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Significant total mass contained in small glaciers

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Abstract

A single large glacier can contain hundreds of millions of times the mass of a small glacier. Nevertheless, small glaciers are so numerous that their contribution to the world's total ice volume is significant and may be a notable source of error if excluded. With current glacier inventories, total volume errors on the order of 10 % are possible at both global and regional scales. However, errors of less than 1 % require glaciers that are smaller than those available in some inventories. Such accuracy requires a global list of all glaciers and ice caps (GIC) as small as 1 km^2, and for regional estimates requires substantially smaller sizes. For some regions, volume errors of less than 5 % require a complete list of all glaciers down to the smallest conceivable sizes. For this reason, sea-level rise estimates and other total mass and total volume analyses cannot ignore the world's smallest glaciers without careful justification.

1 Introduction

The world's largest glaciers dwarf the world's smallest glaciers by six or more orders of magnitude, and one large glacier (circa 10^5 km^2) can contain up to 100 million times more ice mass than the smallest glacier (circa 10^{-1} km^2). Although such an overwhelming volume would suggest that a few of the world's largest glaciers may contain the bulk of the world's ice mass, it is equally reasonable to ask if the rest of the glaciers are so numerous that they contain as much or more total ice. After all, for each single large glacier there are hundreds of thousands of smaller glaciers.

For the purposes of sea-level rise estimates and other analyses that depend on glacier inventories, this question of mass distribution is more than academic and in a warming climate could have important engineering and political consequences. We might ask, for example, if the estimated one-quarter to one-third of the total sea-level rise due to melting glaciers and ice caps (cf., Radić and Hock, 2011, 2010; Bahr et al., 2009; Meier et al., 2007) will be dominated by the few largest glaciers, or if sea level will

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rise faster in response to many smaller glaciers. And while the world's glacier inventories have become increasingly thorough and accurate, the very smallest glaciers are still the ones that are most likely to be overlooked (cf., Radić and Hock, 2010, Table 3). It is entirely possible that the smallest glaciers' sea-level contribution could be underestimated or misunderstood. With the smallest glaciers rapidly melting and possibly disappearing over the next few decades (Mernild et al., 2011; Radić and Hock, 2011), their potentially rapid sea-level contribution should not be summarily dismissed.

Moreover, most calculations of sea-level rise from glaciers and ice caps rely on an estimate of the total volume of land ice, either on a region by region or on a global basis (e.g., Mernild et al., 2011; Radić and Hock, 2011; Bahr et al., 2009). For glaciers and ice caps, the most recent and complete calculation finds a total volume of 0.60 ± 0.07 m sea level equivalent, but by necessity this estimate must scale up from incomplete inventories (Radić and Hock, 2010). An upscaling generally assumes that the estimated total mass is biased most significantly by missing large glaciers (for example, Radić and Hock (2010) found that the largest glacier was excluded from each of the regional inventories of Alaska, Arctic Canada, and Greenland), but it is also possible that a predominance of smaller glaciers could alter the total sea level equivalent while simultaneously shifting the overall mass distribution towards smaller glaciers. At the very least, any future upscaling of incomplete inventories would benefit from knowledge about the correct distribution of glacier mass.

The following analysis shows that when calculated in a reasonable way, the very largest glaciers do contain almost twice the mass of all the remaining glaciers combined. This matches our intuition but also suggests that care is warranted before dismissing smaller glaciers as irrelevant. For example, examined in another light, this implies that over one-third of the glacier mass is contained in the rest of the world's not-quite-so-large and smaller glaciers. Potentially, this sizeable one-third fraction could translate to many centimeters of rapid sea-level rise, contributing the bulk of their roughly 0.2 m sea-level equivalent on a time scale of decades.

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Furthermore, an analysis that uses only the world's largest glaciers will depend on the manner in which these large glaciers are tallied. A typical analysis divides glaciers by orders of magnitude so that they are placed into bins of size 0.1 to 1 km^2, 1 to 10 km^2, 10 to 100 km^2, etc. The largest glaciers are usually in a bin from 1000 to 10 000 km^2, and the following derivations show that this bin contains roughly 65 % of the total mass. However, an equally common and valid set of bins uses powers of 2 (cf., Radić and Hock, 2010; Raper and Braithwaite, 2006; Dyurgerov and Meier, 2005) and in that case, the following analysis shows that the largest glaciers (in the largest bin) contain only 27 % of the total mass. If we abandon order-of-magnitude bins altogether, and if we simply total the mass in an arbitrary selection of the world's largest glaciers, then this selection may have an even smaller fraction of the total ice volume. This again will depend on the number of glaciers selected, although choosing the world's largest 100 or so glaciers will probably span an order of magnitude (by a factor of 10) based on data from the admittedly incomplete World Glacier Inventory (Haeberli et al., 1989; NSIDC, 1999) and from an expanded but still partial inventory (Cogley, 2009). Regardless, it is not defensible to assume that smaller glaciers contain an irrelevant fraction of the total glacier and ice cap (GIC) mass.

