Glacier volume and glacier bed topography estimation of the tropical glacier Huayna West.

V. Moya Quiroga[1], A. Mano[2], Y. Asaoka[1], K. Udo[2], S. Kure[2] and J. Mendoza[3]

[1]{Graduate School of Engineering, Tohoku University, Sendai, Japan}

[2]{International Research Institute of Disaster Science, Tohoku University, Sendai, Japan}

[3]{Instituto de Hidraulica e Hidrologia, Universidad Mayor de San Andres, La Paz, Bolivia}

Correspondence to: V. Moya Quiroga (moyav@potential1.civil.tohoku.ac.jp; vladyman@hotmail.co.uk)

Abstract

Glacier retreat will increase the sea level and decrease the fresh water availability. Glacier retreat will also induce morphologic and hydrologic changes due with the formation of glacial lakes. Hence, it is important not only to estimate glacier volume, but also the spatial distribution of ice thickness. GlabTop and mass turnover ice-flow mechanics (MTIFM) are practical approaches for estimating spatially distributed glacier thickness. However, they depend on some parameters that must be calibrated. Although there are some suggestions for the calibration of those parameters, such suggestions are based on studies on mid and high latitude glaciers. Unfortunately, there are no studies about the application of those methods to tropical glaciers. The present study applied GlabTop and MTIFM to the tropical glacier Huayna West. Then, a Monte Carlo analysis was performed to the whole range of the volume estimations considering the possible BSS values. The different confidence intervals were compared and related to volume estimations from V-A method. The volume estimations within a confidence higher than 70% provide volume estimations close to the V-A estimations suggested for tropical glaciers. The GlabTop estimations were compared with estimations from MTIFM. Both approaches provide a volume about 0.034km^3 and show the formation of future glacial lake. GlabTop is more sensitive to local slopes estimates a deeper lake. The volume of the Huayna West glacier is bigger than the full capacity of the Tuni reservoir.

1 Introduction

Galciers may be considered as the most important water reservoirs since they store some 68% of total fresh water. Unfortunately, they are retreating (Paul et al., 2007; Ramirez et al., 2001). Glacier retreat will increase sea level rise and decrease water resources (Dyurgeyov and Meier 2005; Vuille et al., 2008). Thus, it is important to estimate glacier volume in order to predict future water availability and sea level rise (Kaser et al., 2010; Raper and Braithwaite 2006).

Glacier volume can be estimated by field measurements using ground penetrating radar (GPR) and radio echo sounding (RES) (Navarro et al., 2001; Andreassen et al., 2012). However, GPR and RES are impractical methods, especially for remote and big glaciers. Hence, new and simpler alternatives for estimating glacier thickness and glacier volume have been developed. One popular and practical approach is the volume area (V-A) relation. V-A assumes a power law scaling relation between glacier area and glacier volume, and is based on ice dynamic constraints due to ice rheology and typical climatic-topographic conditions of glacierized areas (Bahr et al., 1997). Nevertheless, V-A approach is prone to an important degree of uncertainty since it depends on two empirical parameters. Currently, the accuracy of V-A is questioned when applied to small sample of glaciers or single glaciers (Farinotti and Huss 2013; Bahr et al., 2012). Besides, V-A method only provides glacier volume, neglecting the spatially distribution of thickness. Spatially distributed glacier thickness is valuable information since it would allow us to predict the glacier bed topography (GBT). Such GBT is important for modelling the glacier evolution (Huss et al., 2010), for estimating possible changes in the runoff regime (Huss et al., 2008), or for predicting the formation of future lakes (Frey et al., 2010).

In the last years several analytical models were proposed for estimating glacier thickness and glacier volume (Farinotti et al., 2013; Zekollari et al., 2013; Colgan et al., 2012; Michel et al., 2013; Morlingem et al., 2011). One practical approach for estimating glacier ice thickness and glacier volume is the glacier bed topography (GlabTop) approach proposed by Paul and Lindsbauer (2012). The popularity of the GlabTop is increasing rapidly and it has been applied in various studies (Li et al., 2012; Lindbauer et al., 2012; Clarke et al., 2013). GalbTop assumes a plastic behaviour of glacier; glaciers flow easily enough to redistribute mass and prevent stresses from rising above a given limit (Cuffey and Paterson 2012; Nye

1967). Such plastic assumption is supported by field measurements showing that glacier deformation is best reproduced considering a plastic deformation (Kavanaugh and Clarke 2006). GlabTop assumes the glacier thickness as a function of surface slope and basal shear stress. However, GlabTop requires basal shear stress (BSS) which is never measured but estimated in a very approximate way. Hence, the GlabTop estimations are prone to a wide range of uncertainty that should be addressed.

Other analytical approach with rising popularity is the mass turnover ice-flow mechanics (MTIFM) (Farinotti et al., 2009). This approach is based on the mass conservation principle; the mass balance is balanced by the ice-flux divergence and the surface elevation change. However, this method transfers all of its uncertainties into a calibration parameter. If there are no measurements for the calibration, the uncertainties of this approach cannot be addressed. Previous studies have already applied both GlabTop and MTIFM to high latitude glaciers. However, there are no studies about the performance of such methods when applied to tropical glaciers.

In the present study we apply the V-A, GlabTop and MTIFM approaches to the tropical glacier Huayna West. First, glacier thickness and volume were estimated with GlabTop considering different BSS values. Then, a Monte Carlo analysis was performed to the volume estimations. The different confidence of volume estimations were compared with volume estimations from V-A. The estimations with confidence higher than 70% provide a voilume close to the one from V-A suggested for tropical glaciers. Then, MTIFM was applied and its results compared with the ones from GlabTop. The basal shear stress in tropical glaciers tends to be closer the one of maritime glaciers. The main uncertainties in GlabTop are the BSS and the valley support. The GlabTop is more sensitive to possible errors due to local small slopes. With the spatially distributed glacier thickness it was possible to reconstruct the glacier bed topography (GBT). Both approaches GlabTop and MTIFM GBT show the formation of a future lake.

