Dear Dr. J. Bassis,

thank you for your valuable comments to our paper in discussion for TC. We considered them carefully in the revised manuscript which to our believe made it more clearly. In the following please find our answers to the comments.

This manuscript describes the use of a finite element model to simulate the penetration of surface crevasses within ice shelves extending previous semi-analytic treatments of crevasse penetration to include vertical variation in ice properties. Overall, this manuscript treats an interesting and timely problem and despite some of my confusion about some of the modeling descriptions (see below) is a valuable contribution to studies of ice shelf fracture.

Specific comments:

Boundary conditions: 1. Viscous versus elastic loading: The authors assert that **_"The stress field at the ice shelf surface is evaluated from the flow velocity in the ice shelf using Glen's flow law (Glen, 1958)."_** This statement is confusing. Normally, one would suppose that the ice shelf surface is traction free and use the condition that the dot product of the stress tensor with the outward normal vector from the surface must vanish. This doesn't involve specifying any velocities nor the use of Glen's flow law. It may be that the authors are using a viscous rheology to determine a velocity along the rightmost side (i.e., calving front edge) of the domain and turning this into a displacement boundary condition? The reason to do this is unclear since it introduces an inconsistency into their model.

You are right, stating that the dot product of the stress tensor with the outward normal vector from the surface must vanish. However, the stress components at the ice shelf surface acting in the tangential plane do not vanish and are the components evaluated using Glen's flow law. As we do not include the calving front in our model, but look at cracks in the shelf in a sufficient distance from the calving front, we do not convert the velocities at the calving front into displacement boundary conditions. Instead we evaluate the displacement boundary conditions from the given velocity field. For the conversion, we use the principal surface stresses that we can evaluate using the rheological relation of Glen's flow law. As the choice of boundary conditions is a crucial point in the modelling, this topic is discussed in detail in Sec. 3.1.1. To further discuss the influence of the boundary, a figure is added to this section together with more detailed explanations. In general, we assume a displacement loading which is consistent with the velocity profile. As the horizontal velocities are assumed to be constant over thickness, the displacements, as the time integration of the velocities, should as well bare a constant value over the thickness. This is the central idea in the choice of the boundary condition. For the sake of comparability to previous results and analytical models we recalculate the displacement boundary condition in terms of surface stresses (of the uncracked) specimen. As discussed in the revised version of Sec. 3.1.1, this is not a one to one relation for deeper cracks and was not intended to be. It is rather to assign stress loads via Eqn. (19) to the displacement loads. Further remarks on this topic can be found in the response to length effect. In order to be brief, we do not repeat them here.

If we consider ice as visco-elastic in terms of a Maxwell element, there is no viscous OR elastic stress - depending on the time scale, the response of the element to a given load is more viscous or more elastic. This leads to viscous strains, which on the long time scale predominate the elastic strains. However, as we look on short time scales to analyse fracture, we consider the elastic strain.

Exact analytical solutions for stress intensity factors do only exist for infinite domains. As our ice shelf geometry has a finite thickness, we used a semi-analytical solution that can be found in many fracture mechanical books (e.g. Gross and Seelig) to validate the numerical model (Sec. 2.5, manuscript). The physical model is then validated using the results of Rist et al. (2002). We are confident that it is not necessary to use special crack-tip shape functions as the results with a refined crack tip are in good agreement with the semi-analytical results. The refinement studies are described in Sec. 2.5 and Fig. 1c.

We changed the sentence in Sec. 2.4 from "The modelled ice shelf consists of a vertical cut through an "infinite" ice shelf ..." to "The modelled ice shelf consists of a vertical cut through a finite section of an "infinite" ice shelf, …" in order to prevent possible misunderstanding about the proximity of the calving front. The geometry chosen is supposed to be far from the grounding line, as well as from the calving front, so that the influence of those on the crack is negligibly small.

2. Domain size: The authors are using glacier length of 2 km for an ice shelf that is uniformly 250 m thick. Given the assumed symmetry, the 1 km length between the fracture the end of the domain is comparable to the flexural wavelength within the ice. This would lead me to wonder if the results may be contaminated by edge effects from the domain boundaries and would be consistent with the large flexure that the authors observe in the constant stress boundary conditions. The authors should test to make sure that there results are independent of the length of their glacier.

Thank you for this very useful comment that made us rethink about the choice of the domain length.
We would like to emphasize that we are dealing with a multi-scale problem: the smallest length scale is the vertical crack length (or the thickness of the ice shelf), the medium scale is the horizontal model length (size of the computational domain), the macro length scale is the characteristic length of the ice shelf. This is not necessarily the lateral extend of the ice shelf. In order to have scale separation the wave length fluctuations of fields in the ice shelf have to be larger than the size of the computational domain. The surface stresses in the un-cracked ice shelf may vary over tenth of km. In order to achieve consistency, the length of the model domain has to be smaller to justify the application of a lateral constant stress or displacement state based on the surface velocities/stresses.
In addition, the model length has to yield results that are in good agreement with semi-analytical studies. For the validation we used stress boundary conditions and an aspect ratio thickness to length, which with approx. 1/10 satisfies the beam aspect ratio. The comparison with semi-analytical results showed that an aspect ratio ½ already yields satisfying results.
Taking constant displacement boundary conditions implies that the stress state at the crack tip depends on the model domain length, e.g. increases with the model length. The limiting stress intensity factor for an infinite model length can only be evaluated by applying the equivalent stress boundary condition.
We added a few sentences about the model length dependence in Sec. 3.1.1, where the dependence on different boundary conditions is analysed. Please find a revised version of Sec. 3.1.1 and the mentioned additional figure attached to the answer to your comment.

