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Investigating the dynamics of bulk snow density in dry and moist conditions using a one-dimensional model

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Abstract

Snowpack is a complicated multiphase mixture with mechanical, hydraulic, and thermal properties, highly variable within the year in response to climatic forcings. Bulk density is a macroscopic property of the snowpack used, together with snow depth, to quantify the water stored. In seasonal snowpacks, the bulk density is characterized by a strong non-linear behaviour due to the occurrence of both dry and wet conditions. In literature, bulk snow density estimates are obtained principally with multiple regressions, and snowpack models have put the attention principally on the snow depth and snow water equivalent. Here a one-dimensional model for the temporal dynamics of the bulk snow density has been proposed, accounting for both dry and moist conditions. The model assimilates the snowpack to a two-constituent mixture: a dry part including ice structure, and air, and a wet part constituted by liquid water. It describes the dynamics of three variables: the depth and density of the dry part and the depth of liquid water. The model has been calibrated and validated against hourly data registered in two SNOTEL stations, Western US, with mean values of the Nash-Sutcliffe coefficient $\approx 0.90\text{--}0.92$.

1 Introduction

Snowpacks and glaciers provide water supply to more than a sixth of the global population (Barnett et al., 2005). In Western United States, the snowpack is the principal source of water supply, about 50 %–70 % of the annual precipitation in the mountainous regions of the Western United States falls as snow and is stored in the snowpack. The dynamics of snowpack is strongly dependent on temperature variability. Hydro-climatological data, relative to the period 1950–1999, indicate a decline of snowpack in much of the Western United States (Pierce et al., 2008). Recent studies indicate that future scenarios of warming temperature will inevitably alter the distribution and magnitude of snowpack in many areas (Barnett et al., 2005).

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The snowpack is a multiphase mixture of three constituents: ice, liquid water, and air, subject to climatic forcings. The ice crystals are organized in cellular structures, or porous matrices, which are the skeleton of the snowpack, and principal responsible of its mechanical properties. The liquid water, produced by melting and rainfall phenomena, occupies the available spaces within the snowpack and modifies the hydraulic properties of the solid structure (DeWalle and Rango, 2008). In literature, snowpack models can be distinguished, according to the number of layers considered, in (1) single-layer models (see e.g. Tarboton and Luce, 1996; Jansson and Karlberg, 2004; Ohara and Kavvas, 2006), (2) two-layer models (Marks et al., 1998; Koivusalo et al., 2001), and (3) multi-layer models (see e.g. Anderson, 1976; Jordan, 1991; Bartelt and Lehning, 2002; Zhang et al., 2008; Rutter et al., 2008; Kelleners et al., 2009). The choice of a single-layer, rather than a multi-layer, model is dependent on the specific problem, one wants to address. For example for the modeling of avalanches, it is important a detailed description of the snowpack, layer by layer, and thus a multi-layer model occurs. On the other side, for the evaluation of the water resources stored within the snowpack, a global description of the snowpack is sufficient and consequently a single-layer, or a two-layer, model can be satisfying for the purpose. In particular, to quantify the water stored in the snowpack, the bulk density, together with the snow depth, is used.

The temporal dynamics of the bulk snow density is characterized by a strong non-linear behaviour, especially at the beginning of the accumulation season, and at end of the melting season (Mizukami and Perica, 2008), in dependence of the status of the snowpack, dry or wet, the first occurring principally during the accumulation season, while the second during the melting season. Modelling the bulk snow density both in dry and moist conditions allows to make multi-year simulations. Estimates of the bulk density of the snowpack are operated principally via multiple regressions on variables including snow depth, temperature, site altitude, wind velocity (Meløysund et al., 2007; Bavera and De Michele, 2009; Jonas et al., 2009; Bavera et al., 2012), with values of the determination coefficient up to ≈ 0.70 . One-dimensional snowpack models

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2 Methods

2.1 Definitions

Let the snowpack be composed by two constituents only, a dry part including ice structure, and air, and a wet part including liquid water. In this way we will be able to follow the dynamics of snowpacks in wet and dry conditions. Under sub-freezing conditions, i.e. temperature smaller than 0 °C, the liquid water is absent, and the snowpack will be referred as dry. Conversely for air temperature greater than 0 °C, the liquid water is present and the snowpack will be referred as moist.

