**The Cryosphere
Discussions**

Interactive comment on "Manufactured solutions and the numerical verification of isothermal, nonlinear, three-dimensional Stokes ice-sheet models" *by* W. Leng et al.

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Response to Comments by Ed Bueler

Summary of the manuscript: The motivation for this manuscript is to improve the testing of 3D numerical ice sheet models that describe the ice by the isothermal, Glen law Stokes equations. This manuscript constructs exact solutions to a generalization of the Stokes equations, in the sense that the momentum equations and the surface kinematical equation have extra terms so that the artificially constructed functions ("manufactured solutions") really are solutions of these modified Stokes equations. The specific

construction here corrects the construction in a closely-related work (Sargent & Fastook, 2010). Then the manufactured solutions are used as verification tests for the parallel, finite element Stokes solver built by the same authors (Leng et al., 2012 JGR), and convergence under grid refinement is shown for their model. Major comments and conclusions: This paper both corrects errors in the Sargent & Fastook paper and actually demonstrate convergence of a numerical model under grid refinement, which was not done by Sargent & Fastook. Because the correction of errors is an important aspect of this paper, the claimed errors in Sargent & Fastook should be stated specifically. The reader needs to know where in Sargent & Fastook is the first error, and what equations are in error. The construction of the manufactured solution would be much clearer if errors in equations (23) and (50) in this manuscript are corrected, and certain odd aspects inherited from Sargent & Fastook are corrected (esp. wrong sign on pressure). The presentation of convergence is minimal, and figures are not used effectively to show the locations and/or character of the largese errors. More significantly, much effort is made to manufacture time-dependent solutions with moving surface, but then grid refinement beyond the initial time (i.e. at some $t > 0$) is not shown, and the reader might wonder if the model achieves it. It is not clear (at least in the manuscript) whether the code which demonstrates grid refinement is open source, which would be advantageous for claims of grid refinement. The best motivations for publication of this manuscript are that it corrects errors in a Cryosphere publication, and that grid refinement of a Stokes solver is actually shown. However, it is only appropriate to publish a carefully-revised form of the manuscript which addresses these several flaws.

Specific and constructive comments:

title: The title is too long. "Manufactured solutions and the verification of three dimensional Stokes ice-sheet models" would be long enough.

Answer: The title is changed as suggested.

abstract lines 7-8: The phrase "and other model parameters" is either worth expanding, or it should be removed. Perhaps "the geometry of the ice sheet, basal sliding parameters, and the ice softness."

Answer: The phrase is expanded as suggested.

abstract lines 9-10: Break run-on sentence. Perhaps "... tests. The upper surface is altered ..."

Answer: Changed as suggested.

abstract lines 12-14: Note "excellent agreement" is in the eye of the beholder. No argument is made that it is "excellent" in the paper. What is shown is more important anyway, namely convergence. So I suggest: "Results from the computational model show convergence under grid refinement using the manufactured analytic solutions."

Answer: Changed as suggested.

page 2690, lines 24-26: This sentence citing Alley and IPCC is unnecessary, as no Cryosphere reader has missed these ideas.

Answer: The sentence is removed.

page 2691, lines 6-7: The phrase "is generally accepted as the gold standard for the

modeling of" is silly. The reader who is interested in anisotropy or fracturing or etc will disagree anyway. And it is common that simpler models explain more in some situations, too. The phrase "is the standard non-shallow description of", or similar, is surely sufficient.

Answer: Changed as suggested.

page 2691, lines 6-24: This is a reasonable summary of the situation. pages 2691-2692: This may be the most important sentence in the current manuscript: "However, due to essential errors in their solution method for this key part in the three dimensional case, the Sargent and Fastook (2010) manufactured solutions .. for the 3-D Stokes model are incorrect." If this is an accurate assessment of the Sargent and Fastook (2010) solutions then the current manuscript needs to be published, even as an expanded erratum. Thus the authors *owe the reader a more precise statement*! What is wrong (i.e. which equations in Sargen & Fastook) and which equations here are corrections?

Answer: We now clearly point out exactly where in the Sargent and Fastook paper the error was made and where our derivation differs from theirs.

page 2692, lines 17-18: The ending "... in a low Reynolds-number flow" is redundant and unnecessary; a Stokes flow is the zero Reynolds-number limit.

Answer: Removed as suggested.

page 2693, equations (1)-(3): These equations appear to be inherited from Sargent &

Fastook, and I think they have a nonstandard sign on pressure. E.g. compare Pattyn et al 2008. In particular, $\sigma_{ij} = \tau_{ij} - p\delta_{ij}$ is, I believe, the standard sign convention. Thus the trace of σ_{ij} is - 3 p so that compression (i.e. diagonal of nsigma are negative numbers) is a positive pressure. If this is indeed the nonstandard choice it should be clearly stated. Better, it should be fixed to match the standard literature.

