on surface elevation (using barometer methods) and ice thickness (from seismic surveys) were gathered. One focus of this work is Franklinbreen, the largest outlet glacier, which is located on the northwestern side of the ice cap and has recently accelerated (Pohjola et al., 2011; Braun et al., 2011).

2.1 Digital elevation models of surface and bedrock topography

As surface elevations we use the digital elevation model (DEM) from the Norwegian Polar Institute (NPI) (1 : 100 000, 1990, UTM zone 33N, WGS 1984) which is based on topographic maps derived from aerial photography. This DEM is completed with the International Bathymetric Chart of the Arctic Ocean (IBCAO Jakobsson et al., 2008) for surrounding sea floor. A comparison between this NPI DEM (1990) and the SPOT-Spirit DEM (2007) shows a difference of less than 10 m on average over Franklinbreen (manuscript in preparation, Pettersson et al.). This misfit is within the uncertainty of the two DEMs and hence the same DEM can be used for all our simulations spanning the period 1995 - 2011.

The bedrock data is a combination of ground-based impulse radar and airborne radio-echo soundings (Pettersson et al., 2011). The ground-based radar was deployed in the central part, while the airborne radar covers outlet glaciers and frontal areas. The interpolated combined DEM has a resolution of 500 m and a vertical resolution of 25 m. A cross validation (Isaaks and Srivastava, 1989) of the DEM shows reasonable low residuals within 55% of the total area of the ice cap (see Pettersson et al. (2011) for further details). Despite a partial inaccuracy due to the relative sparse bedrock data, the first order variability in the bed topography, i.e. major subglacial valleys and peaks, are captured correctly, which is most relevant for modelling the whole ice cap. Exceptions are the south western tip (Idunbreen) as well as the lower parts of Bodleybreen and Rijpbreen where errors in the DEM may be significant as discussed further in Sect. 4.1.2 and 5.3.1B.
2.2 Remote sensing data of surface velocities

The inversion modelling technique used to derive basal friction parameters requires input of measured horizontal surface velocities. Tandem Phase ERS-1/2 1 day SAR scenes were acquired between December 1995 and January 1996 (1 day interval) and surface ice velocities were calculated using SAR interferometry (InSAR) over most of VSF apart from small areas over the lower part of the fast outlets glaciers, where dual-azimuth offset-tracking was employed (henceforth “1995 velocities”, Pohjola et al., 2011; Schäfer et al., 2012). Four ALOS PALSAR scenes were acquired between January 2008 and March 2008 with 46 days time interval and velocities calculated using offset-tracking (henceforth “2008 velocities”, Pohjola et al., 2011). For 2011, an ERS-2 SAR data stack acquired in March/April with a 3 days time interval processed with a combined InSAR and tracking approach similar to the 1995 data (Pohjola et al., 2011) is used (referred as “2011 velocities”, unpublished data). In all cases the vertical components of the velocities have been neglected during the calculation of horizontal velocities.

The displacement error in the InSAR data is 2 cm, which corresponds to a velocity error of $7 \text{ m yr}^{-1}$ for Tandem ERS-1/2 SAR data (1 day time interval) and $2 \text{ m yr}^{-1}$ for 3-days ERS-2 SAR data (Dowdeswell et al., 2008). By considering a matching error estimate of 1/10th of a pixel, the precision of offset-tracking is about $10 \text{ m yr}^{-1}$ for the 2008 ALOS PALSAR data separated by a temporal interval of 46 days (Pohjola et al., 2011). In the 2011 ERS-2 dataset (Fig. 1C), dual-azimuth offset-tracking was considered in the northern part of Vestfonna for the fast flowing glaciers and here the matching error is estimated to be about $35 \text{ m yr}^{-1}$; in the southern part SAR data of only one orbit is available. Consequently InSAR processing could not be used, and the error of range-azimuth offset tracking is very large, on the order of $130 \text{ m yr}^{-1}$. The 2008 and 2011 data sets do not cover the ice cap completely, and data gaps have been filled by interpolation and smoothing, except for the south-western corner (a region of slow flow), where the 1995 data were used to fill a larger data gap (neglecting possible variations in Gimlebreen).

The surface velocities are presented in Fig. 1 (before interpolation and patching). We observe in all three data sets the two very different flow regimes: slow ice flow over the central area of...
the ice cap and high velocities in the outlet glaciers. Between 1995 and 2008 a net speed-up of at least 100% (doubling of speed) in the Franklinbreen outlet can be seen (Pohjola et al., 2011), which levelled in during 2008 until 2011. The southern branch continued to accelerate slightly, while the northern branch decelerated (Fig. 1). Franklinbreen is the outlet glacier showing the biggest changes since 1995, reaching speeds comparable to other fast flowing outlet glaciers in 2008/2011, which, nevertheless, are modest compared to other Svalbard surging glaciers (Hagen et al., 1993).

2.3 Thermal boundary conditions

Different thermal boundary conditions are required in the model, one being surface or air temperature. Svalbard’s climate has a maritime character with cooler summers and warmer winters than is typical at such a high latitude (Möller et al., 2011). Mean monthly air temperatures on VSF do not exceed +3 °C, and winter monthly means fall between −10 °C and −15 °C with minimum values of the order of −25 °C to −40 °C (Möller et al., 2011). A lapse rate approach is used in the current study to prescribe the unaltered surface temperatures

\[ T_{\text{surf}}(x) = T_{\text{sea}}(x) - \gamma S(x) \]

(1)

at the surface elevation \( S(x) \). We use a lapse rate \( \gamma = 0.004 \text{K m}^{-1} \) (Wadham and Nuttall, 2002; Wadham et al., 2006; Schuler et al., 2007). This value is close to the one adopted in other studies: \( \gamma = 0.0044 \text{K m}^{-1} \) (Pohjola et al., 2002). Liljequist (1993) found a slightly larger lapse rate of \( \gamma = 0.005 \text{K m}^{-1} \) from measurements between the summit of Vestfonna (known as Ahlmann summit) and the 1957/58 IGY station at Kinnvika. Data from the atmospheric model WRF (Skamarock et al., 2008; Hines et al., 2011) during 1989–2010 gives a mean lapse rate of 0.0042 K m\(^{-1}\) with variations up to 30% corresponding to up to 1 K in the different directions (B. Claremar Uppsala, personal communication, 2013) and confirms our chosen value.

The mean air temperature at sea level, \( T_{\text{sea}}(x) \), is estimated according to Eq. (1) from data collected during 2005 to 2009 at various weather stations on Austfonna and Vestfonna (Schuler et al., 2007; Möller et al., 2011). We find a mean annual temperature of −7.7 °C and a mean winter temperature of −14.5 °C at sea-level.
In addition, the geothermal heat flux is an important basal boundary condition. Contrary to Schäfer et al. (2012) who assumed a geothermal heat flux of 63 mW m\(^{-2}\) typical for post-Precambrian, non-orogenic tectonic regions (Lee, 1970), we take the value of 40 mW m\(^{-2}\), which is motivated by the measured gradients of profiles obtained by deep drilling on the Nordaustlandet ice caps (Zagorodnov et al., 1989; Ignatieva and Macheret, 1991; Motoyama et al., 2008). In the case of Nordaustlandet, ground surface temperature changes in the uppermost 1–2 km of the bedrock are most likely still influenced by the cold of the Weichselian period, explaining this lower measured value of 40 mW m\(^{-2}\) and leading to good simulations of an observed (via deep drilling) temperature profile on VSF (Motoyama et al., 2008), as explained in Sect. 4.3.

3 Model description

The model equations are solved numerically with the Elmer/Ice model. It is based on the open-source multi-physics package Elmer developed at the CSC – IT Center for Science in Espoo, Finland, and uses the finite element method (Zwinger et al., 2007; Gagliardini and Zwinger, 2008; Gagliardini et al., 2013). More details on the model implementation can be found in Schäfer et al. (2012); Gagliardini et al. (2013). Numerical parameters used in our study are summarised in Table 1.

3.1 Forward model

The ice is modelled as a non-linear viscous incompressible fluid flowing under gravity over a rigid bedrock. The force balance (quasi static equilibrium) is expressed by the Stokes equations. The rheology of the ice is described by Glen’s law assuming isotropic behaviour. The temperature dependency of the deformation rate factor, \(A(T, T_{pm})\), is described by the Arrhenius law (Paterson, 1994) with parameters as in Table 1 where \(T\) is the temperature and \(T_{pm}\) the pressure melting point. The evolution of free surface \(S\) is governed by a kinematic boundary
condition prescribing the climatic mass balance (Cogley et al., 2011) as vertical component. $S$ is assumed to be a stress-free surface, i.e. $\tau \cdot n = 0$ where $n$ is the normal unit vector.