Let consider a control volume of snowpack V , of unitary area and height h . Let V_S be the volume occupied by the porous matrix of height h_S , V_W the volume of liquid water of height h_W , and V_P the volume of pores within the ice matrix. Let $n = V_P/V_S$ indicate the porosity, and $\phi = V_W/V_P$ the degree of saturation of the ice matrix. From a general point of view the control volume can be expressed as $V = V_S + \langle V_W - nV_S \rangle$, and similarly can be done for the height $h = h_S + \langle h_W - nh_S \rangle$, where $\langle \cdot \rangle$ are Macaulay brackets, providing the argument if this is positive, otherwise 0. In normal conditions, i.e., dry, sub-saturated, and saturated, we have that $V = V_S$. Figure 1 reports a sketch of the snowpack in dry (1a) and wet condition (1b).

Let $\theta = V_W/V$ indicate the volumetric water content. Let M_D be the dry mass of snowpack including ice and air, and M_W the liquid water mass. Clearly the mass of snowpack $M = M_D + M_W$. Let indicate the bulk density of dry mass with $\rho_D = M_D/V_S$, the density of water with $\rho_W = M_W/V_W = 1000 \text{ kg m}^{-3}$, and the bulk density of snowpack with $\rho = M/V$. Consequently, ρ can be calculated as $\rho = (\rho_D h_S + \rho_W h_W)/h$. As range of variability, $\rho_F \leq \rho_D \leq \rho_{ICE} = 917 \text{ kg m}^{-3}$, and $\rho_F \leq \rho \leq \rho_W \text{ kg m}^{-3}$, where ρ_F is the density of the fresh snow (generally between 50 kg m^{-3} and 200 kg m^{-3}). Accordingly the porosity will be calculated as $n = (1 - \rho_D/\rho_{ICE})$.

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2.2 Equations of the snowpack

The dynamics of the snowpack is described through a set of mass balance, momentum, energy and rheological equations. In the next, we write the equations in the integral form. We simplify the energetic description of the snowpack as follows: (1) we assume that the constituents are in thermal equilibrium so that it is necessary only one energy balance equation. (2) Following Kondo and Yamazaki (1990), we consider a bilinear behaviour of the temperature $T(z)$ with the depth $z \in [0, h_S]$ of the snowpack. If the air temperature $T_A < 0$ then $T(z) = T_A - a_T(z - h_S)$ for $h_S \geq z \geq z_0$, and $T(z) = 0^\circ\text{C}$, for $z_0 \geq z \geq 0$, where $a_T \approx 0.033$ [°C/mm], and z_0 is the maximum value of z characterized by a temperature equal to 0°C . Conversely, if $T_A \geq 0$ then $T(z) = 0, \forall z$. Thus the depth averaged temperature of the snowpack is $T_S = \frac{1}{h_S} \int_0^{h_S} T(z) dz$. In this way, we use the air temperature as the sole index to describe the heat exchange between the snowpack and the atmosphere (Anderson, 1976; Ohmura, 2001).

2.2.1 Mass balance equations of the snowpack

15 The mass balance equations in the integral form, respect to V , for M_D and M_W are

$$\frac{dM_D}{dt} = \mathcal{P}_S + \mathcal{F} - \mathcal{M} - \mathcal{S} \quad (1)$$

$$\frac{dM_W}{dt} = \mathcal{P}_R - \mathcal{F} + \mathcal{M} - \mathcal{O} - \mathcal{E}. \quad (2)$$

20 In Eq. (1), \mathcal{P}_S is the incoming mass flux due to snow events, \mathcal{F} , \mathcal{M} and \mathcal{S} are mass fluxes due to changing phase phenomena, respectively refreezing, melting, and sublimation. In particular \mathcal{F} and \mathcal{M} are the exchanging terms between M_D and M_W .

25 In Eq. (2), \mathcal{P}_R is the incoming mass flux due to rain events, \mathcal{O} and \mathcal{E} are the outgoing mass fluxes respectively due to water outflow and evaporation phenomena. In the next, we will consider the case of a snowpack overlying an impermeable boundary (horizontal or with a small slope). This because it represents the condition investigated

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in the case study: the snowpack dynamics over a snow-pillow. Consequently, as water outflow we will consider only the water movement in horizontal direction, or parallel to the impermeable boundary (slope flow), not modelling the water percolation from the top to the bottom of the snowpack. We will assume also that the liquid water is accumulated in the lower part of the snowpack, thus having two zones, one unsaturated layer and one saturated layer, as indicated in Fig. 1b. In addition, as first approximation, we will neglect the refreezing, sublimation and evaporation terms.