Answer: The pressure sign has been changed in all relevant formulations.

page 2693, equation (5): This equation seems also to be inherited from Sargent & Fastook, and it is not obviously correct. Now, it *is* correct, because of a particular method of using incompressibility, but why not write it so it is manifestly the matrix norm, i.e. the second invariant? Why confuse readers? The form in Leng et al. (2012; JGR) is correct and manifestly so.

Answer: The viscosity formula is now changed to matrix form. In the appendix for calculation of the viscosity, we still use the shorter version to save computation cost.

page 2693, last line: This defines Ω_t at a fixed time t, and it should define a subset of R^3 (i.e. space only, not space-time). If I am correct then the definition of this set should *not* have "$t \in [0,t_{max}]$" inside the curly brackets. The condition could be stated as "at each time $t \in [0,t_{max}]$ we define the set ..." in the preceding sentence.

Answer: Changed as suggested.

page 2694, equations (8)-(10): Up to the sign convention on pressure p, previously mentioned, these equations are correct. Are they stated with the $1/r_s$ factor because

that is the way the equations are scaled numerically? More generally, how are the equations scaled before numerical solution? (This may be addressed somewhere in Leng et al. (2012; JGR), which would be fine.)

Answer: The normal of the surface s is $(-\frac{\frac{\partial s}{\partial x}}{r_s}, -\frac{\frac{\partial s}{\partial y}}{r_s}, 1)$, where $r_s = \sqrt{1 + (\frac{\partial s}{\partial x})^2 + (\frac{\partial s}{\partial y})^2}$. In this paper, we do use the characteristic length L or aspects ratio δ to scale the equation.

page 2696: It looks to me that equations (15)-(17) are correct. Have Sargent & Fastook made an error at this point, or does it occur later?

Answer: These equations are correct; they made mistakes later as is now pointed out in the paper.

page 2697, equation (23): This is either wrong or makes the assumption that the surface s(x,y,t) and bed b(x,y) elevation functions do not depend on y. This would seem not to be true of the solution shown in Figure 1. What is going on? I think (23) is in error.

Answer: This is a typo and that equation has been removed. Thanks.

page 2701: State that the manufactured solution is available in C form in the supplement.

Answer: We added such a statement in the paper.

Answer: We removed the discussions in the paper about the size of the compensation terms since we find it is too complicated to draw a conclusion on this issue. We list explicit formulas for calculating them in the added Appendix.

Answer: Changed as suggested.

$$u(x, y, z, t) = c_x \left[1 - \left(\frac{s - z}{s - b} \right)^4 \right],$$

Answer: Yes, this is a typo, it has now been corrected. Thanks.

<div align="center">C1927</div>

Answer: We greatly rewrote Section 4 about verification of the manufactured solution and the numerical Stokes ice-sheet model. We change the section title 4.2 (originally 4.1) to "Convergence of the Stokes solver for the ice-sheet dynamics". We note that the accuracy of the time-dependent ice-sheet flow model depends not only on the Stokes solver, but also the solver for the free surface equations. It is quite complicated to balance all issues. Some numerical data and discussions about the model accuracy during the whole ice-sheet flow simulations are added in Section 4.3 (along with some error figures and curves) using the grid of size $80 \times 80 \times 20$. Overall, our finite element Stokes ice-sheet model performed quite well.

To run the simulation on a $160 \times 160 \times 40$ grid and smaller time step will surely improve the model results, but unfortunately we do not have enough CPU time to do that (may take a couple of months' continuous run). It is also obvious not a main goal of this paper, so we give up on that.

<div align="center">C1928</div>

Answer: We have added some related explanations and discussions in the paper about the errors in Table 1.

Explanations of the unit: "In Table 1, the error measure is the L^2 norm of the solution absolute error, and the units are (ma^{-1} km^3) for velocity and (10^5 Pa km^3) for pressure."

Explanations of the convergence rate: "Suppose that the error reduces with the grid refinement as $\|u_h - u\|_{L^2} = O(h^r)$, where h is the maximum element size of the grid and u_h is the numerical solution on such a grid, then r is called the *rate of convergence*. The convergence rates in the Table 1 are calculated between consecutive pairs of grids, e.g., the errors on a series of uniformly refined grids are $e_1, e_2, \ldots, e_K$, then the corresponding convergence rates are $\log_2(\frac{e_{i-1}}{e_i})$, $i = 2, \ldots, K$ respectively."

Answer: Now we show pictures of the exact solutions and the solution error, separately.

Answer: We have re-drawn all these figures using larger text for labels and bars as suggested.

Answer: This is a very insightful comment. We now reselect almost all figures. We first show illustrations of the exact solutions in Figures 3 and 4, and then present figures of the simulation errors in the remaining Figures 4,5,6.