More importantly, we can ask if there is a size below which glaciers become irrelevant to the total volume of all glaciers. Similarly, we can ask what the relative error would be if an analysis only includes glaciers larger than a certain size – for example, a selection of only the world's very largest glaciers, or all glaciers larger than 100 km^2. The following shows that 1 % and sometimes even 10 % errors in the total volume would necessitate inventories with surprisingly small ice masses, in some cases pushing the semantic boundary between glaciers and snow patches.

2 Assessing the mass distribution

2.1 Scaling relationships

Our calculation of total volume (or equivalently mass) uses two power-law scaling relationships. Number-size scaling gives the number of glaciers that have any particular area. Volume-area scaling converts each glacier's area to its volume. Combined, these power laws give the total volume of glaciers that happen to have a particular area (for example, the total volume of all glaciers of size 100 km^2). Integrating can then give the total mass of any range in glacier sizes, such as the total mass of the glaciers from 1000 to 10 000 km^2. Scaling relationships for ice caps will be considered separately at the end of the analysis, and obviously this study is not discussing the massive Greenland and Antarctic ice sheets whose volume, kinematics, dynamics, and contributions to sea level are always calculated separately (e.g., Pfeffer, 2011; Rignot et al., 2011).

Let $V(S)$ be the volume of a glacier of size or surface area S. Data and theory support a power-law relationship of the form

$$V(S) = cS^{\gamma} \tag{1}$$

where $\gamma = 1.36$ is derived from data (Chen and Ohmura, 1990) and $\gamma = 1.375$ is derived from theory (Bahr et al., 1997). The scaling constant c will disappear and become irrelevant.

Let $N(S)$ be the number of glaciers of size S. Data and theory support a power law of the form

$$N(S) = bS^{-\beta} \tag{2}$$

and data shows $\beta = 1.92 \pm 0.06$ as derived from the world glacier inventory (Cogley, 2009) sampled for 10 different regions (Figs. 1 and 2). The exponent $\beta = 1.9$ is also in good agreement with the theoretically predicted value of 2.0 derived from percolation theory (Bahr and Meier, 2000). Refinements of this scaling exponent would change the

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final mass totals and the error estimates but not the general conclusions. The scaling constant b will disappear and become irrelevant.

A theoretical analysis shows that the $N(S)$ power-law relationship should be multiplied by an exponential tail, or in other words, a more rapid decrease than a power law at the largest glacier sizes (Bahr and Meier, 2000). This exponential decay occurs only when the largest glaciers in a region are so big that they are bumping up against the size of the region being considered. In effect, the largest glaciers would be limited in size by the area of the region or mountains in which they can grow. That tail is increasingly irrelevant in a world of shrinking glaciers, and as expected, data from the 10 regional glacier inventories do not show this tail (Fig. 1). For this reason, and for clarity in the mathematics, we will assume that the tail is irrelevant. If anything, this assumption would over-estimate the total mass of the largest glaciers and make the following arguments and conclusions stronger.

At the smallest sizes, the data in Fig. 1 show a deviation from the power law (Eq. 2) suggesting fewer glaciers than predicted by the strict power law. The following analysis includes an adjustment for this contingency, but several considerations suggest that these smallest glaciers may be underrepresented in the inventory data. Certainly the smallest glaciers are the hardest to count. In many regions, these smallest glaciers are blurring the division between snow patches and glaciers which makes their numbers particularly hard to assess. Data also show that snow patches have a power-law distribution (Shook and Gray, 1996), making it less likely that small glaciers should deviate from a power law but then suddenly resume power-law behavior for snow patches that are only slightly smaller. In addition, theory suggests that the scaling exponents for glaciers and snow patches should be the same (Bahr and Meier, 2000). Furthermore, we applied an automated "flowshed" algorithm (Clarke et al., 2012) on the most recently compiled glacier mask for Western Canada (Bolch et al., 2010) to derive the sizes and numbers of glaciers in ten different subregions of British Columbia (Figs. 3 and 4); the mean scaling exponent is $\beta = 1.95 \pm 0.06$ in agreement with data and theory. Additionally, relative to the distributions presented in Fig. 1, there is no deviation

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from the power law at the smallest sizes derived from the flowshed algorithm (Fig. 3).