At the lower boundary $B$, a linear friction law (Weertman law) in the form of a Robin boundary condition (Greve and Blatter, 2009) is imposed

$$t \cdot (\tau \cdot n) + \beta(x, y)v \cdot t = 0,$$

where $\beta$ is the basal friction parameter, $t$ a unit vector in the tangential plane aligned with the basal shear stress, $v_\parallel$ and $\tau_\parallel$ are the components of the velocity $v$ and stress components $\tau$ parallel to the bed at the base. We assume zero basal melting ($v \cdot n = 0$). The basal friction parameter field $\beta(x, y)$ will be inferred in this study from surface velocities using an inverse method (Sect. 3.3).

On the lateral boundaries the normal stress is set to the water pressure exerted by the ocean $p_w$ for elevations below sea-level, else we assume stress-free condition.

### 3.2 Temperature model

Schäfer et al. (2012) use a temporally fixed depth dependent temperature profile (here referred to as depth dependent temperature profile)

$$T(x) = T_{surf}(x) + \frac{q_{geo}}{\kappa} D(x),$$

where $q_{geo} = 40 \text{ mW m}^{-2}$ is the geothermal heatflux, $\kappa = 2.072 \text{ W K}^{-1} \text{ m}^{-1}$ a for the temperature range representative heat conductivity of ice and $D(x)$ the ice depth (vertical distance to the surface at a given location $x$ in the ice body). Temporal evolution of the temperature field is governed by the heat transfer equation, which reads

$$\rho c \left( \frac{\partial T}{\partial t} + v \cdot \nabla T \right) = \nabla \cdot (\kappa \nabla T) + Q \text{ with } T \leq T_{pm},$$

where $\rho$ is the ice density and $Q$ a volumetric heat source. The consistent method solving this variational inequality is detailed in (Zwinger et al., 2007; Gagliardini et al., 2013). The depth
dependent temperature profile from Eq. (3) has proved to show the best convergence properties as initial condition. The heat capacity $c$ and heat conductivity $\kappa$ are functions of temperature (Ritz, 1987), turning Eq. (4) into a non-linear problem:

$$c(T) = 146.3 + 7.253T \text{ (unit J kg}^{-1}\text{ K}^{-1})$$ \hspace{1cm} (5)

$$\kappa(T) = 9.828 \cdot \exp(0.0057 \cdot T) \text{ (unit W m}^{-1}\text{ K}^{-1}).$$ \hspace{1cm} (6)

At the upper boundary a Dirichlet condition is imposed on $T$ using the parametrisation described by Eq. (1) in Sect. 2.3. At the bed a jump in the normal component of the imposed geothermal heat flux $q_{\text{geo}} = \kappa \text{ grad} T \cdot n$ is given by the surface production due to friction heat $q_f = v ||\tau ||$.

The volumetric heat source $Q$ comprises strain heat $Q_s = 2\mu \epsilon^*$, where $\epsilon^*$ is the second invariant of the strain rate tensor and $\mu$ the effective ice viscosity, as well as latent heat from firn heating $Q_1$ (latent heat released during refreezing of percolating melt water). $Q_1$ is calculated using the $P_{\text{max}}$ model of Wright et al. (2007) as used by Zwinger and Moore (2009). $P_{\text{max}}$ is defined as the maximum proportion of the annual snowfall which can be retained by refreezing before runoff occurs and was originally chosen to be 0.6 (Reijmer et al., 2012). The depth integrated amount of energy $Q_{l}$, $P_{\text{max}}$ and the annual snowfall $B$ are related through the latent heat of fusion $L$ by

$$P_{\text{max}}B = Q_{l}/L.$$ \hspace{1cm} (7)

The model of Wright et al. (2007) provides a simple, yet realistic method of calculating $P_{\text{max}}$ as a function of the mean annual and winter temperature. The energy to warm the uppermost part of a glacier from the end of the winter to post-refreezing temperatures is estimated and identified with the heat sink available to be filled with latent heat ($Q_{l}$). Following this approach, different characteristic shapes of the time averaged temperature-depth profiles $\bar{T}(d)$ in summer and winter are used (Wright et al., 2007) to calculate the latent heat at each point and each depth.
of the glacier:

\[ \overline{T}(d) = \left( \frac{d}{d_{\text{ice}}} - 1 \right) (\overline{T}_a - \overline{T}_w) + \overline{T}_a, \text{ in the winter,} \]  

\[ \overline{T}(d) = \overline{T}_a \left( 1 - \frac{(d - d_{\text{ice}})^2}{d_{\text{ice}}^2} \right)^{\frac{1}{2}}, \text{ in the summer,} \]  

where \( d \) is the depth below the surface. There are three free parameters in this firn heating parametrisation: \( \overline{T}_a \) and \( \overline{T}_w \) are the annual and winter mean air temperatures respectively set according to Sect. \( 2.3 \). \( d_{\text{ice}} \) is the typical penetration depth of the annual temperature cycle which is kept as a free parameter and tuned to reproduce the measurements in the deep ice core (Motoyama et al., 2008), see Sect. \( 4.3 \).

The resulting non uniform volume heat source is deduced by the difference of internal energy defined by the difference \( \Delta \overline{T}(d) \) between the seasonal profiles Eqs. (8) and (9) and decreases steadily from the surface to the penetration depth, in all layers below \( d_{\text{ice}} \) it is zero:

\[
\begin{cases}
Q_1(d) = c\rho \Delta \overline{T}(d), & \text{if } d \leq d_{\text{ice}}, \\
Q_1(d) = 0, & \text{otherwise}.
\end{cases}
\]  

### 3.3 Inverse model

A variational inverse method (Arthern and Gudmundsson, 2010) is used in this study to infer the spatially varying basal friction parameter \( \beta(x,y) \). It is based on the minimisation of a cost function when solving the Stokes Equations iteratively with two different sets of boundary conditions. The definition of the cost function and the minimisation algorithm follow Gillet-Chaulet et al. (2012) and Jay-Allemand et al. (2011). This approach is similar to Schäfer et al. (2012) but with the addition of a regularisation term (Morlighem et al., 2010; Habermann et al., 2012).

The method iteratively applies a Neumann and a Dirichlet condition at the upper free surface as introduced by Maxwell et al. (2008). In the Dirichlet problem the Neumann free upper surface
condition is replaced by a Dirichlet condition where the observed surface horizontal velocities are imposed

\[ \mathbf{v}_\text{hor}(x) = \mathbf{v}_\text{obs}(x), \forall x \in S, \]  

(11)

where \( \mathbf{v}_\text{hor}(x) \) and \( \mathbf{v}_\text{obs}(x) \) stand respectively for the modelled and observed horizontal surface velocities. In \( z \)-direction, \( (\mathbf{\tau} \cdot \mathbf{n}) \cdot e_z = 0 \) is imposed on \( S \), where \( e_z \) is the unit vector along the vertical. To avoid unphysical negative values, the friction parameter field \( \beta(x, y) \) is expressed as \( \beta = 10^\alpha \) and the minimisation of the cost function is performed with respect to its logarithm \( \alpha \). The cost function, which expresses the mismatch between the two solutions for the velocity field with different boundary conditions on the upper surface \( S \), is given by

\[ J_0(\beta) = \int_S (\mathbf{v}^N - \mathbf{v}^D) \cdot (\mathbf{\tau}^N - \mathbf{\tau}^D) \cdot \mathbf{n} \, dA, \]  

(12)

where the superscripts \( N \) and \( D \) refer to the solutions of the Neumann and Dirichlet problems, respectively. To avoid unphysical small wavelength variations in \( \alpha \) and to ensure to find a stable unique solution, a Tikhonov regularisation term \( J_{\text{reg}} \) penalising the first spatial derivatives of \( \alpha \) is added to the total cost function \( J_{\text{tot}} \)

\[ J_{\text{tot}} = J_0 + \lambda J_{\text{reg}}, \]  

(13)

\[ J_{\text{reg}} = \frac{1}{2} \int_B \left( \frac{\partial \alpha}{\partial x} \right)^2 + \left( \frac{\partial \alpha}{\partial y} \right)^2 \, dA, \]  