2 Study Area

The study area is the Huayna West glacier in the Bolivian Andes (16° 16’ S, 68° 10’ W). It is located at the west side of the Huayna Potosi massif. This 0.783 km^2 glacier accounts for 5.76% of the total area of the Huayna West basin. The Huayna Potosi massif is one of the biggest glaciers in the Royal Cordillera (Figure 1). Huayna glacier is a tropical one located in the tropic of Capricorn, where the climate is characterized by two seasons with a period of precipitation and a dry period (Mote and Kaser 2007). The melting water from the Huayna Potosi glacier flows towards the Tuni reservoir (1.65 km^2) and plays an important role for the water supply of La Paz and El Alto connurbation (Bolivia). This glacier is currently under study within the Glacier Retreat impact Assessment and National policy DEvelopment (GRANDE) project.

3 Methodology

The extent of the Huayna West glacier was delineated using remote sensing data from the sensor Advanced Visible and Near Infrared Radiometer type 2 (AVNIR-2) of the Advanced Land Observation Satellite (ALOS). The ALOS AVNIR-2 images were analysed and processed with the multispectral image data analysis system software (Landgrebe 2005). It is important to note that the ALOS AVNIR-2 image is more recent than the respective image used for the Randolph Glacier Inventory 3.0, which was observed on May the 31st of 2003. Besides, it was not possible to use the GLIMS data base, since such data base does not include this glacier (Raup et al., 2007). Thus, ALOS AVNIR-2 images provide a valid and current image of the study area.

The flow lines were obtained by processing the global digital elevation model (GDEM) provided by the advanced spaceborne thermal emission and reflection radiometer (ASTER). The slope α was assumed equal to the surface slope (Clarke et al., 2013). Although some studies used SRTM data for studying glaciers and estimating glacier volume (Surazakov and Aizen 2006), in the present study we used the GDEM ASTER, since it is a more recent product with better resolution and provides an accurate delineation of the study area. The DEM was processed with the TauDEM algorithm implemented in the GIS software MAPWINDOWS (Tarboton 1997). Figure 2 shows the glacier flow-lines and the Huayna West basin.

Then, glacier thickness was estimated at the flow-lines assuming perfect plasticity and the GlabTop approach (Lindsbauer et al., 2012) described by:

$$h = \frac{\tau}{\rho\, g \sin\theta} \qquad \text{Eq 1.}$$

Where h is the glacier thickness (m), τ is the basal shear stress (kPa), g is the gravity acceleration (9.79 m s^{-2}), ρ is the ice density (900Kg/m^3) and θ is the slope (°). The most popular estimation of basal shear stress is to consider it as a function of the elevation range of the glacier expressed by (Haeberli and Hoelze 1995):

$$\tau = 0.005 + 1.598\varDelta H - 0.435\varDelta H^2 \qquad if\ \varDelta H \leq 1.6 \qquad \text{Eq. 2}$$

$$\tau = 150 \qquad if\ \varDelta H > 1.6 \qquad \text{Eq. 3}$$

Where $\varDelta H$ (km) is the elevation range of the glacier. However, since tropical glaciers have higher mass balance gradients, it is reasonable to expect a higher basal shear stress. Since there is no published literature about BSS on tropical glaciers, the present study considered the BSS for maritime glaciers described by Eq. (4) and Eq. (5) (Haeberli and Hoelze 1995) as the higher possible BSS.

$$\tau = 3\ \varDelta H \qquad for\ \varDelta H\ \leq 0.5 \qquad \text{Eq. 4}$$

$$\tau = 150 \qquad if\ \varDelta H > 1.6 \qquad \text{Eq. 5}$$

Then, the glacier thickness was used as input data for a spatial interpolation by applying a Kriging routine. The glacier outline was used as boundary condition with zero ice thickness. The distributed glacier thickness maps allowed obtaining the glacier volume. The glacier volume was obtained by multiplying each thickness by the area of its grid cell (900 m^2). However, as we have a range of several possible BSS values, we also have several possible volume estimations. The uncertainty of the possible volume was addressed by performing a Monte Carlo uncertainty analysis (MCUA) of the possible glacier volume estimations. The number of trials was obtained by performing a convergence test considering the variation of the mean and standard deviation as the number of iterations increases (Shrestha et al., 2009).

Then, the volume probabilities were compared against volume estimations using the V-A approach (Eq. 6).

$$V = cA^{\gamma} \qquad \text{Eq. 6}$$

Where V is the volume of the glacier (km^3), A is the area of the glacier (km^2), γ is the scaling exponent and c is the proportional constant. This comparison considered 3 assumptions: a) The use of GlabTop with the correct BSS provides the volume of the glacier, b) The use of V-A with the correct coefficients provides glacier volume and c) When applied to the same glacier, the volume estimated by GlabTop should be equal to the volume estimated by V-A. Literature provides several γ and c coefficients. In the present study we considered several coefficients obtained from synthetic data and from empirical data of different climatic conditions.