The following derivations give two bounding cases for the estimates of GIC mass distribution. One case will assume a power-law distribution at all glacier sizes. This gives a lower bound on the size of glaciers necessary to assess the total GIC mass. The second case will give a defensible upper bound under the assumption that power-law scaling fails for glaciers smaller than approximately 1 km^2. In this case, glaciers smaller than ~ 1 km^2 are ignored and considered irrelevant to the total GIC mass. The correct value will lie somewhere in-between the upper and lower bounds.

2.2 Mass contribution of the largest glaciers

While natural to assume that the largest glaciers are most relevant to sea-level rise and other mass-related calculations, the following demonstrates that the total mass of the largest glaciers is significant but not always sufficient. Let $V_{\mathrm{total}_S}(S)$ be the total volume of glaciers of size or area S. This is not the total volume of all glaciers; it is just the total volume of all of the glaciers that happen to have size S. This total volume can be written as

$$V_{\mathrm{total}_S}(S) = N(S)V(S) = bcS^{\gamma-\beta}. \tag{3}$$

Integrating gives the total mass in any range of sizes. Let $S_{\max}$ be the upper end of the largest size bin where bins increase by a factor of n (for example $n = 10$ or 2). Although $S_{\max}$ could be reasonably given a value on the order of 10 000 km^2, $S_{\max}$ can remain arbitrary for the moment. Bins increase and decrease by factors of n, so the lower end of the largest-size bin is $S_{\max}/n$. The volume of all glaciers in the largest size bin, V_{maxbin}, is the integral of $V_{\mathrm{total}_S}(S)$ from $S_{\max}/n$ to $S_{\max}$, or in other words,

$$V_{\mathrm{maxbin}} = \int_{S_{\max}/n}^{S_{\max}} bcS^{\gamma-\beta}\,dS \tag{4}$$

$$= bc\left(\frac{1}{\gamma-\beta+1}\right)S^{\gamma-\beta+1}\Big|_{S_{\max}/n}^{S_{\max}} \tag{5}$$

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$$= bc\left(\frac{1}{\gamma-\beta+1}\right)\left(S_{\max}{}^{\gamma-\beta+1} - \left(\frac{1}{n}\right)^{\gamma-\beta+1} S_{\max}{}^{\gamma-\beta+1}\right). \tag{6}$$

If bins are not used and a fraction f of the largest glaciers are selected instead, then n is simply redefined as $n = 1/f$ where f is between 0 and 1.

If $S_{\min}$ is the smallest glacier, then the total volume of all glaciers V_{total} is

$$V_{\text{total}} = \int_{S_{\min}}^{S_{\max}} bcS^{\gamma-\beta}\, dS \tag{7}$$

$$= bc\left(\frac{1}{\gamma-\beta+1}\right) S^{\gamma-\beta+1}\Big|_{S_{\min}}^{S_{\max}} \tag{8}$$

$$= bc\left(\frac{1}{\gamma-\beta+1}\right)\left(S_{\max}{}^{\gamma-\beta+1} - S_{\min}{}^{\gamma-\beta+1}\right) \tag{9}$$

$$= bc\left(\frac{1}{\gamma-\beta+1}\right) S_{\max}{}^{\gamma-\beta+1}. \tag{10}$$

The last line follows because $S_{\max} \gg S_{\min}$ (by many orders of magnitude). Also note that the derivation remains unchanged whether or not power-law scaling of $N(S)$ applies to the smallest glaciers – in either case, $S_{\max}$ is far greater than $S_{\min}$. Although the final estimate of total volume depends only on $S_{\max}$, all of the smaller glaciers have still been included in the calculation as part of the integration.

A ratio of V_{maxbin} to V_{total} gives the fraction of the total mass in the largest bin.

$$\frac{V_{\text{maxbin}}}{V_{\text{total}}} = 1 - \left(\frac{1}{n}\right)^{\gamma-\beta+1} \tag{11}$$

For bins that increase by factors of 10, the parameters become $n = 10$, $\gamma = 1.36$, and $\beta = 1.9$. So $V_{\text{maxbin}}/V_{\text{total}} = 0.65$, and the glaciers in the largest bin contains 65 % of the total mass. In other words, when glacier sizes are subdivided in this reasonable

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manner, the largest glaciers have the majority of the mass, but the remaining glaciers still have a sizeable 35 % of the total mass.