(14)

where \( \lambda \) is a positive parameter (see Sect. 4.1.1 for its choice). The minimisation of the cost function is thus a compromise between best fit to observations and smoothness of \( \alpha \), determined by the tuning of \( \lambda \). The minimisation algorithm is described by Gillet-Chaulet et al. (2012) and Gagliardini et al. (2013).
3.4 Meshing

Anisotropic mesh refinement is now increasingly used in numerical modelling especially with the finite element method since the mesh resolution is a critical factor. Schäfer et al. (2012) have investigated effects of varying the resolution in the context of this inverse method. Here we use again the mesh established with the fully automatic, adaptive, isotropic surface remeshing procedure Yams (Frey, 2001). A 2-D footprint-mesh was established according to the glacier outline on the 1990 NPI-map and adapted using the metric based on the Hessian matrix of the observed 1995 surface velocities. Horizontal resolution varies between 250 m and 2500 m. Finally the mesh was extruded vertically in 10 equidistant terrain following layers according to the bedrock and surface data. The obtained mesh consists of linear wedge type and hexahedral prism elements. In the simulations involving firn heating, the mesh was extruded vertically in 20 layers with the upper 10 layer thicknesses reducing towards the surface following a power law. The robustness of the total vertical layer number was already verified (Schäfer et al., 2012), doubling the number of boundary layers also lead to robust results in the runs including firn heating.

4 Simulations

In this section we present the setup of our simulations. Four types of simulations are conducted and summarised in Table 2:

- Simulations dealing with the thermo-mechanical spin-up. These serve as starting points for all other simulations (Sect. 4.1). Since an ideal thermo-mechanical spin-up is unfeasible, we describe our alternative initialisation. First, the distribution of the basal friction parameter regulating the velocity field is determined. Then, a purely mechanical spin-up is conducted followed by the calculation of a temperature field making certain assumptions.
– Simulations to investigate the importance and influence of the mechanical heating (strain and friction heating) (Sect. 4.2).

– Simulations to calibrate our firn heating formulation (Sect. 4.3) with a measured temperature record in a deep borehole.

– Short prognostic simulations (Sect. 4.4) aiming to reproduce the observed acceleration of Franklinbreen and identify the underlying driving mechanisms.

4.1 System initialisation

Starting from a DEM purely based on observed data rises an inherent problem since a consistent and instantaneous initial condition for the thermo-mechanically coupled system is required. Thermal initial conditions are critical in modelling of polythermal glaciers or ice sheets because of the energy storage capacity of ice, the low advection/diffusion rates on the glacier and the strong thermo-mechanical coupling via the ice viscosity. An ideal spin-up would demand a transient run starting from deglaciated conditions with a long enough spin-up time requiring realistic forcing (temperature, mass balance) as well as knowledge of the strongly time-varying velocity field. Air temperature and precipitation records might exist over a long enough time, however the temperature distribution at a given instant is driven by the past evolution of advection, diffusion of heat and heat sources, and hence by the past velocity field.

In the absence of such ideal spin-up we decouple inversion for basal friction parameter from the thermo-mechanical problem as detailed in the following sections.

4.1.1 Inverse simulations to derive spatial patterns of the basal friction

The method of Schäfer et al. (2012) is followed with some improvements: correct marine boundary conditions are applied, a regularisation term in the cost function is added and a better minimisation algorithm is used. The best value of the regularisation parameter $\lambda$ in Eq. (13) is determined by L-Curve analysis (Hansen, 2001) from a plot displaying $J_{\text{reg}}$ (smoothness of the
friction parameter) as a function of $J_0$ (match to observations). This analysis is done for simplicity only once using the 1995 velocity dataset and a fixed temperature distribution given by the depth dependent profile (Sect. 3.2) as done by Schäfer et al. (2012). This is justified since the result of the inversion depends only very little on the small thermally induced variations in the ice viscosity as shown in Schäfer et al. (2012). We find $J_0$ is minimised by setting $\lambda = 10^{5.0}$, which also leads to acceptable smoothness in $\beta$ (simulation beta95). Modelled and observed velocities show a close match as in Schäfer et al. (2012).

In a similar way, we conduct inversions for the basal friction parameter with the 2008 and 2011 surface velocity fields (Fig. 2, simulations beta08 and beta11). These runs are conducted also with the 1990 surface DEM, since no complete additional surface DEM is available. Nuth et al. (2010) and Moholdt et al. (2010) have shown from ICESat laser altimetry data that mean elevation changes over VSF were $0.05 \text{ m yr}^{-1}$ and $-0.16 \text{ m yr}^{-1}$ over the periods 1990 to 2005 and 2003 to 2008 respectively. The changes form a complex spatial pattern on VSF, with local values up to $1 \text{ m yr}^{-1}$ in the south. It has been shown (Schäfer et al., 2012) that surface variations of this order (or higher) between 1995 and 2008 or 2011 do not significantly affect the friction parameter fields derived from the inverse method. Some error is however expected from changes in the surface slope resulting from these small surface elevation changes (Joughin et al., 2004). The same value for the $\lambda$ parameter as well as the same inhomogeneous mesh have been used, the latter to facilitate comparison.

An iteration scheme between inversion and temperature calculation has been tested (simulations beta95T and beta08T). The depth dependent temperature profile (Eq. (3)) was used in the first inversion for $\beta$, then steady-state calculations of the temperature field (accounting for friction and strain heating) and inversions were run alternately. The resulting $\beta$ distribution reveals small changes compared to keeping the temperature fixed to the depth dependent profile, showing a certain robustness of $\beta$ towards changes in temperature. Nevertheless, the value of the cost function has been decreased with the iterative scheme (Fig. 3), showing an improved match between observed and computed surface velocities. Convergence of this iteration was assessed through the cost function and stopped once the cost function stabilised. Convergence of the steady-state temperature field was ascertained through visual inspection.
The effect of firn heating on the resulting $\beta$ distribution has been studied separately (simulation $\text{beta95Tfirn}$). This is motivated both by the need to save computing resource (firn heating simulations require higher vertical resolution), and because of the greater uncertainties associated with this heat source compared to the others. It causes very little change to $\beta$. For all further simulations in the current study, the distribution of $\beta$ obtained with the iteration scheme (but with firn heating omitted) is used (Fig. 2a, iteration no. 120 in Fig. 3).

### 4.1.2 Surface relaxation

Remaining uncertainties in the model initial conditions (including uncertainties in the model parameters as well as the domain geometry), lead to ice flux divergence anomalies (Zwinger and Moore, 2009; Seroussi et al., 2011), resulting in a non-smooth vertical velocity field. Because of the importance of the vertical velocity field for advection of cold ice from the surface, and to smooth out these ice flux divergence anomalies, the free surface is relaxed before being used for further simulations. The relaxation simulations lasted for three years under mean present day climatic surface mass-balance (Möller et al., 2011), simulations $\text{surfrelax95}$ and $\text{surfrelax08}$. They were initialised with output from the inversion-temperature iterations $\text{beta95T}$, $\text{beta08T}$.

A short time step (0.1 yr) was chosen to guarantee temporal resolution of artificially strong surface changes induced by the remaining uncertainties (Zwinger and Moore, 2009). Visual inspection of the smoothness and magnitude (Fig. 4) of the vertical velocity field as well as surface elevation adjustments were used to determine the end of the relaxation procedure. The largest changes to the mesh (Fig. 5) occur in the southwestern corner and in some outlet glaciers where there is a significant paucity of bedrock radar data (see Pettersson et al., 2011, for radar coverage). Some other less important changes are visible in northeastern VSF – again in areas with sparsely covered bedrock data.

A more complex spin-up scheme involving an iteration between surface relaxation, inverse method and temperature calculation was also tested for a single combination of surface velocity data and included heat sources in the temperature calculation (simulation $\text{surfrelax95c}$). This procedure requires huge computational efforts and does not lead to visible improvements in the results ($\beta$ field, temperature field, and surface corrections) and is hence not used in this work.
for several different combinations of surface velocity data (3 possibilities) and included heat sources (6 possibilities).

