The Cryosphere Discuss., 6, C2174–C2181, 2012
www.the-cryosphere-discuss.net/6/C2174/2012/

The Cryosphere
Discussions

Interactive comment on "Numerical mass conservation issues in shallow ice models of mountain glaciers: the use of flux limiters and a benchmark" *by* A. H. Jarosch et al.

Anonymous Referee #1

Received and published: 9 November 2012

General comments

This paper describes a mass conservation problem that arises in the numerical formulation of shallow ice approximation models when either there is ice free point at upstream elevation, for example nunatak points within a glacier, or in very steep terrain where the upstream bedrock point may be at higher elevation than the ice surface point below. A method to ensure mass conservation is introduced and applied and then an analytical solution against which the method can be tested is presented and the code is tested. This paper is clear and well written and the results are of importance for numerical simulations as they provide a new analytical solution to test codes.

It would be interesting to know if authors have tested a simple upstream scheme which should ensure mass conservation, at least for the first problem of ice free points within the ice sheet, and probably less computationally demanding than the MUSCL method they suggest in the paper. The upstream scheme would be to make one power of the thickness the upstream thickness (that could be zero), i.e. replace hn+2 with hn+1*hupstream, which would ensure that no ice flux would come out of ice free points. Would this kind of upstream scheme give results of similar quality/accuracy as the more complicated MUSCL, superbee scheme?

There are a number of places in the text that should be clarified and typos in the formulas as well as mistakes or typos in integration that should be corrected, the places I suggest changes or corrections are pointed out below, but careful editing of both the text and formulas should be done before publishing.

Specific comments

Page 4038 line 2 "low-order" is the shallow model not zero order? Do you mean different orders of shallow models? Why not "zero-order"?

line 16 replace "to" at start of line with "for" suggest to replace "planet′s" with "glacier's" also suggest to clarify which ice masses, it is not clear what "These" is referring to here, do you mean mountain glaciers?

Page 4039

Line 3, suggest add "er" to "High"

Line 4 suggest "their high computational demand"

Line 11 "similar formulation" please clarify in what sense similar, a finite element or something else?

Line 15 add e.g. in front of reference, sentence starts with "Many" implying there are many others

Line 19 "such a model“ clarify, not clear what kind of model, you are using finite difference scheme in this papger and above finite element models are discussed

Page 4040

Line 1-2 suggest to rewrite "basic shallow model .. can be cast as“ to something like "the continuity equation together with Glen‘s flow law are the equations solved for the shallow ice model“ , suggest to write Glen′s flow law, rather than just Glen′s law

Equation (1) and (2) have plus between the time derivative of s and gradient of q, but equations (4), (6), (8) and (10) there is a minus, be consistent or define q as negative D*grad s for consistency

Line 7 replace "usual constants“ with something like "A and n are the rate factor and power law constants in the Glen′s flow law“

Line 9 take out "usual“

Line 10 take out "so“

page 4041

line 1 "can be used“ – clarify what you mean here, what can be used? Add "the“ in front of so-called

line 2 "This“ what? Clarify what it is that allows advances? And what do you mean by theoretical advances?

Line 12 "Roughly, these update ... " rewrite it is not clear what "these“ that are referred to are

Equation (6) why is the m in i+1 time step and not i?

Equation (7) later on in text you refer to this as projection step (e.g. line 11 page 4043) or post-processing (e.g. line 4 page 4046), chose one term for it and make it clear when the equation is introduced what it will be referred to later in the text

Page 4042

Line 1 typo in "inequality“. are the "inequality contraints“ given in Eq (2)? please clarify

Line 5 can you give a reference for this statement?

Page 4044

Equation 14 here is plus in front of the gradient of q, and in equations 12 and 13 there is a minus in definition of q, please make sure that the sign is consistent throughout the paper

Equation 15 Why do you introduce y-dimension here (and in eq 14 and 16) all the compuations in the paper are done in x-dimension only so it is not necesssary to add these terms, unless you want to refer to them, however, I cannot find reference to these terms in the text. In the hn+2 term there should be +, not -, between hk,l and hk+1,l as it is the average of the two, Same in Equation 16, replace the minus with a plus in that average term.

Page 4045

Line 15-18 all the stepping is done in x, or k index, but in Figure 2 the stepping is in l index, suggest to change in either place, probably more logical to change l to k in Figure 2

Page 4046

Equation 17, the second h should have k+1 index (not only k)

Page 4047

Line 4, here is projection step mentioned, do you want to call it that, or post processing step?