An additional analysis was performed according to the MTIFM approach (Farinotti et al., 2009). In this analysis it was assumed that the actual mass balance corresponds to the apparent mass balance b_i defined as:

$$b_i = \left\{ \begin{array}{ll} (z_i - z_o)\left.\frac{db_i}{dz}\right|_{abl} f_{debris} & if\ z_i < z_o \\ (z_i - z_o)\left.\frac{db_i}{dz}\right|_{acc} f_{debris} & if\ z_i > z_o \end{array} \right\} \qquad \text{Eq. 7}$$

Where $f_{\mathrm{debris}} = 1$ if the cell is not debris-covered (the present study assumed $f_{\mathrm{debris}} = 1$). Then, the ice-flux q_i at each point i of the flow-line was estimated as the cumulated apparent mass balance of every grid cell that contributes to the ice-flux at that point. $\left.\frac{db_i}{dz}\right|_{abl}$ and $\left.\frac{db_i}{dz}\right|_{acc}$ are the mass gradients for the accumulation zone (0.015 a^{-1}) and ablation zone (0.025 a^{-1}) respectively. Then, the ice thickness h_i at every flow-line point i was calculated according to:

$$h_i = \sqrt[n+2]{\frac{\bar{q}_i}{2FR}\frac{n+2}{\left(CF\rho g sin\bar{\theta}\right)^n}} \qquad \text{Eq. 8}$$

Where ρ is the ice density (900 kg m^{-3}), FR is the flow rate factor (2.4 $x10^{-15}$ kPa^{-3} s^{-1}), CF is the correction factor to be calibrated, $\bar{\theta}$ (°) is the mean slope of the flow-line and n is the Glen flow law exponent ($n = 3$). Then, the estimated thickness was used for a spatial interpolation applying a Kriging routine.

Then, the estimated glacier thickness (GT) was subtracted from the glacier surface elevation (GSE) in order to get the GBT elevation

$$GBTE_i = GSE_i - GT_i \qquad \text{Eq. 9}$$

Where the subscripts i identify the glacier cell. GSE was obtained from the DEM

4 Results and Discussion

Table 1 shows the BSS of flow-line 1 and flow-line 2 and the volume estimation considering the two relations for the BSS. The maximum BSS values obtained with the relation for maritime glaciers are almost twice the BSS for normal continental glaciers; thus, the maximum volume is almost twice the minimum possible volume (Table 2). Such differences represent a wide uncertainty range with almost no practical use and doubtful accuracy. This wide range of uncertainty shows the importance of more studies about BSS in tropical glaciers. The problem is to find the range of possible BSS that provides better volume estimation with an acceptable degree of uncertainty.

The MCUA estimated the glacier volume 5000 times considering 5000 different BSS values within the minimum and maximum limits of table 1. Figure 3 shows that the MC analysis gets stable after 2000 iterations. Table 3 shows the glacier volume estimations and its associated confidence.

The different volume estimations were compared with other estimations considering different suggested coefficients of the V-A relationship. Table 4 shows the coefficients used and their respective volume estimations. Such comparison is better explained by dividing the results into four groups:

In the first group we have the volume estimations that provide a volume lower than the volume estimated with the minimum BSS values. Those lower values could be obtained with the GlabTop approach by applying a lower BSS value. However, the coefficients from those relations were developed for glaciers with different conditions. The coefficients from Meier and Bahr (1996) and Bahr (1997) were obtained from continental Alpine with areas between 0.2 km^2 and 1 km^2. Although the Huayna West glacier is within that area range, the HWG is located in the tropics with different climatic and geophysical conditions. The coefficients from Driedger and Kennard (1986) were obtained by analysing a volcanic glacier in the USA, with climatic and geophysical condition different than the current study area.

In the second group we have the volume estimation between the minimum BSS and the 30% confidence. The volume estimations from this group are similar to the estimations using V-A relations suggested for normal continental glaciers. It is important to note that the volume estimation according to Bodin et al., (2010) is also for an Andean glacier; however, such glacier is located in the Southern Andes out of the tropical range.

The volume estimations between the 30% confidence and the 60% confidence (group 3) are similar to volume estimations with coefficients obtained from synthetic data and glaciers from different locations. Those estimations may be considered as a world average. Thus, when applied to tropical glaciers they may still provide some underestimation of the total volume.

The volume estimations with a confidence higher than 70% are related to volume estimations using coefficients suggested for tropical glaciers. This confirms the assumption that tropical glaciers have a BSS higher than mid-high latitude continental glaciers. The BSS of tropical glaciers is similar to that of marine glaciers. The estimation according to Huss and Farinotti (2012) provides the lowest volume estimation. This estimation can be considered as the minimum probable volume. This minimum estimation can be obtained by applying a BSS of 2.46 ΔH. The other two estimations provide a volume of 0.034 km^3 and 0.035 km^3. The difference between the minimum and maximum volume estimations has a difference of 17%.

Figure 4 shows the respective glacier thickness map according to GlabTop. The thickest area of the glacier is located at some 180 m from the east boundary. This deepest part is elongated with a northeast - southsouthwest direction and a longitude of 370 m. However, such deepest area is located in an area where the slope is 8.5 % (4.85 °). A sensitivity analysis shows that between a slope of 5° and 6°, the thickness error is about 20% (Figure 5). A new glacier thickness was estimated considering a slope threshold of 6°. Figure 6 shows the glacier thickness map estimated with this threshold. In this new map the thickest area of the glacier has the same location, but the thickness is 33% lower.