4.1.3 Thermal initialisation

In the absence of an ideal thermomechanical spin-up, a steady-state temperature field was computed after the mechanical relaxation, even though non-steady-state conditions have occurred on VSF between 1995 and 2008, and probably on longer time scales. We found characteristic timescales to reach such a steady-state to be of the order of several hundreds of years (not shown in this paper). This will lead to over- or underestimations of the temperature depending on the past state of each outlet glacier. Seroussi et al. [2013] addressed the question of thermal initial conditions on the Greenland ice sheet and came also to the conclusion that steady-state temperatures based on present-day conditions are a reasonably good approximation both for calculations of basal conditions and century-scale transient simulations.

4.2 Temperature steady-states including mechanical heating

Strain and friction heating (Sect. 3.2) are effective locally in the outlets and given by the mechanical model. To discuss their influence on the thermal regime of the ice cap, temperature steady states for various combinations of the heat sources are calculated (Fig. 6, simulations 1995ssA, 1995ssSH, 1995ssFH, 1995ss, 2008ss, Table 2). In these simulation only the internal ice temperature is allowed to evolve; the surface temperature, velocity field and geometry are kept fixed. At two locations on Franklinbreen and Frazerbreen (locations of well surveyed measurement sites), in the ablation and in accumulation areas respectively as well as the location of drill hole (locations are indicated in Fig. 6), the simulated temperature-depth profiles are extracted (Fig. 7).

4.3 Calibration of the firn heating formulation

In order to calibrate the parameters (penetration depth, mean annual and winter temperatures) in the applied firn heating parametrisation (see Sect. 3.2), we study the temperature profile at the
location of the drill hole (see Fig. 6 for the exact location). Using only strain and friction heating (simulation 1995ss), the measured temperature profile (Motoyama et al. (2008), recorded in 1995) cannot be reproduced, even close to the bedrock where these heat sources are most effective (Fig. 8 green line compared to the data in red).

In our firn heating formulation we assume that the penetration depth increases linearly from 0 m at the elevation of an average firn line to the maximum penetration depth $d_{\text{ice}}$ at the summit, leading to the effect that firn heating increases with altitude above the firn line. This is also in line with the usual approach of calculating refreezing as a fraction of winter accumulation, which is best described on VSF by an elevation gradient (Möller et al., 2011). In reality the melting should be largest at low (warmer) elevation, but this is at least partially counterbalanced by the formation of ice lenses inhibiting penetration of melt water. Ice lenses are defined as discrete anomalies in density of the firn column observed by density measurements and by geophysical scanning (DEP) and by ocular inspection of the ice facies. The latter separate the ice facies due to difference in void space / air bubble content. These observations are standard when analysing ice cores. Ice lenses are more effective with more melt, thus in lower altitude. Assuming increasing firn heating with increasing firn thickness (altitude) is hence a simplification of these competing effects. The mean elevation of the firn line was digitised from several satellite pictures: Landsat July 1976, September 1988 and August 2006; Spot July 1991 and August 2008; Aster August 2000 and July 2005. Two of these lines (August 2008 and September 1988) have been excluded since the firn lines are located at exceptionally low elevations probably due to abnormally early fresh snow. We observe little change over recent decades, as found by Möller et al. (2013). Since the firn line elevation is approximately uniform over most of the ice cap, a single mean elevation for the firn line is assumed ignoring any other spatial variations. We estimate this mean elevation to be 410 m a.s.l (Fig. 9). This is consistent with estimates of the average equilibrium line elevation (326 m a.s.l., Möller et al., 2013), since on Svalbard glaciers the equilibrium line is typically located significantly lower due to extensive superimposed ice formation (Möller et al., 2011; van Pelt et al., 2012). For future prognostics with climatically varying mass balance the firn line elevation could be parametrised by the elevation of the equilibrium line.
Using the mechanical setup as described in Sect. 4.2, a variety of simulations with different parameters of the firn heating formulation were conducted. Different initial conditions, steady-state and time evolving simulations have been tested and lead to the following conclusions:

(1) The measured inflection in the upper part of the temperature profile is a transient effect occurring on decadal timescales. Fig. 8 illustrates the smoothing of the inflexion and its propagation towards the bedrock when approaching equilibrium.

(2) Significant changes of temperature at the bedrock induced by firn heating occur in simulations over timescales long enough to conduct heat to the bottom (centuries). Such simulations lead to temperature profiles similar to steady-state profiles (Fig. 8), but with a spatially uniform warm or cold shift.

Consequently we hypothesise that the measured borehole profile can be modelled by a succession of a steady-state followed by a time evolving simulation with different surface boundary conditions: low firn heating for the steady state simulation and an increase in firn heating prescribed for the more recent time evolving simulation, which has not yet reached equilibrium at present day.

In order to tune the model to match drill hole observations (Motoyama et al., 2008), we make the hypothesis that boundary condition change can be represented in our firn heating parametrisation by the penetration depth parameter and keep the surface temperatures fixed to the observations of the weather stations as stated earlier. A first run of the model (simulation firnss) to equilibrium temperature using a maximum penetration depth of 13.2 m leads to a good match with the measurements in the lower part of the drill hole, see Fig. 8 (blue line). A prognostic run (simulation firnevol) over 35 yr with an increased penetration depth of 17.6 m starting from this equilibrium state allows for a reasonably good match with the observed peak in the upper layers (black line). Because of advection, on a long enough time-scale, firn heating affects the thermal regime of the whole ice cap including below the firn line. The horizontal distribution of the modelled temperature at the bedrock including firn heating is shown in Fig. 9 and the vertical in Fig. 7 as well as Fig. 8.
4.4 Prognostic simulations over the period 1995–2008

In addition to the steady-state experiments with the different heat-sources, we conduct prognostic simulations with three different temporal evolutions of $\beta$ prescribed to simulate the recent acceleration of Franklinbreen and analyse the connected evolution of all system variables to get a better understanding of the underlying mechanisms. The three simulations are run with full thermomechanical coupling starting from the relaxed 1990 DEM (surfrelax95), forced by mean present day climatic mass balance (Möller et al., 2011), and with constant surface temperature and glacier extent. The three basal friction parameter evolution scenarios are:

1. The basal friction parameter kept constant at the 1995 pattern (simulation $const13$).

2. A sudden switch after five years to the 2008 pattern (which differs from the 1995 pattern mainly by the acceleration of Franklinbreen, simulation $sudden13$).


All simulations span 1995–2008. Each simulation was run three times – excluding and including firn heating ($firnss$ or $firnevol$) to study different scenarios. In the simulation excluding firn heating, temperature is initialised to the 1995 steady-state temperature profile $1995ss$. In the simulations including firn heating temperature is initialised to $firnss$ and $firnevol$ respectively. Changes in surface elevation and basal temperatures are shown in Figs. 10 and 11.

5 Discussion

5.1 Implications of inferred basal friction parameter distributions

The basal friction parameter $\beta$ is a crucial parameter in simulating the thermodynamical regime of VSF as it is a key control on sliding velocities, which govern both friction heating and heat advection. As already shown by Schäfer et al. (2012), use of an inverse method to derive the spatially varying basal friction parameter is largely unaffected by temperature distribution (Fig. 2).
This is because temperature, which impacts on deformation, does not feature in the sliding relation, and in VSF outlet glaciers sliding dominates over deformational velocities. Conversely, the temperature distribution shows high sensitivity to such an inversion since even small changes in the basal friction parameter can introduce important changes in the velocity field, especially the vertical component, which then affects the heat redistribution through advection. Surface relaxation (Sect. 4.1.2, simulations surfrelax95,08) reduces this sensitivity by producing smoother velocity fields.

Variations in the basal friction parameter distribution across the three periods (Fig. 2) are due to large variations in observed surface velocities and indicate the importance of a time evolving basal friction parameter based on the underlying physical processes. When comparing the obtained basal patterns from 1995, 2008 and 2011 (simulations beta95, beta08, beta11), the fine structure of the basal friction parameter in some of the outlet glaciers differs slightly, but the most striking change remains the acceleration of Franklinbreen from 1995 to 2008 featured by a strong increase in basal sliding. The 2011 β-distribution mainly reflects the different changes in velocity pattern in the two branches of Franklinbreen: the northern one is decelerating while the southern one continues to accelerate. In all outlet glaciers a distinct spatial variation of β can be seen, indicating that a sliding law also needs to reproduce these variations, especially since Schäfer et al. (2012) have shown that spatially constant friction parameters specific to each outlet glacier do not allow reproduction of the observed velocity structure within the outlet glaciers. A sliding law based on the presence of temperate ice at the base could not reproduce such a fine structure.