The snow precipitation term can be written as $\mathcal{P}_S = \rho_F s$ where s is the snow precipitation rate, generally expressed in $[\text{m h}^{-1}]$. Following Anderson (1976), in the next, 10 ρ_F will be considered a function of the air temperature only, T_A , at the beginning of the snowy event, $\rho_F = 50 + 1.7(T_A + 15)^{1.5} \text{ kg m}^{-3}$, if $-15 \leq T_A \leq 2^\circ\text{C}$, and $\rho_F = 50 \text{ kg m}^{-3}$ if $T_A < -15^\circ\text{C}$. The term of liquid precipitation is written as $\mathcal{P}_R = \rho_W p$ where p is the rain rate, expressed in $[\text{m h}^{-1}]$.

The melting term can be expressed, following a temperature-index approach 15 (Ohmura, 2001), as $\mathcal{M} = \rho_D / (T_A, M_D)[a + b(T_A - T_\tau)]$, where $/ [-]$ is a binary function equal to 1 if $\{T_A \geq T_\tau, M_D > 0\}$, and 0 otherwise. $a \text{ m h}^{-1}$ and $b \text{ m h}^{-1} \text{ }^\circ\text{C}^{-1}$ are two parameters, and T_τ is a temperature threshold. T_τ is usually assumed equal to 0°C , even if some literature studies show that the model quality can be improved by adopting different thresholds (e.g. van den Broeke et al., 2010). In the next, as first approximation, 20 we will assume $T_\tau = 0^\circ\text{C}$. Physically, a is the melting at $T_A = T_\tau$, while b represents the increase of ablation with the temperature. b is also known as degree-hour factor.

The water outflow, \mathcal{O} , depends on the hydraulic properties of snowpack, which change significantly during the melting season (DeWalle and Rango, 2008). When water moves through the melting snowpack, many observations have showed the existence of preferential flow channels in horizontal and vertical directions (Gerdel, 1954; Marsh and Woo, 1984; Schneebeli, 1995). Thus the hydraulic motion of water through the snowpack is both a “matrix flow” and a “preferential flow” (Waldner et al., 2004) in proportions that depend on the liquid water content. However since this is still an open issue, here, as a first representation of the phenomenon, we model the water

outflow through a kinematic wave approximation following Nomura (1994) and Singh (2001). In particular we assume that $\mathcal{O} = c\rho_W\theta h_W^d$ for $\theta > \theta_r$, otherwise 0, where c [$1\text{m}^{-1}\text{h}^{-(d-1)}$] and d are two constants, and θ_r is the residual water content (value under which the residual amount of liquid water is retained into the ice matrix and only vapour exchanges occur). The residual water content is calculated as $\theta_r = F_C\rho_D/\rho_W$ where F_C is the mass of water that can be retained per mass of dry snow, assumed equal to 0.02, according to Tarboton and Luce (1996) and Kelleners et al. (2009). c is a site-specific parameter depending by factors like slope and altitude of the site. For the exponent d , Nomura (1994) proposed $d = 1.25$. Following Nomura (1994), in the next we will use $d = 1.25$.

Since $M_D = \rho_D h_S$ and $M_W = \rho_W h_W$, after some algebra, Eqs. (1)–(2) can be written as

$$\frac{dh_S}{dt} = -\frac{h_S}{\rho_D} \frac{d\rho_D}{dt} + \frac{\rho_F}{\rho_D} s - I(T_A, h_S)[a + b(T_A - T_\tau)] \quad (3)$$

$$\frac{dh_W}{dt} = p + \frac{\rho_D}{\rho_W} I(T_A, h_S)[a + b(T_A - T_\tau)] - c\theta h_W^d. \quad (4)$$