Applying the MTIFM approach, the glacier volume can be obtained by applying a correction factor *CF* = 0.09 and *FR* 2.4 $x10^{-15}$ kPa^{-3} s^{-1}. The *CF* is lower than the one applied to continental glaciers (Farinotti et al., 2009). This may be because of the mass gradients and the *FR* assumptions. In the present study the apparent mass balance was assumed corresponding to the actual mass balance; such correspondence occurs when the glacier is in steady state. However, in the present study the glacier is not in equilibrium. Considering that the glacier is under retreat conditions, higher mass balance and steeper mass gradients could be expected. The value of *FR* used in the present study corresponds to a basal temperature of 0 °C and steady state, which is not always the case. The MTIFM assumes that the uncertainties of *FR* are transferred into *CF*. Such uncertainty transference assumes that the basal temperature is close to 0 °C (temperate ice). However, such assumption is valid for high latitude glaciers. Low latitude glaciers are composed by temperate ice and cold ice; thus, it may be expected lower basal temperatures (Greeve and Blatter 2009). In the case of basal temperatures of -5 °C the *FR* is reduced to 9.3 x10-15 kPa^{-3} s^{-1} (Greeve 2010). In the case of lower basal temperatures *FR* would be much smaller since it decreases exponentially with temperature (Smith 2001). Table 5 shows that for basal temperatures lower than 5 °C, the thickness may increment between 20% and 47 %. In such cases, the uncertainty transference may not be valid and the calibration of two parameters would provide better results. Figure 7 shows the ice thickness according to MITFM approach.

Using GlabTop 38% of the glacier is less than 30.0 m thick. Using MITMC 33% of the glacier is less than 30.0 m thick. It is important to consider this threshold, since some studies suggest that glaciers begin to flow only when they reach a thickness of 30.0 m (PRI 2013). In the present case all the flow-lines are thicker than 30.0 m. However, some regions of the flow-lines are very close to this threshold with values around 34.0 m. Such regions close to the 30.0 m threshold could be considered in the limits of application both GlabTop and MITMC. Thus, it may be assumed that for smaller glaciers with more advanced retreat process, the GlabTop and MITMC approaches cannot be applied.

The Huayna West glacier has an estimated volume of 0.034 km3. Considering the area of the basin and the area of the Tuni reservoir, such volume can be expressed as water layer equivalent (WLE) of the whole Huayna West basin and the Tuni reservoir (Table 6). The volume of the Huayna West glacier is equivalent to a water layer of 2.5 m over the whole basin. This water layers is 3.4 times higher than the yearly precipitation. Thus, in the hypothetical case that the whole glacier melts at a constant rate in 4.65 years, then during those years the water from melting glacier would equal the water input from precipitation. Relating the glacier volume to the Tuni reservoir, the water layer equivalent is higher than the elevation of the reservoir (18.0 m) (MMAyA 2010). This is because the Huayna West glacier has more water stored than the Tuni reservoir capacity (0.0215 km^3).

Figure 8a shows the reconstructed glacier bed topography (GBT) obtained from GlabTop without any slope threshols. The GBT shows the formation of a future lake in the quadrant C3 once the glacier disappears. This lake has an area of 0.07 km^2 and a maximum depth estimated of 32 m. The area-maximum depth relation of the glacier is a reasonable value that fits reasonably with other estimations. For instance, Sakai (2012) developed a power area-depth relation considering several glacial lakes. Applying such relation to the present lake gives a maximum depth of 26.06 m. Figure 8b shows the GBT obtained from GlabTop including the slope threshold. This map also shows the formation of the lake. However, in this case the lake is much smaller (0.02 km^2) and less deeper (15 m). The GBT estimated according to the MTIFM thickness (Figure 8c) provides the smallest lakes (0.01 km^2). Actually in this case the shore of the lake is connected to the basin outlet in the quadrant B4.

5 Conclusions

Theoretical approaches for glacier volume estimation are influenced by coefficients that depend on local conditions. Although there are suggested valued for mid-high latitude glaciers, there are almost no applications or suggestions for tropical glaciers. This study estimated the glacier volume of the tropical glacier Huayna West by applying two analytical estimations: the Glacier bed topography approach (GlabTop) and the mass turnover ice-flow mechanics approach (MTIFM). Both approaches were estimated considering V-A estiamtions for tropical glaciers.

The most sensible parameter of GlabTop is the basal shear stress (BSS). The Monte Carlo analysis of the possible BSS values shows that for this tropical glacier the BSS is within the upper 30% confidence. It was found that the BSS – elevation range relation of tropical glaciers is closer to the relation of maritime glaciers.

Although local slopes do not have much influence in the overall volume estimation, they have an important influence on the spatial distribution. Ice thickness estimation is more sensitive to slope in slopes smaller than 6°. Slopes lower than 6° may overestimate the ice thickness more than 20%.

The most sensitive parameter of MITMC is the calibration factor. The main difference between the results from GlabTop and MITMC is in the spatial distribution of glacier thickness. GlabTop is more sensitive to the local slopes. MITMC is not sensitive to local slopes. Thus, when applying GlabTop it is important to consider a minimum threshold slope.

The Huayna West glacier has a volume of 0.034 km^3. This volume is higher than the storage capacity of the Tuni reservoir (0.024 km^3).

Glacier bed topography shows the formation of a future glacial lake. The GBT from GlabTop provides a deeper glacial lake than one from MITMC. The estimated area and depth of such lake have a reasonable agreement with dimensions observed at other glacial lakes.

6 Aknowledgements

The authors would like to thank the “Science and Technology Research Partnership for Sustainable Development” (SATREPS) of “Japan Science and Technology Agent – Japan International Cooperation Agency” (JST-JICA). This research is developed within the framework of the GRANDE project, financed by SATREPS

References

Adhikari, S., and Marshall, S. J.: Glacier volume-area relation for high-order mechanics and transient glacier states. Geophys Res Lett, 39(16), 1-6, 2012. doi:10.1029/2012GL052712

Andreassen, L. M., Kjøllmoen, B., Rasmussen, A., Melvold, K., & Nordli, Ø.: Langfjordjøkelen, a rapidly shrinking glacier in northern Norway, J Glaciol, 58(209), 581–593, 2012. doi:10.3189/2012JoG11J014

Bahr, D.B.: Global distributions of glacier properties: A stochastic scaling paradigm, Water resources research, 33, 1669-1679, 1997

Bahr, D.B., Meier, M.F. and Peckham, S.D.: The physical basis of glacier volume-area scaling, J Geophys Res, 102, 20355-20362, 1997

Bahr, D.B., Pfeffer, W.T., Kaser, G.: Glacier volume estimation as an ill-posed boundary value problem, The Cryosphere Discuss, 6, 5405-5420, 2012.