5.2 Interpretation of a temperature profile from deep drilling

As shown in our simulations concerning the calibration of the firn heating formulation (Sect. 4.3, simulations firnss, firnevol), the observed shape of the temperature profile measured in 1995 in the borehole (Motoyama et al., 2008) (Fig. 8) cannot be explained by an equilibrium temperature profile. Our model-supported interpretation requires a recent perturbation away from an earlier (close to) equilibrium state, caused by a change in the surface conditions. This can be motivated by the fact that ice cores elsewhere in Svalbard indicate that periods of firn heating
and percolation have been frequent in the last 500–1000 yr \cite{vandeWal2002, Divine2011}. The proportion of ice lenses which indicate periods of near zero ice surface temperatures increased from 33% during the Little Ice Age to 55% in the 20th century \cite{Pohjola2002}.

\cite{vandeWal2002} came to a similar conclusion when reconstructing the temperature record in the Lomonosovfonna plateau (northeast of Billefjorden/Isfjorden, Spitzbergen). However, they kept the surface temperature as tuning parameter. Their obtained surface temperature is too high and induces a shift of a few Kelvin. Hence model and data fit well in the lower part, but the surface values are unrealistically warm. They conclude a change in surface conditions in the 1920ies from their model and find confirmation for this by comparing to the mean air temperature record at Svalbard airport starting in 1910. Discrepancy between our model-implied change in the 50ies or 60ies and the actual climatic record can be explained by various facts: first, as stated earlier, the uncertainty in the basal friction parameter strongly impacts the evolution of the temperature distribution through advection. Second, only one data set of deep borehole temperatures is available for model calibration. Lastly, our approach might be too simplistic, especially with respect to the assumptions of spatial or elevation dependencies, and the use of penetration depth as the only calibration parameter (surface temperature variations also lead to temperature variations at depth but with different timescales than the penetration depth and certainly have not been constant in the past. It is known for example that there were periods warmer than the first decade of the 21st century \cite{DAndrea2012}).

The calibrated penetration depths (13.2 m and 17.6 m, Sect. 4.3) exceed the expected values, since measured relative densities \cite{Motoyama2008} reach values over 0.85 at 10 m depth and below, i.e. values of ice or ice lenses with very slow percolation. These unrealistically high values for the penetration depth can be explained by the omission of firn layer compressibility in the mechanical model \cite{Zwinger2007}, and imply that our approach should be considered as qualitative rather than quantitative.

With respect to this more recent change in the conditions on the surface (simulation firnevol), our model predicts that combined advective and diffusive processes will take over 100 yr to propagate this signal to the base in the centre of the ice cap (see Fig. 8) and over the outlets (not
shown). Thus for studying basal processes, even for prognostic simulations up to a century, we can neglect the effects of this change in firn heating. Conversion of the latent energy released at the location of the ice core and comparing to snow fall corresponds at the location of the drill hole to a $P_{\text{max}}$ value of 0.9, which is in the expected range (Wright, 2005), increasing confidence in our model.

A discrepancy between our equilibrium profile and the data is also apparent in the middle of the depth profile. We explain this either by the fact that the ice cap had not yet reached the first thermal equilibrium corresponding to the first penetration depth of 13.2 m before the change in surface boundary conditions occurred (see the 50/100/150 yr etc. graphs in Fig. 8 for the shape of such profiles in the lower and middle part) or by our simplified assumption of a constant surface temperature or by the impacts of uncertainties in advection (Sect. 5.1) or the geothermal heat flux. With the model formulation representing latent heat release due to refreezing we are qualitatively able to reproduce the observed profile, indicating that we identified the driving mechanisms behind the measured distribution. Different limitations of this model have been highlighted.

5.3 The role of heat sources for VSF fast flowing outlet glaciers

A comparison between surface and sliding velocities at the bedrock (see Fig. 12a) clearly shows that sliding dominates the ice dynamics at the fast flow areas of VSF, which even holds for increased deformation in the case of by temperature lowered viscosity (Schäfer et al., 2012). Schäfer et al. (2012) further showed that the temperature distribution has little impact on both surface velocities and basal friction parameter obtained with the inverse method. Therefore we focus on the impact of temperature and the respective heat sources on the onset and maintenance of fast flow.
5.3.1 Interpretation of calculated temperature steady-states

Even though the ice cap had probably neither in 1995 nor in 2008 reached a temperature steady-state, examination of the steady-state distributions gives insights in the possible impacts of the heat sources.

A) Strain and friction heating

Strain heat integrated over the whole ice column is of the same order of magnitude as friction heat at the bed, Fig. 12b. It is mainly confined to the shear margins and corresponds well to the few areas of non negligible deformational velocities, Fig. 12a. Friction heat is greatest at the transition of fast and slow flow, i.e. at the margins of the southern fast flowing outlets as well as in the areas of changing basal friction parameter of Franklinbreen, Fig. 12c. Simulation 2008ss shows that it is also important in areas of very high velocities at Franklinbreen, Fig. 13c. Larour et al. (2012) observed a similar spatial distribution of strain and friction heat.

In contrast to Pohjola and Hedfors (2003), who investigated fast flow in Antarctica using a one-dimensional numerical thermodynamic model, the impact of strain heating on the temperature field is found to be very small on VSF, see Fig. 6b and 7, simulation 1995ssSH. Brinkerhoff et al. (2011) found for some Greenland outlets even less impact of strain heating. Friction heating allows temperatures close to pressure melting to be reached in a small area of the base of Franklinbreen (simulation 1995ssFH, Fig. 6c). In 2008ss (Fig. 6d) the temperature distribution shows colder areas, probably resulting from transport of colder ice from central regions towards the outlet. None of the southern outlets reaches a temperate base.

We conclude that friction heat played a role for the onset of the acceleration of Franklinbreen and hypothesise a combined thickness - friction heat feedback: Franklinbreen was in 1990 the thickest of the outlet glaciers and the only outlet glacier featuring some areas with sub-melt sliding (Hindmarsh and Le Meur, 2001), even in the absence of additional heat sources. By its reduced ice velocities it might have thickened enough to allow basal ice to approach pressure melting through insulation and in turn triggering sliding. Thickening of Franklinbreen especially in the lower part between 1990 and 2005 is confirmed by Nuth et al. (2010). However friction heat cannot be responsible for maintaining fast flow in any of the outlets. We also rule
out a simple thickness feedback for the recent reduction in acceleration: No complete surface DEMs from different periods are available. Neither Nuth et al. (2010) nor Moholdt et al. (2010) observed a thinning on Franklinbreen between 1990–2005 or between 2003–2008. Nuth et al. (2010) observe a balanced or slightly positive volume change over Franklinbreen (average of 0.06 ± 0.12 km$^3$ yr$^{-1}$) for the period 1990-2005. Even though their result is subject to a large error, it seems unlikely that the observed recent reduction in acceleration between 2008 and 2011 is driven by a mechanism involving thinning.

**B) Firn heating due to melt water refreezing**

Firn heating is important for the general thermal regime of the ice cap and has larger impacts than friction or strain heating in most of the regions (Figs. 7 and 9, simulation *firnss*). It is not only efficient above the firn line, but with a certain delay by advection also in all other parts of the ice cap. Advection time scales are estimated to be of the order of centuries in our model (Fig. 14). Our model shows - assuming steady-state - an increase in basal temperature in the onset areas as well as over the whole area of all fast flowing outlets (Fig. 9), where we observe a good correlation between temperate base and fast flow.

In the lower parts of some of the southern facing outlet glaciers our calculated temperatures remain unnaturally cold and we observe areas which are cold based yet fast flowing, Fig. 9. This contradiction of observed sliding over very cold bed is a consequence of our approach where the basal friction parameter is inferred from observed surface velocities without direct coupling to temperature. Brinkerhoff et al. (2011) discussed the possibility of sliding over a cold base or underestimation of ice deformation due to neglecting ice anisotropy. Here, the apparent cold based sliding occurs mainly in areas where the bedrock elevation is poorly known as confirmed by the 20 m ice thickness iso-line in the two problematic outlets, Fig. 9. Irregularities in the bedrock data and analysis of the 1995–2008 prognostic simulations (Sect. 5.3.2) confirm this likely local underestimation of the ice thickness; improvement of the bedrock data by some control method should be considered as for example done by Morlighem et al. (2013) or van Pelt et al. (2013).
5.3.2 Simulating the acceleration period 1995-2008

By examining the temperature steady-state distributions, information about temporal evolution is disregarded. While the lower part of the temperature measured at the location of the deep drilling was in 1995 very close to the steady-state firnss, steady-state has not necessarily been reached over the whole ice cap. We conduct simulations of the acceleration period to get further insights into the temporal evolution.