2.2.2 Momentum balance and rheological equations of the snowpack

The momentum balance equation in integral form, relatively to M_D , of height h_S , is

$$\sigma - \rho_D g h_S = 0, \quad (5)$$

where σ is the vertical stress at the bottom of the ice matrix, and g is the gravitational acceleration. Equation (5) is obtained assuming quasi-static conditions of snowpack. Considering the *Maxwell law* as rheological equation to connect the vertical stress σ to the vertical viscous strain rate $\dot{\epsilon}$, $\eta = \frac{\sigma}{\dot{\epsilon}}$, where η is the coefficient of viscosity (Mellor, 1975). The vertical deformation rate can be expressed as a function of the density of ice matrix (Kojima, 1967), i.e. $\dot{\epsilon} = \frac{1}{\rho_D} \frac{d\rho_D}{dt}$. Substituting these last equations in Eq. (5),

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we obtain

$$\frac{d\rho_D}{dt} \bigg|_{\text{comp}} = \frac{\rho_D^2 g h_S}{\eta}. \quad (6)$$

Equation (6) models the dynamics of the bulk density of dry constituent due to compaction. The coefficient η is the product of two components: one due to compaction effect, η_C , and the other to temperature change, η_T . Following Kojima (1967), η_C can be expressed as an exponential function of the snow density, i.e. $\eta_C \propto e^{k_0 \rho_D}$ where k_0 is a constant. Similarly, following Mellor (1975), η_T can be expressed as $\eta_T \propto e^{[k_1(T_\tau - T_S)]}$, where k_1 is a constant, and T_S [°C] is the temperature of the snowpack. Consequently, Eq. (6) can be written, after Kongoli and Bland (2000), Ohara and Kavvas (2006) and Zhang et al. (2008) as

$$\frac{d\rho_D}{dt} \bigg|_{\text{comp}} = c_1 \rho_D^2 h_S e^{[0.08(T_S - T_\tau) - 0.021 \rho_D]} \quad (7)$$

where $c_1 = 0.001 \text{ m}^2 \text{ h}^{-1} \text{ kg}^{-1}$. Equation (7) represents the time evolution of ρ_D as consequence of the compaction. From a general point of view, the temporal derivative of ρ_D can be written as

$$\frac{d\rho_D}{dt} = \frac{d(M_D/V_S)}{dt} = \frac{1}{V_S} \frac{dM_D}{dt} - \frac{M_D}{V_S^2} \frac{dV_S}{dt}. \quad (8)$$

From Eq. (8), it is possible to see how the temporal variability of ρ_D is the sum of two terms: the first one depending on the dry mass variation, and the second one on the dry volume variation. If $\frac{dM_D}{dt} = 0$ then $\frac{d\rho_D}{dt} = -\frac{M_D}{V_S^2} \frac{dV_S}{dt}$ and the variation of ρ_D is due only to the variation of the volume of the dry constituent, as it happens in compaction when no snow events occur. Equation (8) includes as particular case Eq. (7). Snow events entail

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variations of both mass and volume of the dry constituent. From Eq. (8), it is possible to show that the temporal variability of ρ_D due to a snow event is $\frac{d\rho_D}{dt} = \frac{\rho_F}{h_S} s - \frac{\rho_D}{h_S} s$. Similar result is also reported in Ohara and Kavvas (2006). Here we assume that melting phenomena (as well as sublimation ones, that we have neglected here) occur at $\rho_D = \text{const}$, i.e. mass variations balance volume variations. This assumption works well in the case of dry snowpacks, less in moist snowpacks. Consequently, Eq. (8) can be written as

$$\frac{d\rho_D}{dt} = c_1 h_S \rho_D^2 e^{[0.08(T_S - T_\tau) - 0.021\rho_D]} + \frac{(\rho_F - \rho_D)}{h_S} s \quad (9)$$

Equations (3), (4), and (9) represent a system of three differential equations in the three state variables h_S , h_W , and ρ_D , forced by the meteorological variables (p , s , T_A) and with a parsimonious parametrization: three parameters to be calibrated a , b , and c . The other parameters (c_1 , d , T_τ , F_C) are fixed. Once solved the system of Eqs. (3), (4), and (9), we can obtain the dynamics of the other variables of interest and in particular the bulk density of the snowpack ρ .