For bins that increase by factors of 2, the parameters become $n = 2$, $\gamma = 1.36$, and $\beta = 1.9$. So $V_{\mathrm{maxbin}}/V_{\mathrm{total}} = 0.27$. In other words, the largest bin has only 27 % of the total mass, and a significant 73 % majority of the mass is in the remaining glaciers. The choice of n is entirely arbitrary, but $n = 2$ is particularly common, so this example clearly illustrates the danger of assuming that only the largest glaciers contain relevant mass. Depending on what is meant by the largest glaciers and depending on the analysis, the remaining glaciers may contain as much or more mass. In essence, there are so many small glaciers, that their numbers can overtake the largest volume glaciers.

2.3 Mass contribution of smaller glaciers (lower bound)

The following establishes a lower bound for the smallest-sized glaciers needed in both global and regional inventories. In other words, this section considers the lower bound at which small glaciers become irrelevant to the total ice volume (either global or regional) assuming that a power-law scaling applies to $N(S)$ across all glacier scales. (The next section adds a correction to account for any deviations from power-law scaling at the smallest glaciers sizes.)

Consider the total volume of all glaciers V_{total} and an approximation of the total volume V_{approx} that ignores all glaciers below some size S_{min}. Integrating as before, the relative error θ between the volume approximation and the actual volume is

$$\theta = \frac{V_{\mathrm{total}} - V_{\mathrm{approx}}}{V_{\mathrm{total}}} \tag{12}$$

$$= 1 - \frac{\int_{S_{\mathrm{min}}}^{S_{\mathrm{max}}} bcS^{\gamma-\beta}\,dS}{V_{\mathrm{total}}} \tag{13}$$

$$= 1 - \frac{{S_{\mathrm{max}}}^{\gamma-\beta+1} - {S_{\mathrm{min}}}^{\gamma-\beta+1}}{{S_{\mathrm{max}}}^{\gamma-\beta+1}} \tag{14}$$

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$$= \left(\frac{S_{\text{min}}}{S_{\text{max}}}\right)^{\gamma-\beta+1}. \tag{15}$$

For any specified relative error we can solve for the smallest glaciers needed in an inventory

$$S_{\text{min}} = S_{\text{max}}\theta^{1/(\gamma-\beta+1)}. \tag{16}$$

If $\gamma = 1.36$, $\beta = 1.9$, and the world's largest glaciers are on the order of $S_{\text{max}} = 10\,000\,\text{km}^2$, then keeping $\theta \leq 1\,\%$ means $S_{\text{min}} = 0.45\,\text{km}^2$. In other words, all glaciers less than $0.45\,\text{km}^2$ can be excluded from the inventory because they contribute less than 1 % of the total volume. As an order of magnitude estimate, all glaciers smaller than $\sim 1\,\text{km}^2$ can be excluded from the inventory.

Similarly, if errors up to 5 % of the total volume are acceptable, then glaciers smaller than $15\,\text{km}^2$ do not need to be included in an inventory (using $\gamma = 1.36$, $\beta = 1.9$). As an order of magnitude estimate, all glaciers smaller than $\sim 10\,\text{km}^2$ can be safely excluded. At 10 % errors, glaciers smaller than $67\,\text{km}^2$ (or on the order of $\sim 100\,\text{km}^2$) can be excluded from the total volume (using $\gamma = 1.36$, $\beta = 1.9$).

However, if only the world's largest glaciers in the largest bin are used, then $S_{\text{min}} = S_{\text{max}}/n$ and the relative error is

$$\theta = \left(\frac{S_{\text{max}}/n}{S_{\text{max}}}\right)^{\gamma-\beta+1} \tag{17}$$

$$= \left(\frac{1}{n}\right)^{\gamma-\beta+1}. \tag{18}$$

For bins that increase by factors of $n = 2$, the relative error is 73 % (consistent with the fraction of mass outside the largest bin, previously derived). For bins that increase by factors of $n = 10$, the error is 35 %. For most applications, neither of these errors will be acceptable.

For many regions of the world, the largest glaciers may be orders of magnitude smaller than the globally relevant order of 10 000 km^2. The largest glaciers in the Alps, for example, are on the order of 100 km^2, and the largest glaciers in the Brooks Range (Alaska) are on the order of 10 km^2. To keep relative errors at 1 % or at any other value, the smallest inventoried glaciers within that region must shrink. For example, in the Alps, all glaciers larger than 0.004 km^2 must be included to keep errors at 1 % or less (using $\gamma = 1.36$, $\beta = 1.9$). For the Brooks Range, all glaciers larger than an improbably small 0.0004 km^2 must be included, effectively implying that all glaciers must be inventoried to keep errors in the region's total volume below 1 %.