We first highlight some general observations of such prognostic simulations with different friction parameter evolution scenarios (Sect. 4.4, Fig. 2) with the simulations without firn heating (const, sudden, lin13) focusing on Franklinbreen. Simulations sudden13 and lin13 exhibit a clear thinning of the onset area of Franklinbreen relative to const13 (Fig. 10), which is more pronounced (up to 25 m compared to 20 m) in simulation sudden13 resulting from the time integrated ice flux which is higher the sooner the velocities increase. The pronounced increase in thickness at the terminus of Franklinbreen can be interpreted at least partly as a model feature caused by fixing the lateral extent of the ice cap and neglecting enhanced mass loss due to calving. Other outlet glaciers, especially Rijpbreen and Bodlebreen, are highly influenced by errors in the bedrock DEM and thus not discussed.

As expected, velocities, friction and strain heat remain constant during simulation const13. In simulation sudden13 a sudden jump after the change in the basal friction parameter is observed, while in simulation lin13 the variables change synchronously with changes in the basal friction parameter. The 2008 variables are very similar in simulations sudden13 and lin13. The velocities are not only faster than in 1995, but show the two pronounced branches of Franklinbreen. Friction heat values are fairly high up to the centre of the ice cap, also emphasising the developing two branches; Fig. 12 compared to Fig. 13. Strain heat increases at the lateral margins of Franklinbreen over the whole ice depth, Fig. 13.

Temperature also remains unchanged in simulation const13. The final temperatures of simulations sudden13 and lin13 are again very close, although different from 2008ss. Ice temperature changes are not restricted to the bed. In simulation sudden13, the temperature adjusts smoothly to a similar pattern as in simulation lin13; no sudden jump in basal temperature is visible in
spite of the step change in basal friction parameter. Taking into account a few years for adjustment we see good agreement for all variables after this 13 yr period with simulations sudden13 and lin13, suggesting that the simulations are robust towards details of how changes in the basal friction parameter occur.

We compare different temperature fields: (i) the steady-state temperature field obtained with the 1995 basal friction parameter (simulation 1995ss), (ii) the steady-state temperature field obtained with the 2008 basal friction parameter (simulation 2008ss) and (iii) the final temperature distribution of the 1995–2008 prognostic simulation with linear changing basal friction parameter (simulation lin13). Firstly, we observe that the results of lin13 and 2008ss show differences in the central, slow flowing areas. This implies that the ice cap is not in steady state and decadal periods are not sufficient to reach a steady state. Secondly, the large-scale structure of temperatures is rather similar around Franklinbreen. During the 13 year prognostic simulation (lin13) a warming with respect to the initial conditions taken from run 1995ss of about 2-3 K at Franklinbreen’s lateral margins as well as a slight warming in its fastest area (texture Fig. 11a, b) can be observed. Those 13 years evolution are not sufficient to reproduce the cold area between the two branches visible in the steady state simulation 2008ss (Figs. 6 and 11a). A continuation of the lin13 simulation beyond 13 years with constant friction parameter shows that a steady state result similar to the run 2008ss is reached after about 100 years (result not shown). This is a quicker equilibration than the central area. We explain the colder areas of the steady state 2008ss in comparison to the end of the 13 year prognostic run lin13 by the fact that the friction heating - thickness feedback is compensated by cold ice advected into the fast flow region from the catchment area. Under the assumption that a temperate base is needed to allow for sliding, it becomes clear that another mechanism is needed to sustain the temperate base underneath the fast flow as already observed in Sect. 5.3.1A.

In what follows, we limit the discussion to one of the simulations 1995 to 2008 including firn heating (lin13firnss rather than lin13firnevol) since the transient effects are insignificant, not reaching the base within relevant time scales. The additional heat source due to firn heating does not significantly alter velocities and surface elevation or how the different variables adjust to changes in the basal friction parameter (sudden jump versus smooth adjustment). The
final strain heat distribution is very similar, though friction heat is slightly greater further up in the catchment areas of the outlets (Fig. 13). The “sliding area”, defined by temperatures above 271 K, i.e. temperatures close enough to pressure melting to allow for sliding (Hindmarsh and Le Meur, 2001) remains almost constant in shape and size (Fig. 11b). It is however challenging to interpret the changes occurring to this “sliding area”, which is essential for temperature based parametrisations of sliding laws as implemented by Seddik et al. (2012) or Dunse et al. (2011). In such laws sliding coefficients are mainly defined by the existence of a temperate base, but they are not based on physical mechanisms and would require a precise knowledge of the temperature distribution as initial condition which is highly uncertain for ice caps like VSF.

As in the steady-state simulations including firn heating firnss, a good correlation between fast flow and temperate base is produced throughout the full 13 years. This leads us to the conclusion that firn heating is the main driver for maintaining fast flow by supplying the fast flowing outlets with enough warm ice from the interior of the ice cap. We speculate that this convected front of increased ice temperature from firn heating had already reached the southern outlets well before 1995, explaining the observed fast velocities, but only reached Franklinbreen at around 1995.

There are several challenges to our hypothesis. First of all, we are not able to judge to what extent the initial quiescent state, given by the steady state solution 1995ss that excludes firn heating, reflects the de-facto existing englacial temperature field at this time. Secondly, a steady state including firn heating (firnss, Fig. 9) certainly overpredicts the contribution of warm ice advected to the outlets, since Franklinbreen was not yet fast flowing in 1995. Only a time evolving friction law over much longer times coupled to a hydrology model accounting for additional heat transfer through melt water flow would be able to fully capture this heat redistribution. We, nevertheless, conducted an additional simplified transient simulation starting from the initial state of 1995ss with fixed friction parameters (beta95T = those obtained for 1995) but with firn heating that on infinite timescales should lead to the same distribution as obtained with the steady state run firnss instantly applied to the system. This simplistic simulation neglects all climatic changes leading to variations in firn heating during the last centuries as well as the heat redistribution through meltwater. The evolution of the area of warm base is depicted in Fig. 15. We observe that changes in englacial temperature caused by excess firn heating take
a few centuries before a significant area of the base of Franklinbreen becomes temperate. We
equally observe that different areas of the ice cap approach the steady state firnss with differ-
ent timescales implying that it is possible that temperature at the drilling hole location could
have been quite close to firnss in the nineties, while it was not the case over the whole area
and the catchment area of Franklinbreen. Typical time scales (several centuries) for this advec-
tion process to reach the lower part of the outlets can be deduced from Fig. 14. By choosing a
fixed set of friction parameters, these values certainly are not exact, especially since the feed-
back between warming and increase of sliding velocity and hence shorter advection times is
neglected. The advection times for the southern outlets are certainly underestimated since they
are calculated with already high sliding velocities in these outlets, which are unrealistic before
the arrival of the increased ice temperatures. However, it provides us with a crude estimate of
the advection timescales involved. These timescales (several centuries) exclude the possibility
that changes in firn heating during the 20th century (as for example in firnevoll) could be respon-
sible for the trigger of fast flow at Franklinbreen. Nevertheless, Figs. 14 and 15 also show that
heat released earlier by refreezing, the existence of which we infer from the the temperature
profile measured in the deep borehole (steady state firn heating run firnss), could have started
contributing to maintain the temperate base in the late nineties. Consequently, we would ex-
pect that at some point the acceleration at Franklinbreen will stop and velocities level out, just
as it occurred decades earlier on the southern outlet glaciers of VSF. One explanation for the
vast delay of Franklinbreen can be seen from Figs. 14 and 15 which document a significantly
longer (in comparison to other outlet glaciers) convection timescale for ice from the interior to
reach Franklinbreen. This is in line with greater distance of the firn line relative to Franklin-
breen (Fig. 9). In addition, we cannot conclude about the role of a possible initial trigger for the
fast flow through a thickness - friction heat feedback and resulting changes in advection times
compared to the arrival of the convected front of warm ice. Furthermore, several distinguishing
features of Franklinbreen have to be taken into account: Franklinbreen might have been surging
in 1956 (Hagen et al., 1993), even though this was questioned by Sneed (2007) because of the
lack of geomorphological evidence. It has a much bigger catchment area and probably a larger hy-
draulic head, it is longer and probably flatter, the flow is split in two branches and additionally
complicated by a Nunatak, it terminates in a long cold fjord with more fresh water and is less prone to calving.
The question about the delayed speed-up of Franklinbreen could only be fully answered when including hydrology in the model to get a full picture of the water and heat redistribution throughout the ice cap, in particular since the englacial temperature distribution affects the englacial hydrology and vice-versa.