3 Data and results

As case study we have considered two weather stations of the snowpack telemetry (SNOWTELE) network: S1) Thunder Basin station (ID = 817) in Washington, $48^\circ 31' \text{N}$, $120^\circ 59' \text{W}$, with an altitude of 1300 m a.s.l., and S2) Brooklyn Lake station in Wyoming (ID = 367), $41^\circ 22' \text{N}$, $106^\circ 14' \text{W}$, with an altitude of 3100 m a.s.l. In these locations, precipitation regime is characterized by a winter maximum and summer minimum with a maximum in snow accumulation during spring. Hourly data of air temperature T_A [$^\circ\text{C}$], accumulated precipitation [mm], snow depth h [mm], and snow water equivalent SWE [mm] are available. Unfortunately no measurements of liquid water content are collected. We have selected data relatively to the period 1 October 2007–30 September 2011 in S1, and 1 October 2006–30 September 2011 in S2 (data are available at the

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website: <http://www.wcc.nrcs.usda.gov/snow/>). We have operated this selection within the period of observation (2006–2011 both for S1 and S2), because these years do not suffer of technical problems like: (1) presence of periods where data are available but only at a coarser resolution, or (2) significant percentage of lacking data. A filtering operation of SNOTEL data has been also done: (i) negative values of h were eliminated; (ii) absolute hourly increments of h greater than 60 cm were removed; (iii) positive (negative) increments of h followed by negative (positive) values of equal entity were considered erroneous and removed; (iv) a temperature filter has been applied to remove flutter phenomena in snow depth series. The filtering operation has led to discharge 9.5 % of data in S1, and 5 % in S2. Then values of the bulk snow density ρ are obtained from those of h and SWE.

The model (Eqs. 3, 4, 9) is designed to be driven by inputs of air temperature T_A and precipitation P . A time step of 1 h has been used in model's runs and comparisons with observed data. Air temperature is used to infer the snow temperature. Precipitation data input are obtained, as hourly increments, from the time series of accumulated precipitation for rain (ρ) and from the time series of snow depth for snow (s). As for rain, a critical temperature of 0 °C has been imposed to T_A . Rain precipitation inputs are therefore derived by evaluating the hourly differences between total precipitation and snow precipitation increments, and imposing that any positive difference (with $T_A > 0^\circ\text{C}$) is a rain event. As quality control of precipitation data, any negative value, and any negative increment of cumulative precipitation has not been considered. We found a snow/precipitation ratio $\approx 67\%$ for S1 and $\approx 78\%$ for S2. Figure 2(3) reports, for two years, in panel (a) the air temperature data, and in panel (b) the precipitation data (where the snow is in red and the rain is in blue), for S1(S2).

Model's parameters (a , b , c) are calibrated using the least square method on the first year of data (i.e., 2007–2008 for S1, and 2006–2007 for S2), while the other years (i.e., 3 for S1, and 4 for S2) are considered during the validation phase. We found the following estimates $a = 0.00011 \text{ m h}^{-1}$, $b = 0.00042 \text{ m h}^{-1} \text{ }^\circ\text{C}^{-1}$, $c = 0.11 \text{ m}^{-1} \text{ h}^{-(d-1)}$ for S1, and $a = 0.0001 \text{ m h}^{-1}$, $b = 0.00056 \text{ m h}^{-1} \text{ }^\circ\text{C}^{-1}$, $c = 0.51 \text{ m}^{-1} \text{ h}^{-(d-1)}$ for S2. Estimates