Baraer, M., Mark, B., McKenzie, J., Condom, T., Bury, J., Huh, K.-I., Portocarrero, C., Gomez, J. and Rathay, S.: Glacier recession and water resources in Peru’s Cordillera Blanca. J Glaciol, 58(207), 134–150, 2012. doi:10.3189/2012JoG11J186

Bodin, X., Rojas, F., and Brenning, A.: Status and evolution of the cryosphere in the Andes of Santiago (Chile, 33.5°S.). Geomorphology, 118(3-4), 453–464, 2010. doi:10.1016/j.geomorph.2010.02.016

Clarke, G.K.C., F. Anslow, A. Jarosch, V. Radic, B. Menounos, T. Bolch, and Berthier, E.: Ice volume and subglacial topography for western Canadian glaciers from mass balance fields, thinning rates, and a bed stress model. J. Climate., 26, 4282-4303, 2013.

Colgan, W., Pfeffer, W. T., Rajaram, H., Abdalati, W., and Balog, J.: Monte Carlo ice flow modeling projects a new stable configuration for Columbia Glacier, Alaska, c. 2020. The Cryosphere, 6(6), 1395–1409, 2012. doi:10.5194/tc-6-1395-2012

Cuffey, K.M., and Paterson, W.S.B.: The physics of glaciers, Butterworth-Heinemann/Elsevier, Amsterdam, Netherlands, 2010.

Driedger, C.L. and Kennard, P.M.: Glacier volume estimation on cascade volcanoes: an analysis and comparison with other methods, J Glaciol, 8, 59-64, 1986.

Dyurgerov, M., and Meier, M.F.: Glaciers and the Changing Earth System: A 2004 Snapshot. Occasional Paper 58, Institute of Arctic and Alpine Research, University of Colorado, Boulder, CO, 118 pp, 2005

Farinotti, D. and Huss, M.: An upper-bund estimate for the accuracy of volume-area scaling, The Cryosphere Discuss, 7, 2293-2331, 2013

Farinotti, D., Huss, M., Bauder, A., Funk, M., Truffer, M.: A method to estimate the ice volume and ice-thickness distribution of alpine glaciers, J Glaciol, 55, 422-430, 2009.

Farinotti, D., Corr, H., and Gudmundsson, G. H.: The ice thickness distribution of Flask Glacier, Antarctic Peninsula, determined by combining radio-echo soundings, surface velocity data and flow modelling. Ann Glaciol, 54(63), 18–24, 2013. doi:10.3189/2013AoG63A603

Frey, H., Haeberli, W., Linsbauer, A., Huggel, C., and Paul, F.: A multi-level strategy for anticipating future lake formation and associated hazard potentials, Nat. Hazards Earth Syst. Sci., 10, 339–352, 2010

Greve, R.: Dynamics of ice sheets and glaciers, Institute of low temperature science, Hokkaido University, Sapporo, Japan, 2010.

Greve, R. and Blatter, H.: Dynamics of ice sheets and glaciers, Springer, Berlin, Germany, 2009.

Haeberli, W., and Hoelzle, M.: Application of inventory data for estimating characteristics and regional climate-change effects on mountain glaciers: a pilot study in the European Alps, Ann Glaciol, 21, 206-212, 1995.

Huss, M., Farinotti, D., Bauder, A., and Funk, M.: Modelling runoff from highly glacierized alpine drainage basins in a changing climate, Hydrological Processes, 22 3888–3902, 2008. doi:10.1002/hyp

Huss, M., Jouvet, G., Farinotti, D., and Bauder, A.: Future high-mountain hydrology: a new parameterization of glacier retreat, Hydrology Earth and System Sciences, 14, 815-829, 2010. doi:10.5194/hess-14-815-2010

Huss, M., and Farinotti, D.: Distributed ice thickness and volume of all glaciers around the globe. J Geophys Res, 117, 1-10, 2012. doi:10.1029/2012JF002523

Kaser, G., Hardy, D.R., Molg, T., Bradley, R.S., and Hyera, T.M.: Moder glacier retreat on Kilimanjaro as evidence of climate change: observation and facts, International journal of climatology, 24, 329-339, 2004. DOI: 10.1002/joc.1008

Kavanaugh, J. L., and Clarke, G. K. C.: Discrimination of the flow law for subglacial sediment using in situ measurements and an interpretation model. J Geophys Res, 111, 1-20, 2006. doi:10.1029/2005JF000346

Klein, A.G. and Isacks, L.: Alpine glacial geomorphological studies in the central Andes using Landsat thematic mapper images, Glacial geology and geomorphology, 1998, rp01, 1998.

Landgrebe, D.: Multispectral land sensing: Where from, Where to?, IEEE Transactions on Geoscience and Remote Sensing, 43, 414-421, 2005.

Li, H., Ng, F., Li, Z., Qin, D., and Cheng, G.: An extended “perfect-plasticity” method for estimating ice thickness along the flow line of mountain glaciers. J Geophys Res, 117, 1-11, 2012. doi:10.1029/2011JF002104

Linsbauer, A., Paul, F., and Haeberli, W.: Modeling glacier thickness distribution and bed topography over entire mountain ranges with GlabTop: Application of a fast and robust approach. J Geophys Res, 117, 1-17, 2012. doi:10.1029/2011JF002313

MMAyA (Ministerio de medio ambiente y agua): Inventario nacional de presas [National dams inventory], La Paz, Bolivia, 2010. [in spanish]

Meier, M.F., and Bahr, D.B.: Counting glaciers: Use of scaling methods to estimate the number and size distribution of the glaciers in the world, edited by: Hanover, N. H., CRREL Spec. Rep., US Army, 89–95, 1996.