For regions dominated by ice caps, the volume-area curve is less steep with data supporting a scaling exponent of $\gamma = 1.22$ and a theoretical value of $\gamma = 1.25$ (Bahr et al., 1997). As with glaciers, theory predicts $\beta = 2.0$, but inventories are not sufficiently complete to derive an observational value. Using only the theoretically derived exponents and S_{max} on the order of 10 000 km^2, all ice caps larger than 0.0001 km^2 must be included to keep errors at or below 1 %. Errors below 10 % require inventories to include all ice caps as small as 1 km^2. Clearly the smallest ice caps are always significant when calculating total ice cap volume in any region.

Table 1 shows the size of the smallest glaciers that are necessary for inventories of 10 different regions around the world. For 1 % errors in total regional volume, some regions have only barely sufficient inventories. However, published inventories are generally acceptable at larger error thresholds such as $\theta \leq 10$ %. In all cases, it is not acceptable to choose only the largest glaciers when estimating the total volume.

2.4 Mass contribution of smaller glaciers (upper bound)

If the smallest glaciers deviate from power-law scaling, then we can apply a correction to the previous estimates. This correction assumes that all glaciers smaller than $S_{deviate}$ do not contribute to the total volume, in which case V_{total} becomes an integral from $S_{deviate}$ to S_{max}. Because every region contains glaciers smaller than $S_{deviate}$, this gives an upper bound on the smallest glaciers S_{min} that need to be included when calculating

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the total ice volume.

Integrating for the total volume gives

$$V_{\text{total}} = bc\left(\frac{1}{\gamma-\beta+1}\right)\left(S_{\max}{}^{\gamma-\beta+1} - S_{\text{deviate}}{}^{\gamma-\beta+1}\right). \tag{19}$$

And the relative error becomes

$$\theta = 1 - \frac{S_{\max}{}^{\gamma-\beta+1} - S_{\min}{}^{\gamma-\beta+1}}{S_{\max}{}^{\gamma-\beta+1} - S_{\text{deviate}}{}^{\gamma-\beta+1}}. \tag{20}$$

Solving for $S_{\min}$,

$$S_{\min} = \left(S_{\max}\theta^{\gamma-\beta+1} + (1-\theta)S_{\text{deviate}}{}^{\gamma-\beta+1}\right)^{1/(\gamma-\beta+1)}. \tag{21}$$

The second term is a correction to the original equation that assumes power-law scaling across all glacier sizes. The correction term becomes small and irrelevant for large relative errors θ (Fig. 5).

Data suggest that the deviation from a power law happens at approximately $S_{\text{deviate}} = 1\,\text{km}^2$ in most regions of the world (Fig. 1). For typical values of $\gamma = 1.36$, $\beta = 1.9$, and $S_{\max} = 10\,000\,\text{km}^2$, keeping $\theta \leq 1\,\%$ means $S_{\min} = 3.1\,\text{km}^2$ which to an order of magnitude is $\sim 1\,\text{km}^2$. Similarly, for 5 % errors, $S_{\min} = 25\,\text{km}^2$ or an order of magnitude of $\sim 10\,\text{km}^2$. At 10 % errors, $S_{\min} = 87\,\text{km}^2$ or an order of magnitude of $\sim 100\,\text{km}^2$. These order of magnitude estimates are the same as when power-law scaling applies to the smallest glaciers. In effect, Fig. 5 shows that any deviation from power-law scaling is unlikely to change the order of magnitude of the error except for the smallest errors ($\lesssim 0.1\%$), and the deviation is relevant only for relative errors smaller than approximately 5 %.

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3 Conclusions

Glacier and ice cap areas span six or more orders of magnitude, but the smallest of these glaciers are much more numerous. As a result, the vast numbers of the smallest glaciers can have a significant total mass. When calculated in a reasonable manner (dividing glaciers into size bins that increase by factors of 10), the largest glaciers have roughly twice the mass of all the remaining glaciers combined. Although the largest glaciers may therefore have the majority of the mass, this leaves approximately one-third of the total volume in the remaining glaciers, and shows that significant care is warranted when assessing the relative contributions of different-sized glaciers and ice caps to sea-level rise (or to any other analysis). As an example, the dynamic response time of the smallest glaciers can be a hundred times faster than that of the largest glaciers (Jóhannesson et al., 1989), so faster rates of sea-level rise could be expected if the total mass of the small glaciers is deemed significant as suggested here.

The total volume of all the world's glaciers can be estimated to within any specified tolerance as long as glacier inventories are sufficiently complete at both the largest and the smallest sizes. If 10 % errors are acceptable, then current inventories are adequate. However, many regional glacier inventories may be incomplete at the smallest glacier sizes, and these small glaciers are notably relevant when demanding small errors. For example, global volume errors less than 1 % require inventories to include glaciers down to at least 1 km^2.