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of a and b are abundant in literature (WMO, 1965; Braithwaite, 1995; DeWalle and Rango, 2008), it is not so for values of c . Generally a and b are expressed respectively in $[m d^{-1}]$ and $[m d^{-1} C^{-1}]$, and their conversion to hourly time scale can be done considering 12 h as the effective day time. For snowpack, DeWalle and Rango (2008) gave for b the range $0.0002\text{--}0.0004\text{ m h}^{-1} C^{-1}$, while WMO (1965) the interval $0.000083\text{--}0.00058\text{ m h}^{-1} C^{-1}$. For glaciers, Braithwaite (1995) estimated $a = 0.00025\text{ m h}^{-1}$, $b = 0.00067\text{ m h}^{-1} C^{-1}$. Our estimates are very close to the ones given in literature for snowpacks, and smaller than the estimates provided for glaciers as expected. An estimate of c is found in Nomura (1994), who provided a value of $1/6$. This estimate is quite close to the value obtained in S1 (0.1) and not too far from the value found in S2 (0.5). The system of Eqs. (3, 4, 9) has been solved numerically using the forward Euler finite-difference scheme. A fixed time step, Δt , of one hour (congruent with the data series resolution) has been used. The modeled values of the state variables at the time instant $t + 1$ have been calculated using values at the previous time step t , and considering that the time derivatives are calculated as $(f(t + 1) - f(t)) / \Delta t$, where $f = h_S, h_W, \rho_D$. As initial values, we set the state variables h_S and h_W at zero, if at the beginning of the first water year no snowpack is present, as in the case of seasonal snowpacks, and the calculation of dry density has been conditioned to the existence of snowpack, i.e. $h_S > 0$. Eq. (9) has numerical problems because it is not defined for $h_S = 0$ (as already pointed out by Ohara and Kavvas, 2006). To avoid this inconvenience, the second term of Eq. (9) is calculated as $\frac{(\rho_f(t) - \rho_D(t))}{h_S(t) + s(t)\Delta t} s(t)$. Note that, in this way, it is possible, from one side evaluating the new snow event effect on the density dynamics with an updated snow depth, and from the other, keeping the benefits of an explicit finite-difference scheme. Clearly an implicit scheme could provide a more accurate evaluation of the dynamics, but with longer computation times. Thus we obtain time series of the variables h_S , h_W , and ρ_D . From this we calculate time series of h , ρ , and SWE, which can be compared with observed data. Figure 2(3) shows the comparison between data and model for S1(S2), relatively to the year of calibration, and the last year of validation. In particular, panel (c) reports h , panel (d) ρ , and panel (e)

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SWE. For sake of clarity we have reported also h_W in panel (c), and ρ_D in panel (d). We have calculated the Nash-Sutcliffe model efficiency coefficient, R^2 , for each year, and for each of the three variables: h , SWE, and ρ . For the year of calibration, we found that for S1, the Nash-Sutcliffe for the snow depth, $R_h^2 = 0.98$, for SWE $R_{SWE}^2 = 0.98$, and for ρ $R_\rho^2 = 0.96$, while for S2 $R_h^2 = 0.84$, $R_{SWE}^2 = 0.78$, and $R_\rho^2 = 0.87$. For the years of validation, we have calculated the mean value of the Nash-Sutcliffe coefficient: $\bar{R}_h^2 = 0.92$, $\bar{R}_{SWE}^2 = 0.90$, $\bar{R}_\rho^2 = 0.90$ for S1 and $\bar{R}_h^2 = 0.92$, $\bar{R}_{SWE}^2 = 0.92$, $\bar{R}_\rho^2 = 0.92$ for S2. Note that these values of the Nash-Sutcliffe coefficient are obtained keeping constant the parameters' values along all the simulation period. The model presents good performances, as in calibration, as in validation phase, in both the two sites. In particular, it is interesting to note that, for S1, the model's performances in calibration and validation phases are quite equivalent, although a slight reduction in the average of the Nash-Sutcliffe coefficient can be observed in the validation period. For S2, we found that the performances in the validation period are better than the ones in the year of calibration. According to our opinion, this is due to the fact that during the year of calibration, the problems of flutter, not completely eliminated with the filter, have led to overestimate the snow deposition, and thus to worst performances respect to the ones found during the years of validation. From panel (d) we can appreciate the differences between ρ and ρ_D . For S2, located at 3100 m.a.s.l., ρ and ρ_D curves are very close, except for the last part of the melting season, indicating that ρ_D can give a good approximation of ρ during the accumulation and in the first part of the melting season. This is also supported by a small value of the average value (over the year of calibration) of the water content 7 %, and by the fact that the condition $\theta < n$ is verified in 99 % of the year. On the contrary, for S1 located at a lower altitude, 1300 m a.s.l., ρ and ρ_D curves are in general different one from the other, and in this case ρ_D cannot be considered an approximation of ρ . In this case the average value of the water content is 12 % and the condition $\theta < n$ is verified in 71 % of the year. Lastly, we have compared the predictions of θ with the volumetric liquid water content observations by Techel and Pielmeier (2011) during the melting season of 2010 in an Alpine environment. Although Techel