6 Conclusions

We present the basal friction parameter distribution obtained on the VSF ice cap for 1995, 2008 and 2011 from the inversion of observed velocity fields using Elmer/Ice. An acceleration between 1995 and 2008 on one of the outlet glaciers (Franklinbreen) is reflected in a pronounced temporal variation of the distribution of the basal friction parameter. The observed drastic change of inversely determined basal friction parameters between these snapshots (Sect. 5.1) renders prognostic simulations using a single in time constant set of these parameters highly inaccurate. Instead, models incorporating the whole complexity of physics involved in basal sliding (hydrology, thermodynamics, till-deformation) have to be developed for simulations of future scenarios of the ice cap. Coupling the inversion to temperature simulations affects the obtained basal friction parameter distributions only marginally because of the small influence of the viscosity-temperature dependency.

The temperature profile measured in a deep borehole in the accumulation zone has been successfully modelled and qualitatively explained by a recent change in climatic forcing at the surface influencing the heat produced through firn heating (Sect. 5.2). Qualitative interpretations of our firn heating experiments are limited by the uncertainty in the basal friction parameter distribution which comes into play through its influence on advection. These sources of uncertainty are: uncertainties of the DEMs, which impact via basal friction parameter (Sect. 2.1); the temperature and hence viscosity used during inversion (Sect. 4.1.1); different approaches for surface relaxation (Sect. 4.1.2). Recent deviations from an equilibrium profile triggered by increased firn heating can explain the current shape of the upper part of the profile, but the much
earlier existence of firn heating explains the warm temperatures observed deeper in the ice in the central thick slow flowing areas of the ice cap. Simulated timescales for temperature changes caused by increased firn heating due to changing surface conditions to reach the ice base are upwards of a century.

A temperate base is a prerequisite to allow for sliding in the fast flow areas. Friction heating and strain heating significantly contribute to heat production in the fast flowing outlets, the former superseding the latter. Nevertheless, in steady state simulations accounting for only these two heat sources, Franklinbreen was the only outlet to develop a temperate base (Sect. 5.3.1A). Taking observed surface elevation changes into account, we conclude that a combined thickness - friction heating feedback could be the trigger for the onset of Franklinbreen’s recent acceleration. In order to reach pressure melting point at the base of all outlet glaciers we need to include the effects of firn heating in the accumulation zone and the subsequent transport of the heat into the fast flowing outlets by advection (and possibly meltwater). Thus, we identify firn heating as the heat source responsible for the persistence of the observed fast flow over all outlets (Sect. 5.3.1B). Prognostic simulations (Sect. 5.3.2) confirm that the inflow of such warm ice is necessary to maintain a temperate base. From these simulations we also conclude that the englacial and basal temperature distribution is not in equilibrium; timescales are estimated at decades to centuries to reach steady-state. However, because of the low contribution to velocity of ice deformation compared to sliding, the impact of the temperature dependency of the viscosity is small. This justifies thermal steady-state assumption during the inversion for the sliding parameter or more generally for the initialisation of temperature fields in prognostic simulations where the focus is not on an accurate and detailed temperature regime. Various speculations on the delayed onset of the fast flow of Franklinbreen compared to the other southern outlets can be made. This question could only be fully answered when including hydrology in the model to get a full picture of the water and heat redistribution throughout the ice cap.

We conclude that sliding laws based only on the temperature distribution (especially the extent of the area close to pressure melt) cannot capture the full complexity of all processes related to the production and transport of heat including the role of hydrology and sediments. Also such an approach cannot reproduce the fine structure in the basal friction parameter distribution ob-
tained from inversion, which shows variations even within temperate domains at the bedrock. In addition, reducing the friction parameters to a temperature dependency will be very prone to errors in the initial conditions (obtained from spin-up) of its input variable.

Authors’ contribution

M. Schäfer designed and conducted the simulations, F. Gillet-Chaulet implemented the inverse method, R. Gladstone contributed to experimental setup, R. Pettersson and V. A. Pohjola provided data, T. Strozzi provided the remote sensing data of the surface velocities, T. Zwinger helped with the numerical setup. All authors have contributed to, seen and approved the manuscript.

Acknowledgements. We would like to thank all partners for access to data and constructive comments, we would like to mention especially the German and Norwegian teams for access to their AWS data (Roman Finkelnburg, Marco Möller, Dieter Scherer, Christoph Schneider, Thomas Vikhammar Schuler and others) as well as Björn Claremar and Jon Jörpeland for discussion about their results from WRF simulations. We acknowledge CSC – IT Center for Science Ltd. for the allocation of computational resources. The velocity data were produced with support of the EU FP6 INTEGRAL Project (1995), the ESA DUE GLOBGLACIER Project (2008) and the EU FP7 CRYOLAND Project (2011). We also would like to thank Ilmo Kukkonen for valuable discussions on the geological background. This work was performed at the Arctic Centre at the University of Lapland through funding of the Finnish Academy, the projects SvalGlac and SVALI. R. Pettersson and V. A. Pohjola were funded by the Swedish Science Council. This publication is contribution number 25 of the Nordic Centre of Excellence SVALI, Stability and Variations of Arctic land Ice, funded by the Nordic Top-level Research Initiative. Martin Truffer and two anonymous reviewers provided detailed suggestions that greatly improved the manuscript. Edited by: Dr. Eric Larour

References


Fig. 1. Surface Velocities from remote sensing data in December/January 1995/96, December 2008 and December 2011 (original data) (left side). To the right the location of the ice cap in Svalbard (middle), surface (top) and bedrock topography (bottom) are illustrated. FB indicates the location of Franklinbreen, SB Sabinenbreen, RB Rijpbreen, BB Bodleybreen, AB Aldousbreen, FzB Frazerbreen, IB Idunbreen and GB Gimblebreen.
Fig. 2. Distribution of the basal friction parameter in 1995 when coupling to temperature (upper line, (a), simulation $\text{beta95T}$). The other three figures show a zoom over Franklinbreen (using the depth dependent temperature profile) in 1995 (b), 2008 (c) and 2011 (d) (simulations $\text{beta95}$, $\text{beta08}$, $\text{beta11}$). Note that only the south-western corner has been patched with 1995 data in the 2008 and 2011 data sets. All figures are in logarithmic scale (units are $\text{MPa yr m}^{-1}$).
Fig. 3. Evolution of the total cost function $J_{tot}$ (Eq. (13), log scale, scaled by $10^9$) when iterating inverse method and temperature steady-state calculation (1995 velocities, simulation beta95T). The graph is cut at $1.6 \times 10^9$ for better visibility, higher values in the first iterations are thus not visible. In orange the evolution of the cost function when not coupling with temperature is shown.
Fig. 4. Evolution of maximum (and minimum) values of mesh adjustments and vertical velocities during the surface relaxation *surfrelax95* to determine the end of the procedure.
Fig. 5. Vertical corrections (m) of the topographic data (1995 basal friction parameter field) at the end of the surface relaxation (simulation `surfrelax95`), to the left absolute corrections, to the right relative to the initial ice thickness.
Fig. 6. Basal temperature field (°C, relative to pressure melt): (a) advection only \((1995ssA)\), (b) adding of strain heating \((1995ssSH)\), (c) adding of friction heating for 1995 \((1995ss)\), (d) advection, strain and friction heating for 2008 \((2008ss)\). The position of the ice core is indicated in red, the position of the two locations used in Fig. 7 in white.
Fig. 7. Temperature profiles in chosen locations (temperatures relative to pressure melt), (a) in the ablation zone of Franklinbreen and (b) in the accumulation zone of Frazerbreen: (1) depth dependent profile (Sect. 3.2) assumed in Schäfer et al. (2012) (red), (2) profile including advection, but no additional heat sources (green) (1995ssA), (3) profile including advection and strain heating (blue) (1995ssSH), (4) profile including advection and friction heating (pink) (1995ssFH), (5) profile including advection and both strain and friction heating (light blue) (1995ss), (6) profile including also firn heating (orange), which is identical for the equilibrium state firnss and the following time-evolving simulation firnevol. The locations of these profiles are indicated in Fig. 6.
Fig. 8. Temperature profile as measured (red) in a drill hole (Motoyama et al., 2008) (temperatures relative to pressure melt), the position is indicated in Fig. 6 (red dot). The dotted black line corresponds to the depth dependent profile, the green line corresponds to the modelled temperature field including only strain and friction heating. The dark blue line is the result of the equilibrium firn heating simulation with the first set of parameters (percolation depth of 13.2 m, simulation firnss), which acted as initial profile for the succeeding transient runs. The orange, cyan and pink profiles illustrate the evolution of temperature in the succeeding time evolving simulation with the second set of parameters (percolation depth of 17.6 m) evolving towards a new equilibrium (grey line). The best fit to the data among these profiles after 35 yr is highlighted in black (simulation firnevol).
Fig. 9. Basal temperature field (°C, relative to pressure melt, 1995): adding of firn heating (simulation *firnss*). This figure remains unchanged whether the non-equilibrium part (simulation *firnevol*) of the firn heating is added or not. The position of the ice core is indicated in red, the position of the two locations used in Fig. 7 in white. The firn line is drawn in purple, the 20 m ice thickness iso-line in light grey. The latter is a good indicator for underestimated ice thickness in areas where fast sliding seems to be in strong contradiction with too cold basal temperatures.
Fig. 10. Change in surface elevation $\Delta S(m)$ during the prognostic 1995–2008 simulations for the three scenarios: (a) constant basal friction parameter, simulation $const13$, (b) sudden jump after 5 yr, simulation $sudden13$ and (c) linear change in basal friction parameter, simulation $lin13$. 
Fig. 11. In a zoom over the Franklinbreen area the change in the extent of the “sliding area” (basal temperatures above 271 K) is shown. To the left, light blue corresponds to the extent of 1995ss, dark blue to 2008ss and pink to lin13. The temperature distribution as texture is the 1995ss temperature distribution (temperatures relative to pressure melt).