Line 5, "likely to acquire ..“ how? Because m is positive? Clarify

Line 14 "Consider a vanishingly small ice thickness in cell k,l“ what do you mean, is h=0 in k,l or is it vanishing?

Equation 18 is confusing, what is the x index on the q? Why is there not hk,l in the averageterm? Why is there both s and b in the gradient of the surface? Should the upper index on b in the first term be i+1?

Line 20 do you mean the downstream cell (k+1,l)?

Page 4049

Equation 21 check if the index on the first h after the equal sign should not be k-1,l

Equation 22 check if the index on the first h after the equal sign should not be k,l

Line 15 rewrite the text the two flux terms are from Eq. 20 and 19, not only 20

Page 4050

Equations 30 and 31 the first has a plus sign, the other has minus sign in front of the gradient q term, should define q so it is clear that it is negative of the D grad s term

Page 4051

Equation 32 the first term should have dx (the index in k is changing) and the second term should have dy (the index l is change), again, why do you include y-dimension if you do not use it? Suggest to simplify all equations and drop the l index, since it is not used

Line 14-15 suggest to change "analytically computable solutions“ to "analytical solutions“

Page 4052

Line 6 here the accumulation rate is called a, but in the text above it is m(dot), be consistent

Line 7 sentence is a little strange with occupied and occupying, suggest to rewrite to

something like "a continous ice region for the interval ...“

Equation 37 should the index on the latter x be m, rather than s?

Line 15 why not lower limit 0<x<xm?

Page 4053

Equation 44 check the integration, I would think that the power on the right hand side should be n/(2n+2) (as you have also in equation (46) and rather than 4 in the denominator I get n*2(1/n), I may well be wrong, but check carefully, there seems to be confusion of n and 2 in the power term, leading to both cases

Equation 45 same comment as above

Page 4055

Line 13 do you mean exact or analytical solution?

Line 16 could you use the defined subscript x(page 4052 subscript x is defined as "ordinary derivative“) to note the derivative of Q here? The superscript "Q‘ “ has not been defined

Equation 50 please check integration of equation 49, I am missing an (n-1) term, or the term that is the integration of the last bracket in equation 49, not sure how you end up with only the two terms that come out of the multiplication of the last bracket in equstion 50 with the rest. Again, I might be wrong, but please check again.

Page 4056

Line 3, again replace a with m(dot)

Eqution 51 check the algebra, I get xm(-1/n) in the denominator rather than xm(2n-1)/n, I may be wrong, please check. To be consistent replace x with x' on the right hand side, or drop the marker on x on the left hand side

Equation 52 is correct (apart from possibly the power of xm, see comment above) if (44) is right, my suspicion is, however, that the power on the bracket should be n/(2n+2) and 4 in the denominator should be replaced by n*2(1/n), see comment above

Page 4057

Line 5 add equation 12 into the bracket

Page 4058

Line 2 replace increasing with decreasing

Line 4 suggest to replace "recreate“ with "create“

Line 5 replace "of“ with "for“ 50 000 years

Line 10 add "s“ in described - not clear what is meant here with modification, please clarify the text

Line 18 why this particular number of years? Explain or clarify

Line 24 What do you mean by "outperform“ does it perform better?

Page 4059

Line 19 "best“ have you tried "all“ and thereby can conclude it is the best, or is it the best of the ones you tested?

Line 22 suggest to take out "case of“ do you want to use "exact“ or "analytical“ solution as in line 3 page 4060? Suggest to only use one term throughout the paper

Line 23 suggest to replace "with“ with "against“

Page 4060

Line 3, see above is it exact solution?

Line 4 take out s in equations

Line 5 do you mean finite difference scheme as above?

Equation A1 why do you have x superscript on the q here? Both the index and the denominator indicate x direction

Page 4061

Line 1 "projection step“ – what are you referring to here? It is not clear and Mahaffy should be referenced

Equation A4 should the second equation have equal sign, rather than < ?

Line 9 suggest to replace "To be definite“ with something like "to take an example“ Numbering has gone strange, Eq. A5 and A6 is only one quation. Shouldn‘t there be a power (n-1) on the || term?

Line 22 Eq. A5 has only left hand side

Equations A7 and A8 is only one equation. Again, should there be a power (n-1) on the first gradient term?

Figure 2 suggest to replace l with k

Figure 3 and figure 4 Suggest to indicate somehow the resolution of the solution, with points on the curves, or small delta x symbol

---

Interactive comment on The Cryosphere Discuss., 6, 4037, 2012.