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The Cryosphere Discussions

Interactive comment on "Manufactured solutions and the numerical verification of isothermal, nonlinear, three-dimensional Stokes ice-sheet models" *by* W. Leng et al.

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Response to Comments by Anonymous Referee #1

We thank the referee very much for his insightful comments which help us a lot in revising the paper.

We first would like to address the essential differences between our paper and the work by Sargent and Fastook (2010). In this paper we did follow the idea of Sargent and Fastook for the solution manufacture procedure. However, staring From equation (26)[1] in Section 3.1, we first corrected the mistake in Sargent and Fastook's work and present a correct solution process to a first-order partial differential equation which is the key part to construction of the general form of the analytic 3D ice-sheet Stoke solution. Based on the correct general solution formula, then a specific solution of the ice-sheet flow under a given geometry and boundary conditions was also derived in Section 3.2 (the one in Sargent and Fastook's work is also wrong since their general solution formula is wrong). Finally in Section 4 we presented numerical verification of the proposed analytic solution for ice-sheet flow using our parallel finite element 3D Stokes ice-sheet model. We think numerical verification is very important in this paper for demonstrating correctness of the constructed solution since errors in theoretical derivations and analysis sometimes are hard to find. Together with the codes for the analytic solution we provided in the supplemental material, people could test their 3D ice-sheet Stokes models in accuracy and efficiency. The above works are hard and tedious, it took us quite long time to figure them out and we finished all testing jobs just a couple months ago. That is also why we only "extruded" the correct 2D analytical solution of Sargent and Fastook 2010 to a third dimension and used it to verify output from their Stokes ice sheet model in our 2012 JGR paper. We think without any doubt our works clearly can not be covered by a comment to Sargent and Fastook (2010).

- In order to make clear the mistakes of Sargent and Fastook (2010) in solving the first-order partial differential equation and the correct solution process, we briefly explain below.

  Generally, to solve a first order partial differential equation such as

$$\frac{dy}{P(y,z,v)} = \frac{dz}{Q(y,z,R)} = \frac{dv}{R(y,z,v)}, \tag{1}$$

  certain first integral function like $\phi(y,z,v)$ needs to be found which satisfies

$$P(y,z,v)\partial_y\phi + Q(y,z,v)\partial_z\phi + R(y,z,v)\partial_v\phi = 0. \tag{2}$$

[1]The blue number means the equaiton index in our paper.

If two such first integral functions ϕ_1, ϕ_2 have been found, then the general solution to (1) is $\theta(\phi_1, \phi_2) = 0$.

In the special case, to solve the first order partial differential equation (26), which is in the form of

$$\frac{dy}{1} = \frac{dz}{f(y,z)} = \frac{dv}{g(y,z,v)},$$

one first integral could be found in

$$\frac{dy}{1} = \frac{dz}{f(y,z)}. \quad (3)$$

Since this equation does not contain v, it is easy to deduce $\phi_1 = \frac{z-b}{s-b}$ as in equation (31) . The other first integral function could be found in

$$\frac{dy}{1} = \frac{dv}{g(y,z,v)}. \quad (4)$$

However, we must be careful because this equation contains z. One big mistake of Sargent and Fastook (2010) is that they solve (4) as an ODE of variable of (y, v), but in fact dz must also be taken into account.

The correct way to solve (4) would be – first substitute z term with y term, then solve the ODE of variable (y, v), then substitute back z, as is done in (34) . In this way the integral of (34) satisfies (2).

To further demonstrate the solving process, let's solve a simple problem

$$\frac{\partial v}{\partial y} + y\frac{\partial v}{\partial z} - z = 0, \quad (5)$$

and here the characteristic equation is

$$\frac{dy}{1} = \frac{dz}{y} = \frac{dv}{z}.$$

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According to

$$\frac{dy}{1} = \frac{dz}{y},$$

one integral is $z - y^2/2 = C_1$. *For*

$$\frac{dy}{1} = \frac{dv}{z}, \tag{6}$$

the other integral is NOT $v - yz = C_2$, *clearly* $v = yz$ *is not solution to (5). Instead, substitute* $z - y^2/2 = C_1$ *into (6), and solve it for variables* (z, v), *we get*

$$v - y^3/6 - C_1 y = C_2.$$

Then substitute C_1 *back, we get*

$$v - y^3/6 - (z - y^2/2)y = C_2,$$

or

$$v - yz + y^3/3 = C_2,$$

then the general solution is $v = yz - y^3/3 + F(z - y^2/2)$, *where* F *is an arbitrary function.*

- **About the compensatory terms.** The compensatory terms are computed by substituting u, v, w, p to corresponding Stokes equations and the top/bottom surface boundary conditions (just in order to keep left sides and right sides of these equations equal). In the Sargent and Fastook (2010) paper, the authors listed needed derivatives of variables (not the final explicit formulas) for computing the compensation terms. In our method, it is easy to calculate them by using symbolic operations of the software "MAPLE". Since the formulas are too long, we don't want to list them in the paper. We generated the C code from "MAPLE" calculation output and used it in our numerical tests. The codes are provided in the separate supplementary material.

We also note that in order to verify the manufactured solution of the modified Stokes equation, one can check that whether u, v, w satisfy kinematic boundary conditions (6) and (7), and incompressible condition (4), since the velocity component u, v, w are derived from these three equations.

- **About the sliding boundary condition (12)-(13).** We thank the referee for pointing out this mistake. We adopted this part from Sargent and Fastook (2010) without careful checking. The correct one should be $t \cdot \tau \cdot n = -\beta^2 v \cdot t$, not $\tau \cdot n = -\beta^2 v$. We have corrected this part in the revision. We also would like to note that it almost does not affect the results of the paper at all (except the compensation terms added in this two boundary condition equations): *as pointed out above, the velocity component u, v, w in the general form of the analytic solution are derived from equations (4), (6) and (7); in the construction of the specific solution which is then tested in numerical experiments, we used the zero-velocity boundary condition on the whole bottom surface.*

- **About the lateral boundary.** We have revised it as the referee commented.

- In equation (50), there is a typo (a "1-" is missed) and it should be $u(x, y, z, t) = c_x \left[1 - (\frac{s-z}{s-b})^4\right]$ (it comes from equation (37) with $\gamma_1 = c_{bx} = 0$). This answers the Referee's question about equation (50) and Figure 3 (the value of u is about 46 not zero).

- We have redrawn Figure 3 as requested so that the same range of values are used in the color bar for each of subfigures in one row.

- We have redrawn Figure 4 as requested and now present all velocity components at the cross section $y = L/4$.

- We further find two more typos, in both equations (26) and (32), there should be a minus sign before dv.

TCD

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