Some regions such as the Alps need to include even smaller glaciers to obtain an accurate estimate of the total regional volume. Without especially large glaciers to bias and shift the mass distribution upwards, the Alps (and similar regions) need inventories that are complete down to the smallest scales that could conceivably be called a glacier. Errors of less than 1 % in the Alps (certainly relevant for regional water resource planning) would require an inventory of all ice masses down to 0.004 km^2. At these scales, the difference between glaciers and snow patches becomes blurred, and the inventory must be effectively 100 % complete.

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As a practical measure, the relative error θ in total ice volume can be estimated easily from the largest and smallest glaciers used in an analysis.

$$\theta = \left(\frac{S_{\min}}{S_{\max}}\right)^{\gamma-\beta+1} \tag{22}$$

If power-law scaling does not apply to the world's smallest-sized glaciers, then this should be modified (as detailed above) for relative errors less than roughly 5 %. However, the differences at an order of magnitude scale are generally irrelevant, and this formulation should be a reasonable estimate to errors under most circumstances.

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Table 1. A calculation of the smallest glaciers in an inventory that would be necessary for relative errors in total volume that are less than or equal to 1 %, 5 %, and 10 %. Calculations are for 10 different glacierized regions around the world (Fig. 2) with inventories (Cogley, 2009) that are greater than 90 % complete. For regions dominated by ice caps, calculations use the theoretically derived values of $\gamma = 1.25$ and $\beta = 2.0$ (note the sensitivity of these regions to small ice caps due to the smaller volume-area scaling exponent). For regions dominated by glaciers, calculations use $\gamma = 1.36$ and $\beta = 1.9$. The maximum size glacier is estimated by order of magnitude. Results are also presented as order of magnitude estimates in km^2.

Region	Order of magnitude of largest glacier in inventory (km^2)	Minimum size S_{min} necessary for 1 % error in total volume (km^2)	S_{min} for 5 % error (km^2)	S_{min} for 10 % error (km^2)
Caucasus	100	0.01	0.1	1
Central Europe	100	0.01	0.1	1
Central Asia (North)	1000	0.1	1	10
Central Asia (South)	1000	0.1	1	10
Central Asia (West)	1000	0.1	1	10
New Zealand	100	0.01	0.1	1
North Asia	100	0.01	0.1	1
Russian Arctic	10 000	0.0001	0.1	1
Scandinavia	100	0.01	0.1	1
Svalbard	1000	0.00001	0.01	0.1

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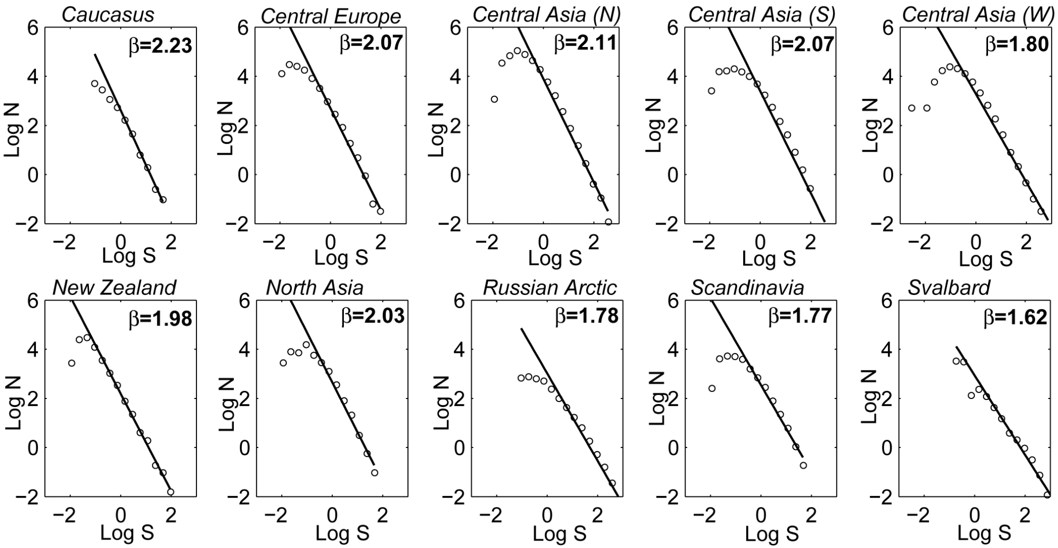

Fig. 1. Log-log plots of glacier size distribution, $N(S)$, for 10 regions around the world. Only regions with greater than a 90 % complete glacier inventory from Cogley (2009) are included. The location of each region is mapped in Fig. 2. The scaling exponent β (in other words, the slope of the regression line in each plot) is shown for each region assuming that glaciers in the four smallest size bins are ignored.