3. It looks like all computations were performed for a 250 thick ice shelf. It may be helpful to readers to report crack penetration depths as the ratio of crack penetration depth to ice thickness. I suspect that if the authors can perform the same simulation for a range of ice thicknesses then they will find that the ratio of crevasse penetration depth to ice thickness may be constant for a given set of parameters. If it is not, then this would be useful to report.

As we are using depth dependent material parameters (density, Young's modulus) the stress intensity factor can only be a function of the crack depth and thickness and not of only the ratio crack depth to ice thickness. Additionally, the density variation will always occur in the upper 70 to 100 m, independent of the ice thickness.

Section 2.4: I had a very hard time following the description of the boundary conditions used. I suspect that the authors treatment is correct and the misunderstanding is on my part, but it might be helpful to other readers if the authors can clarify the following questions.
1. It is not clear to me why the authors want to neglect bending stresses in the formulation. One often neglects these stresses our of convenience, but this does not imply that they are not present.

We do not neglect bending stresses within our ice shelf geometry. The geometric constraint of depth constant displacements exactly causes a bending moment as reaction to this constraint, thus bending stresses are taken into account. However, as the model geometry represents a finite cut out of an infinite ice shelf we have to prevent the vertical boundaries from bending more than the entire ice shelf would allow. We allow boundary displacements in vertical direction, which leads to bending stresses. We changed the sentence in Sec. 2.4 from "…, as those would allow bending." to "… as those would allow tilting of the vertical boundaries." .

2. "Sufficiently long rectangular domain" is vague. Can the authors confirm that they have performed domain size tests to determine if their results are independent of the length of the domain?

We believe that the discussion of the size effect in the revised version of Sec. 3.1.1 clarifies also this point. The vague expression was removed in the revised version.

3. "The stress field at the ice shelf surface is evaluated from the flow velocity in the ice shelf": It would be helpful if the authors explained how the authors are computing the viscous flow field. For instance, are they solving the full Stokes equations in the interior of the domain subject to traction boundary conditions on three sides?

The stress field is calculated using the velocity field of Braun at al. (2009) as stated in Sec. 3.1.2. In the cited paper the velocities are estimated using interferometry.

4. Glen's flow law depends on temperature and. How does the assumed ice temperature influence the authors results?

The temperature in the ice shelf influences the rate factor, that is used to evaluate the surface stresses using Glen's flow law. As we are only interested in a range of possible stresses and not in one specific stress, the influence of the temperature at this point has no influence on the conclusions of our results. The influence of the temperature on the ice density is included in the densification model of Herron and Langway. The dependence of the elastic parameters on the temperature is hardly investigated and therefore not simulated in the present study.

Of course, the ice surface is traction free, i.e. as stated in the beginning the stress vector normal to the ice shelf surface is zero. Nevertheless, the in-plane stresses at the surface are non-zero. We tried to avoid this misunderstanding by rephrasing the relevant sentences: "…, the magnitude of the boundary displacement Δu on one side of the model ice shelf is related to the horizontal stress σ at the ice shelf surface by …"

We solved the conversion problems.

We chose a qualitative plot with an equal colour range for both cases to show the influence of different boundary conditions. As the maximum tensile stress is at the crack tip and therefore singular in both cases, we do not think that actual numbers would increase the statement of this plot.

We would like to thank Dr. Bassis for the very constructive and helpful comments that helped us to improve the paper.

3.1.1 Boundary conditions and scale effects

Figure 3 (a) shows two uncracked example geometries of a homogeneous isotropic body, i.e. $\rho = const.$, with different horizontal BCs (left: constrained, right: unconstrained). Additionally, the vertical displacement at the basal boundary is constrained in both configurations, $u_z(z=0)=0$. The geometries are solely loaded by gravity. Figure 3 (b) shows the resulting the normal stress σ_{xx} for different Poisson's ratios and BCs. The stress component σ_{zz}, that can also be referred to as the ice overburden pressure, is identical for all simulated Poisson's ratios and BCs. It equals the horizontal stress for the constrained boundaries and $\nu = 0.499$ (dot-and-dashed line). The horizontal stress component σ_{xx} is affected by both, the changes in the Poisson's ratio and the BCs. A Poisson's ratio of $\nu = 0.499$, approximating an incompressible material, leads to a cryostatic stress state for the horizontally constrained body, meaning that for every material point in the body, the normal stress components are equal and shear stresses vanish. The value $\nu = 0.499$ is a good approximation of the incompressible case ($\nu = 0.5$), which numerically can not be treated with the applied constitutive law as it yields singularities in the stiffness matrix. For $\nu = 0.3$, the body is compressible. This leads for the horizontally constrained body to stresses σ_{xx} which are less than half of the stress component σ_{zz}. For a horizontally unconstrained body, the horizontal stress component σ_{xx} is identical zero. These results show that the assumption of incompressibility overestimates the crack closing pressure due to the weight of the ice by approximately a factor two.