Michel, L., Picasso, M., Farinotti, D., Bauder, A., Funk, M., and Blatter, H: Estimating the ice thickness of mountain glaciers with an inverse approach using surface topography and mass-balance. Inverse Problems, 29, 1-23, 2013. doi:10.1088/0266-5611/29/3/035002

Morlighem, M., Rignot, E., Seroussi, H., Larour, E., Ben Dhia, H., and Aubry, D.: A mass conservation approach for mapping glacier ice thickness. Geophys Res Lett, 38, 1-6, 2011. doi:10.1029/2011GL048659

Mote, P. W. and Kaser, G.: The Shrinking Glaciers of Kilimanjaro: Can Global Warming Be Blamed? A “poster child” for climate change starves for snow and sublimates, Am. Sci, 95, 318–325, 2007.

Navarro, F. J., Macheret, Y. Y., and Benjumea, B.: Application of radar and seismic methods for the investigation of temperate glaciers. J Appl Geophys, 57(3), 193–211, 2005. doi:10.1016/j.jappgeo.2004.11.002

Nicholson, L., Marin, J., Lopez, D., Rabatel, A., Bown, F., Rivera, A.: Glacier inventory of the upper Huasco valley, Norte Chico, Chile: glacier characteristics, glacier change and comparison with central Chile, Ann Glaciol, 50, 111-118, 2009

Nye, J. F.: Plasticity solution for a glacier snout, J Glaciol, 6, 695-715, 1967.

Paul, F., and Lindsbauer, A.: Modeling of glacier bed topography from glacier outlines, central branch lines, and DEM, International journal of geographical information science, 26:7, 1173-1190, 2012.

PRI (Paleontological Research Institution): Glaciers, http://www.priweb.org/ed/TFGuide/NE/glaciers/glaciers.pdf, last access, October 2013.

Radic, V., and Hock, R.: Regional and global volumes of glaciers derived from statistical upscaling of glacier inventory data, J. Geophys. Res., 115, 1-10, 2010. doi:10.1029/2009JF001373.

Ramirez, E., Francou, B., Ribstein, P., Descloitres, M., guerin, R., Mendoza, J., Gallaire, R., Poyaud, B., Jordan, E.: Small glaciers dissapearing in the tropical Andes: a case-study in Bolivia: glaciar Chacaltaya (16° S), J Glaciol, 47, 187-194, 2001.

Raper, S. C. B., and Braithwaite, R. J.: Low sea level rise projections from mountain glaciers and icecaps under global warming. Nature, 439, 311–313, 2006. doi:10.1038/nature04448

Raup, B.H.; A. Racoviteanu; S.J.S. Khalsa; C. Helm; R. Armstrong; Y. Arnaud (2007). "The GLIMS Geospatial Glacier Database: a New Tool for Studying Glacier Change". Global and Planet Change, 56:101-110. (doi:10.1016/j.gloplacha.2006.07.018)

Sakai, A.: Glacial lakes in the Himalayas: A review on formation and expansion processes, Global environmental research, 16, 23-30, 2012.

Shrestha, D.L., Kayastha, N. and Solomatine, D.: A novel approach to parameter uncertainty analysis of hydrological models using neural networks. Hydrology and Earth System Science, 13, 1235-1248, 2009.

Smith, G.: McGill paleoclimate model ice sheet sensitivity to ice flow rate and discharge parameters, Department of Atmospheric and Oceanic Sciences, McGill University, Montreal, Canada, 2001.

Surazakov, A.B., and Aizen, V.B.: Estimating volume change of mountain glaciers using SRTM and map-based topographic data, IEEE transactions on geoscience and remote sensing, 44, 2991-2995, 2006

Tarboton, D.G.: A new method for the determination of flow directions and upslope areas in grid digital elevation models, Water Resour Res, 33, 309-319, 1997.

Vuille, M., Francou, B., Wagnon, P., Juen, I., Kaser, G., Mark, B. G., & Bradley, R. S. (2008). Climate change and tropical Andean glaciers: Past, present and future. Earth-Science Reviews, 89(3-4), 79–96. doi:10.1016/j.earscirev.2008.04.002

Zekollari, H., Huybrechts, P., Fürst, J. J., Rybak, O., and Eisen, O.: Calibration of a higher-order 3-D ice-flow model of the Morteratsch glacier complex, Engadin, Switzerland. Ann Glaciol, 54, 343–351, 2013. doi:10.3189/2013AoG63A434

List of Tables

Table 1. Main characteristics of the flow-lines

Flow-line	1	2
ΔH [km]	0.455	0.329
τ minimum [kPa]	64.2	48.4
τ maximum [kPa]	136.5	98.7
L [m]	1043.6	720
$\bar{\alpha}$ [°]	23.55	24.57

Table 2. Huayna West glacier maximum and minimum volume estimations using the maximum and minimum basal shear stress values

Using	Volume [km^3]
τ maximum [kPa]	0.035
τ minimum [kPa]	0.017

Table 3. Huayna West glacier volume estimations according to different confidence levels

Confidence	Volume [km^3]
10 %	0.019
20 %	0.020
30 %	0.023
40 %	0.025
50 %	0.026
60 %	0.028
70 %	0.030
80 %	0.032
90 %	0.034

Table 4. Huayna West glacier volume estimation according to different coefficients of the V-A method. Notes: (*) The estimation is defined by the shape. (**)The estimation is defined by the slope.