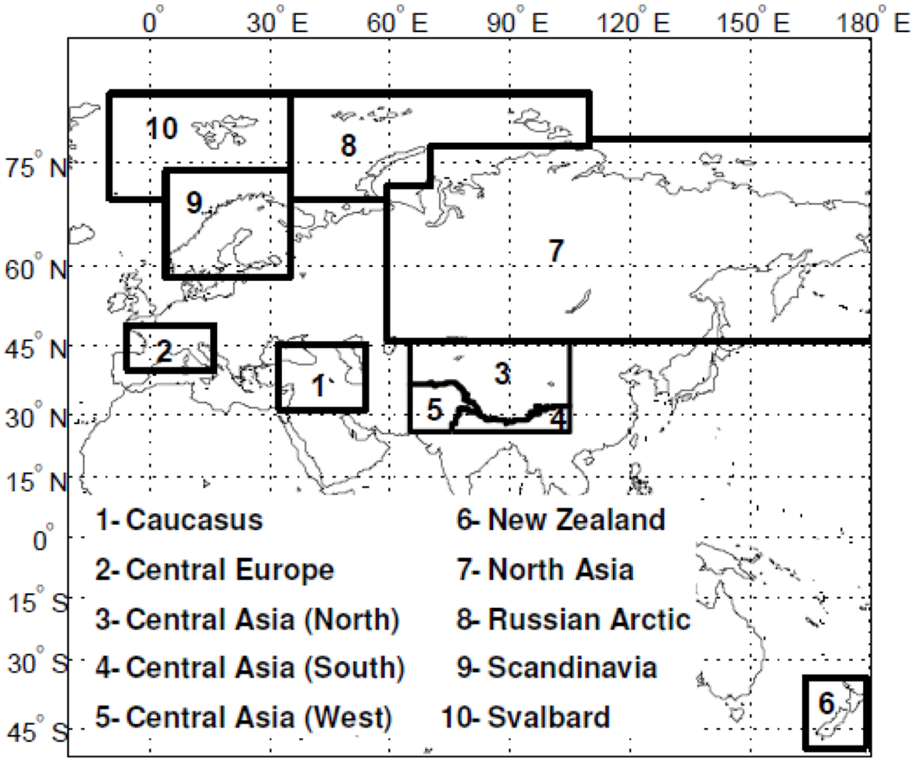

Fig. 2. Locations of the 10 glacierized regions whose glacier distributions are presented in Fig. 1 and Table 1.

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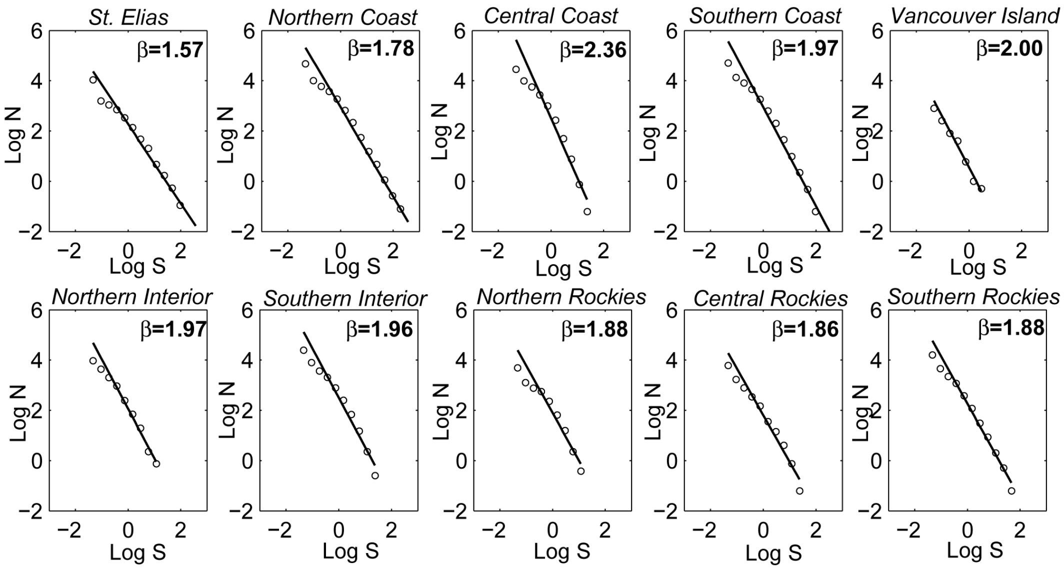

Fig. 3. Log-log plots of glacier size distribution, $N(S)$, derived from a numerical flowshed algorithm for 10 different subregions of British Columbia. The location of each subregion is identified in Fig. 4. The scaling exponent β (in other words, the slope of the regression line in each plot) is shown for each region.