In a next step, we apply different BCs on a cracked geometry to analyse the effect of the type of boundary condition on the stress concentration at the crack tip as well as possible dependencies on the length of the model domain. To ensure comparability to semi-analytical results, the simulations are performed without additional gravity or bottom pressure. Figure 4a shows, that simulations with pure stress boundary conditions on a domain of 2km length and 250m depth (green drawn through line) reproduce the semi-analytical results (black circles), which assume infinitely long domains (Gross

and Seelig, 2011). Further simulations with different depth/length ratios and stress BCs indicated, that a ratio larger than $\frac{1}{2}$ is sufficient to yield a crack tip field that is independent of the disturbance introduced by the boundaries. Different results can be expected for constrained boundaries (dashed and dotted lines) where the dependence on the domain length affects the stress intensity at the crack tip in two ways. To identify the factors that induce a length dependence, the cracked domain is on the first hand loaded by displacement BCs Δu and on the second hand loaded by stress BC σ with the additional constraint $\frac{\partial u}{\partial z} = 0$, which prevents tilting and bending of the vertical boundaries. The relation between the applied Δu and σ is evaluated via Eq. 19 using the uncracked geometry. The displacements Δu are kept constant for all crack depths. The dashed and the dotted lines in Fig. 4a show the resulting stress intensity factors $K_I(d)$ considering different domain length l for displacement and stress BCs, respectively. For large crack depths d, a length dependent difference between the constrained (dotted line) and the unconstrained stress BC represented by the drawn through line becomes obvious. The difference is caused by the crack closing bending moment induced by constrained boundaries. For l reaching infinity, the influence of the constraints on the deflection in proximity of the crack becomes negligibly small and the K_I converge to the solution of the unconstrained problem with stress BCs.

The difference between the dashed and the dotted lines for geometries of equal length l results in the compliance effect of deep cracks loaded by constant displacement BCs. If we imagine the ice shelf as a spring, the compliance due to the crack growth can be understood as a reduction of the spring stiffness. A reduction in stiffness yields a reduction in the resulting stresses and therefore decreasing stress intensity factors. The compliance effect becomes less obvious for values of l approaching infinity and $\lim_{l\to\infty}(K_I(d))$ yields the solution of the unconstrained problem with stress BCs.

The results show, that constrained boundaries imply a dependence on the length of the cracked domain. For infinitely long domains the influence on the type of BC vanishes and the solution of the pure stress BC is reached. The crack closing bending

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moment as well as the compliance due to the application of displacement BCs are reasonable qualities of the chosen model, considering an ice shelf with a given vertically constant velocity field.

Cracks in ice shelves are small-scale defects - crack depth and crack opening are small in comparison to the lateral extend of the ice shelf. As we are interested in the local effect of the crack we have to choose a model size that is large in comparison to the crack length but small in comparison to the characteristic length of the ice shelf. In addition, the stress field in the shelf can vary over length scales of less than 10km for which reason it is unrealistic to model cracked domains with lengths of more than 10km. To this end, we chose a finite domain length of $l = 2$km for all further simulations, taking the length dependence due to displacement BCs into account.

Figure 4b illustrates the deviation in the resulting K_I for different BCs and realistic loading due to additional volume forces ($\rho = $ const.) and water pressure at the bottom of the shelf.

The applied BCs consist of pure tension (a), the equivalent displacement boundary condition given by (19) (b) and the superposition of tension and horizontal pressure (c). As for $\nu = 0.3$, the horizontal pressure is not equivalent to the ice overburden pressure (see Fig. 3 (b)), it has to be evaluated from the horizontal reaction forces of the horizontally constrained body. For a small load of 100kPa and resulting shallow cracks, the difference between the BCs (b) and (c) is marginal. Case (a), with pure tension represents a totally different loading case. Even though the body is loaded by volume forces, they do not influence the horizontal stresses. The stress intensity factors for higher loads are on the other hand more sensitive to the choice of the BC as (d) and (e) demonstrate. Figure 4 (c) shows a qualitative plot of the horizontal normal stress σ_{xx} and the deformed shape (exaggerated presentation, scaled by a factor 100) for stress BCs and equivalent displacement BCs (no application of gravity or water pressure at the bottom boundary). As displacement BCs prevent the ice from bending, a bending moment that works against the crack opening is induced and the stress intensity at the crack tip, especially for deeper cracks, is smaller.