Group	Source	*c*	γ	Volume [km^3]
G1	Meir and Bahr (1996)	0.02	1.36	0.014
G1	Bahr (1997)	0.02	1.375	0.014
G1	Driedger and Kennard (1986)	0.0218	1.124	0.017
G2	Bahr et al., (1997)	0.0276	1.36	0.019
G2	Bodin et al., (2010)	28.5	0.357	0.020
G3	Adhikari and Marshall (2012)**	0.0336	1.3835	0.024
G3	Adhikari and Marshall (2012)*	0.0353	1.328	0.025
G3	Radic and Hock (2010)	0.0365	1.375	0.026
G3	Nicholson et al., (2009)	39.09	0.6009	0.026
G4	Baraer et al., (2012)	0.04088	1.375	0.035
G4	Klein and Isacks (1998)	0.048	1.36	0.034
G4	Huss and Farinotti (2012)	32.7	0.31	0.047

Table 5. Sensitivity of MTIFM thickness estimation with lower basal temperature

Temperature [°C]	Thickness increment [%]
0	0.00
-2	7.22
-5	20.92
-10	47.01

Table 6. Volume of the Huayna west glacier expressed in terms of a water layer equivalent considering the whole basin and the Tuni reservoir

Volume [km^3]	Basin WLE [m]	Reservoir WLE [m]
0.047	3.46	28.48
0.034	2.53	20.85

List of figures

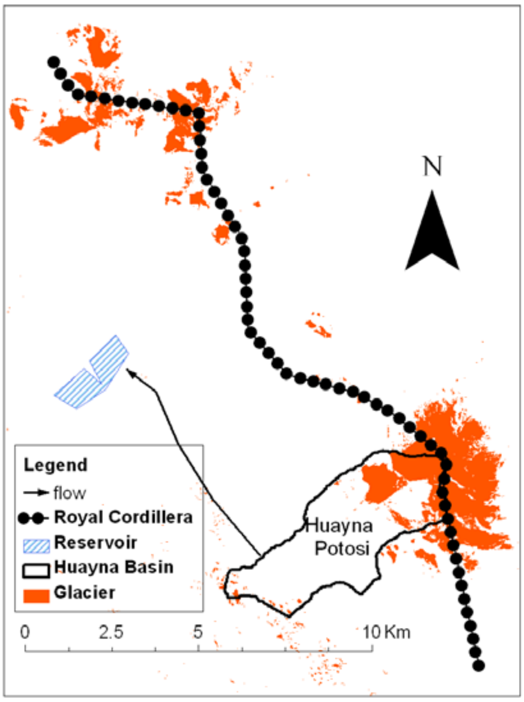

Figure 1. Huayna Potosi basin and the main glaciers of this area of the Royal Andes. The Huayna Potosi is the biggest glacier in this regions of the Royal Andes. The water from Huayna Potosi basin flows towards the Tuni reservoir.

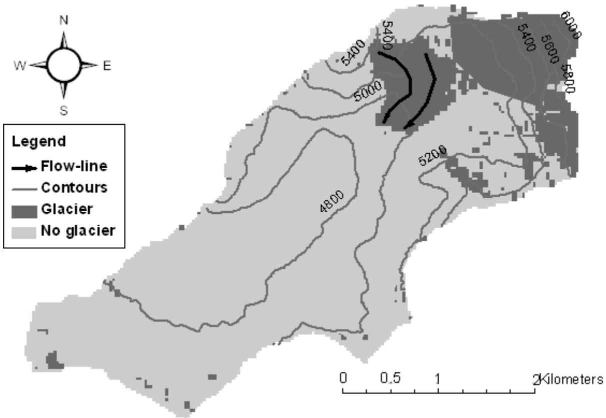

Figure 2. Huayna Potosi basin and the Huayna West glacier. The figure shows the two flow-lines of the glacier.

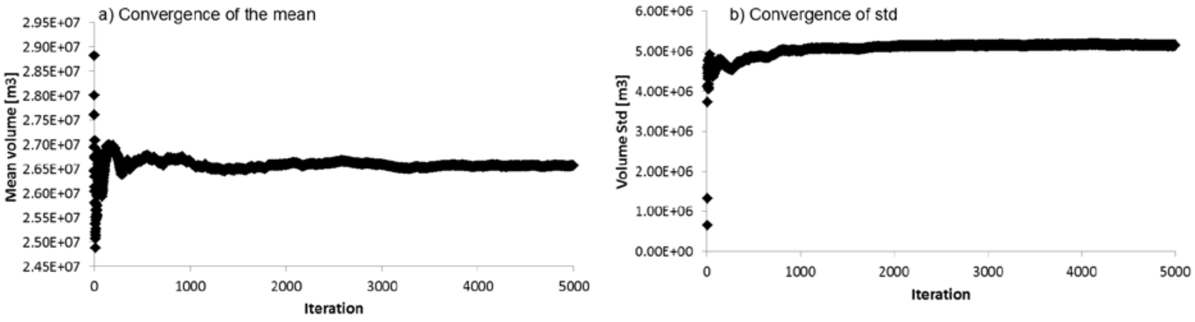

Figure 3. Convergence of the Monte Carlo analysis. Bothe the mean and the standard deviation (std) converge stabilize after 2000 iterations.

