Derivation of **Equation 20**

$$f_{g,m} = \frac{V_g\rho_g}{V_g\rho_g + V_f\rho_f} = \frac{\rho_g}{\rho_g + \frac{V_f}{V_g}\rho_f} = \frac{(1-\phi_g)\rho_p}{(1-\phi_g)\rho_p + \frac{V_f}{V_g}(1-\phi_f)\rho_p}$$

$$V_f = V_g \frac{(1-\phi_g)(1-f_{g,m})}{f_{g,m}(1-\phi_f)}$$

$$f_g = \frac{V_g}{V_g + V_f} = \frac{1}{1 + \frac{(1-\phi_g)(1-f_{g,m})}{f_{g,m}(1-\phi_f)}} = \frac{f_{g,m}(1-\phi_f)}{f_{g,m}(1-\phi_f) + (1-\phi_g)(1-f_{g,m})}$$

Where $f_{g,m}$ and f_g are mass and volumetric fraction of gravel, respectively; ρ_g, ρ_f and ρ_p are bulk densities of pure gravel, pure fine mineral, and particle density (assuming the same for gravel and fine mineral), respectively; V_g and V_f are volume of pure gravel and fine mineral, respectively; $\phi_g$and ϕ_f are porosity of pure gravel and fine mineral, respectively.

Table 2. Comparison of soil thermal properties calculated in three different sets of schemes (the CLM, CKJ and CKJ-G, see Table 1) for 130 cm sand (with and without gravel) with 0^o and 10^o slope during winter (December-Feburary, Frozen) and summer seasons (June-August, Unfrozen) over the period of 2003-2011.

Slope	Schemes	Thermal Conductivity (W m^{-1} K^{-1})		Volumetric Heat Capacity (MJ $m^{-3}K^{-1}$)		Thermal diffusivity (10^{-6} m^2s^{-1})	
		Unfrozen	Frozen	Unfrozen	Frozen	Unfrozen	Frozen
0^o	CLM	3.17	5.25	2.88	2.07	1.10	2.54
	CKJ	1.94	3.13	2.82	2.07	0.69	1.51
	CKJ-G	2.66	3.21	2.50	2.11	1.06	1.52
10^o	CLM	1.28	2.67	2.00	1.64	0.64	1.63
	CKJ	1.54	1.82	2.00	1.63	0.77	1.11
	CKJ-G	2.02	1.60	1.76	1.83	1.03	0.87