To the right (same colour scale), light colour’s present the “sliding area” when neglecting firn heating (lin13), dark colours when including it (lin13firnss), blue lines represent the start of the 13 yr simulation and pink the value at the end. As texture the temperature at the end of the simulation when neglecting firn heating is drawn (lin13) (temperatures relative to pressure melt).
Fig. 12. (a) Difference between surface and bedrock velocities, i.e. the velocity due to deformation in 1995. Vertically integrated strain heat (b) and friction heat at the bedrock (c) in the simulation 1995ss.
**Fig. 13.** Zoom over Franklinbreen. Distribution of the mechanical heat sources at the end of the prognostic simulation without firn heating (*lin13*, (a) and (c), distributions identical to *2008ss*) and with firn heating (*lin13firnss*, (b) and (d)). In pink the contour of 271K and in yellow the iso-velocity line of 150 m yr⁻¹ is drawn.
Fig. 14. Typical advection time scales estimated from backward streamlines obtained with the backward Runge-Kutta method implemented in ParaView (Ahrens et al., 2005) assuming the basal friction parameter $beta95T$. The firn line is depicted in orange, as texture the temperature field $1995ss$ is drawn.
Fig. 15. Temporal evolution of the temperature field during a simplified transient simulation from 1995ss towards the steady-state firnss with the friction parameter beta95T. The firn line is shown in violet, the drill hole corresponds to the red dot. The evolution of the “sliding area” (271 K) from 1995ss to firnss is shown in rainbow colours (black area; blue, green, yellow, red contour lines) 100 years apart (dark green for example corresponds to 500 years). As texture the surface elevation is depicted.
Table 1. Numerical parameters used in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>magnitude of gravitational acceleration</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>density of sea water</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of ice</td>
</tr>
</tbody>
</table>

Arrhenius law as in Paterson (1994), temperatures relative to pressure melting point

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Temperature Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>pre-factor</td>
<td>$3.985 \times 10^{-13}$ Pa$^{-3}$ s$^{-1}$ T $\leq -10^\circ$ C $1.916 \times 10^{3}$ Pa$^{-3}$ s$^{-1}$ T $&gt; -10^\circ$ C</td>
</tr>
<tr>
<td>$Q$</td>
<td>activation energy</td>
<td>$-60$ KJ/mol$^{-1}$ T $\leq -10^\circ$ C $-139$ KJ/mol$^{-1}$ T $&gt; -10^\circ$ C</td>
</tr>
</tbody>
</table>
Table 2. Summary of simulations

<table>
<thead>
<tr>
<th>Description</th>
<th>Section</th>
<th>Figures</th>
<th>name</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inverse Simulations to derive basal friction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-curve analysis</td>
<td>1995</td>
<td>4.1.1</td>
<td>beta95</td>
</tr>
<tr>
<td>depth dep. temp. field</td>
<td>1995</td>
<td>4.1.1</td>
<td>beta08</td>
</tr>
<tr>
<td>depth dep. temp. field</td>
<td>2008</td>
<td>4.1.1</td>
<td>beta11</td>
</tr>
<tr>
<td>depth dep. temp. field</td>
<td>2011</td>
<td>4.1.1</td>
<td></td>
</tr>
<tr>
<td>iterated with temp. s. state</td>
<td>1995, no firn heating</td>
<td>4.1.1</td>
<td>beta95Tfirn</td>
</tr>
<tr>
<td>iterated with temp. s. state</td>
<td>2008, no firn heating</td>
<td>4.1.1</td>
<td>beta08T</td>
</tr>
<tr>
<td>iterated with temp. s. state</td>
<td>1995, including firn heating</td>
<td>4.1.1</td>
<td></td>
</tr>
<tr>
<td><strong>Surface relaxation - mechanical spinup</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>present day mean cmb</td>
<td>using beta95T</td>
<td>4.2</td>
<td>surfrelax95</td>
</tr>
<tr>
<td>present day mean cmb</td>
<td>using beta08T</td>
<td>4.1.2</td>
<td>surfrelax08</td>
</tr>
<tr>
<td>complex spin-up</td>
<td>iterating with beta95T</td>
<td>4.1.2</td>
<td>surfrelax95c</td>
</tr>
<tr>
<td><strong>Temperature steady-states</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>advection only</td>
<td>beta95T</td>
<td>4.2</td>
<td>5.3.1</td>
</tr>
<tr>
<td>advect. + SH</td>
<td>beta95T</td>
<td>4.2</td>
<td>5.3.1</td>
</tr>
<tr>
<td>advect. + FH</td>
<td>beta95T</td>
<td>4.2</td>
<td>5.3.1</td>
</tr>
<tr>
<td>advect. + FH + SH</td>
<td>beta95T</td>
<td>4.2</td>
<td>5.3.1 5.3.2</td>
</tr>
<tr>
<td>advect. + FH + SH</td>
<td>beta08T, start surfrelax08</td>
<td>4.2</td>
<td>5.3.1 5.3.2</td>
</tr>
<tr>
<td><strong>Firn heating calibration runs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>steady-state part</td>
<td>depth 13.2 m</td>
<td>4.3</td>
<td>5.2 5.3.1 5.3.2</td>
</tr>
<tr>
<td>evolutive part</td>
<td>35 yr with depth 17.6 m</td>
<td>4.3</td>
<td>5.2</td>
</tr>
<tr>
<td><strong>Prognostic simulations, 13 years</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no change in $\beta$</td>
<td>beta95T no firn-heating</td>
<td>4.4</td>
<td>5.3.2</td>
</tr>
<tr>
<td>sudden change in $\beta$</td>
<td>beta95T, beta08T, no firn-heating</td>
<td>4.4</td>
<td>5.3.2</td>
</tr>
<tr>
<td>linear change in $\beta$</td>
<td>beta95T, beta08T, no firn-heating</td>
<td>4.4</td>
<td>5.3.2</td>
</tr>
<tr>
<td></td>
<td>beta95T, beta08T, as in firnss</td>
<td>4.5</td>
<td>5.3.2</td>
</tr>
<tr>
<td>linear change in $\beta$</td>
<td>beta95T, beta08T, as in firnevol</td>
<td>4.4</td>
<td>5.3.2</td>
</tr>
</tbody>
</table>