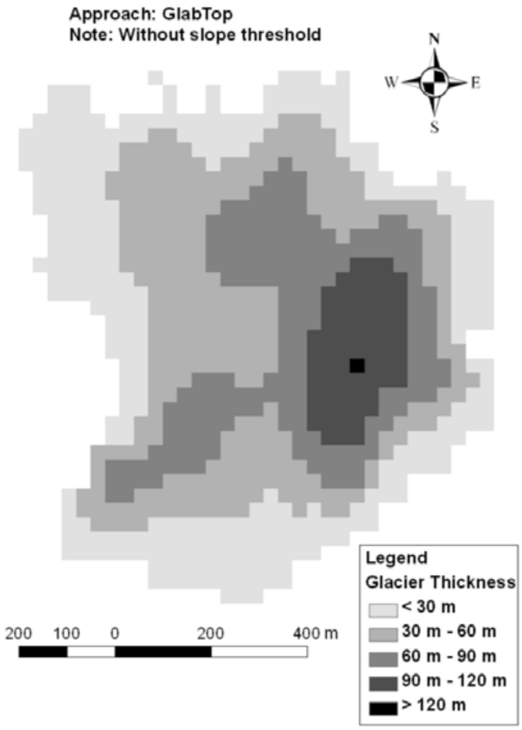

Figure 4. Thickness map of the Huayna West glacier according to the GlabTop methodology without any slope threshold

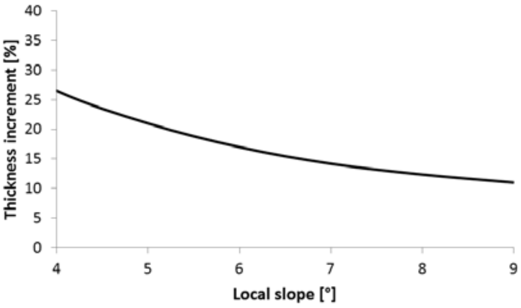

Figure 5. Sensitivity of the GlabTop thickness estimation to the local slope. For slopes lower than 6° the thickness estimations increase more than 20%.

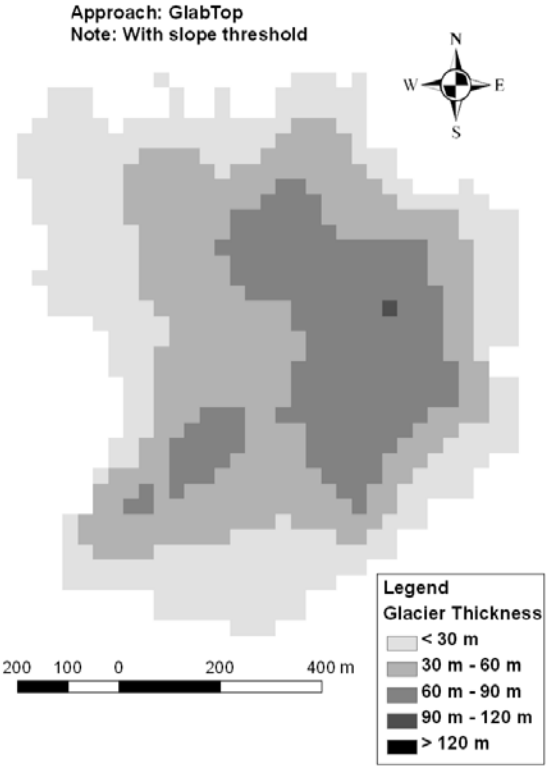

Figure 6. Thickness map of the Huayna West glacier according to the GlabTop methodology and applying a slope threshold of 6°.

Figure 7. Thickness map of the Huayna West glacier according to the MTIFM approach.

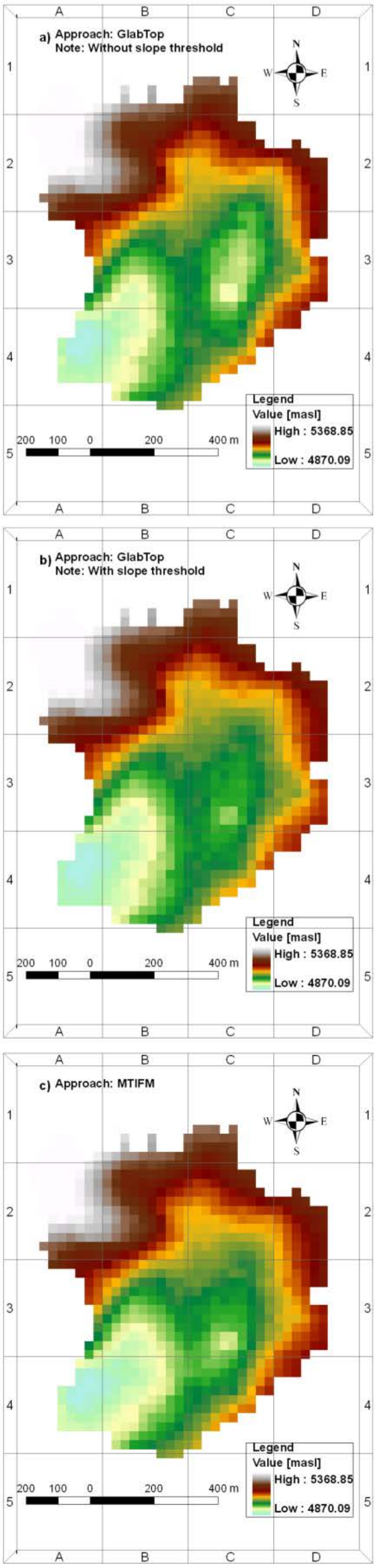

Figure 8. Reconstructed glacier bed topography of the Huayna west glacier according to the thickness estimations from: a) GlabTop without slope thresholds, b) Glabtop with a slope threshold of 6° and c) MTIFM. In the three cases there is a lake in the quadrant C3. The case

a) gives the deepest lake. In the case c) the lake is connected to the outlet of the basin in quadrant B4