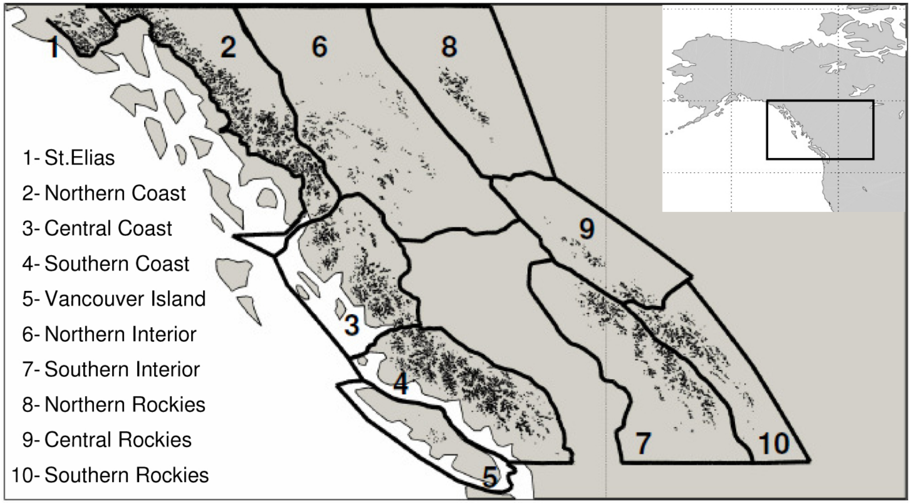

Fig. 4. Locations of the 10 subregions of British Columbia whose glacier distributions are plotted in Fig. 3. Black dots on the map correspond to the locations of glaciers within each region.

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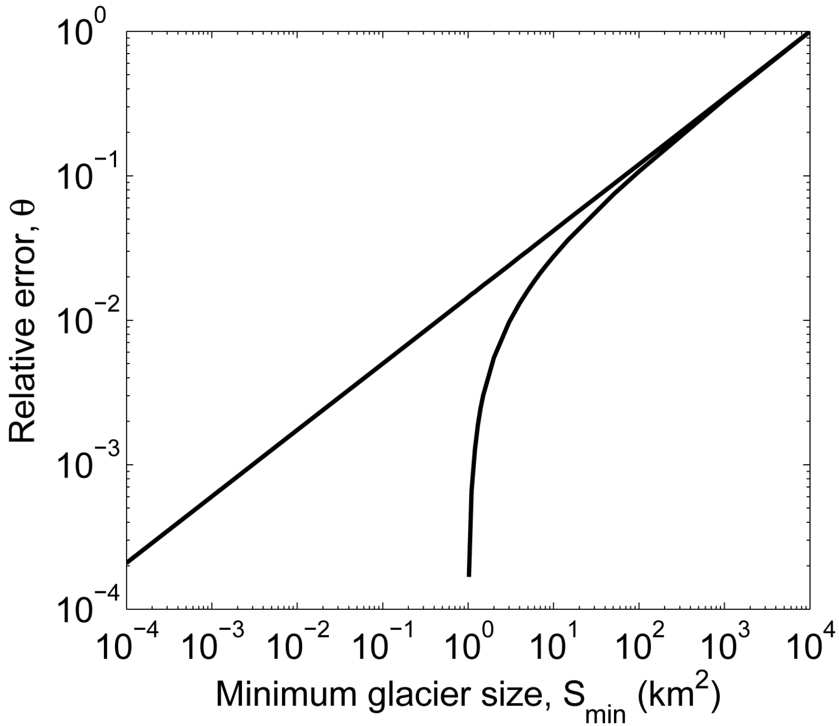

Fig. 5. The relative error in total volume calculated as a function of the minimum glacier size. The top curve shows the error when power-law scaling applies to $N(S)$ at all glacier sizes. The bottom curve shows the error when power-law scaling does not apply to glaciers smaller than $S_{deviate} = 1\,km^2$. Note that the difference is small (a fraction of an order of magnitude) for relative errors larger than roughly 5 %. Calculations for this plot use $\gamma = 1.36$, $\beta = 1.9$, and $S_{max} = 10\,000\,km^2$.

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Significant mass in small glaciers

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