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and Pielmeier (2011) provide a description of the local volumetric content of a snowpack throughout its depth, it is possible to relate our global estimations with Techel and Pielmeier (2011) ranges of variation. For example, on 3 April 2010, we predict a volumetric water content of roughly 5.4% in S1, and 2.3% in S2, which are similar to those measured by Techel and Pielmeier (2011), which lie in a range between 0% and 7% throughout snow depth. On 17 April 2010, we predict a liquid water content of 10% for S1, and 4.5% for S2, while Techel and Pielmeier (2011) data are greater than 4–5%, with many local peaks of 10%, being intermediate the simulated values in S1 and S2. Probably, this is due to the fact that the height (2210 m) of the site considered by Techel and Pielmeier (2011) is intermediate between the altitude of S1 and S2. This comparison is enforced by noting that in Techel and Pielmeier (2011) any measured data greater than 10% has been cut to 10%, because of some instrumental uncertainties. As a consequence, we think that a precise modeling of bulk snow density (as shown in this contribute) could help answering the open issue of Techel and Pielmeier (2011) work about quantitatively measuring the liquid water content in snow, by observing that the difference between dry density and bulk density is directly ascribable to the water content itself.

4 Conclusions

We have presented a one-dimensional model for the dynamics of bulk snow density in dry and wet conditions, where the snowpack is represented as a two-constituent mixture: a dry part including ice structure, and air, and a wet part constituted by liquid water. The model includes mass balance equations of dry and wet constituents, momentum balance and rheological equations for the dry part, and a simplified energetic description of the snowpack. The model results in a system of three differential equations in the variables, depth and density of the dry part and depth of liquid water, forced by precipitation and air temperature data input, with a parsimonious parametrization: only three parameters to be calibrated. The model has been tested against hourly

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data registered in two SNOTEL stations: Thunder Basin station for 2008–2011 period, and Brooklyn Lake station in the period 2007–2011, located at different altitude. The model shows a good agreement with data of snow density, snow depth and SWE, not only in calibration, but also in validation phase, with mean values of the Nash-Sutcliffe coefficient in the range [0.90, 0.92]. Improvement of performances could be obtained including within the model refreezing, sublimation and evaporation terms. The model seems suitable to predict the snowpack dynamics starting from hydroclimatic inputs. The general good capacity of the simulations in reproducing measured snow density confirms our preference for a global one-dimensional model, which avoids the local incongruities in snow density modeling during the snowmelt season, as pointed out by Koivusalo et al. (2001). This analysis will be extended to the other stations of SNOTEL network in order to make other tests on the model's performances and to investigate the variability of the calibration parameters, especially for the site-specific parameter c . In addition, a validation of the liquid water content dynamics is necessary, and will be the object of a future study.

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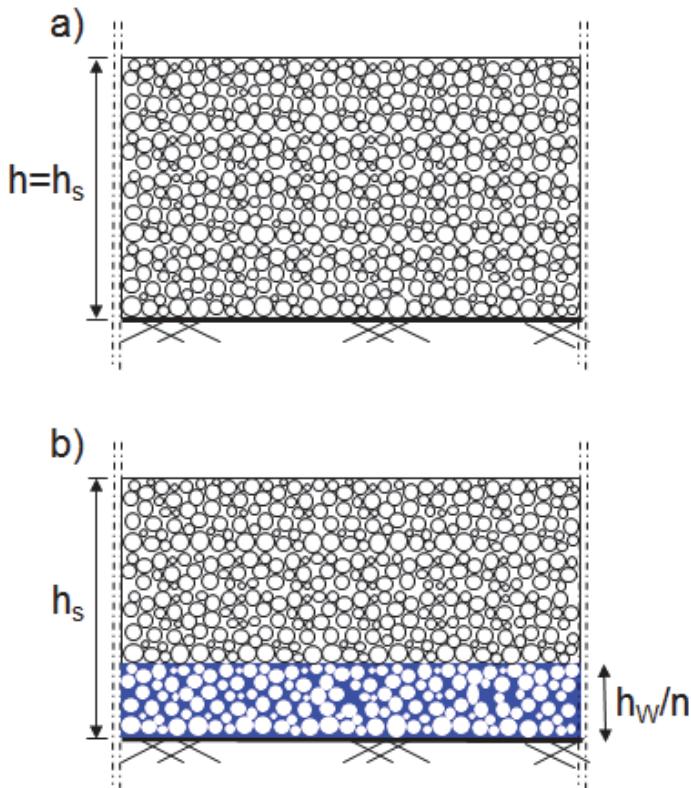


Fig. 1. Snowpack in dry condition panel **(a)**, and in wet condition panel **(b)**.

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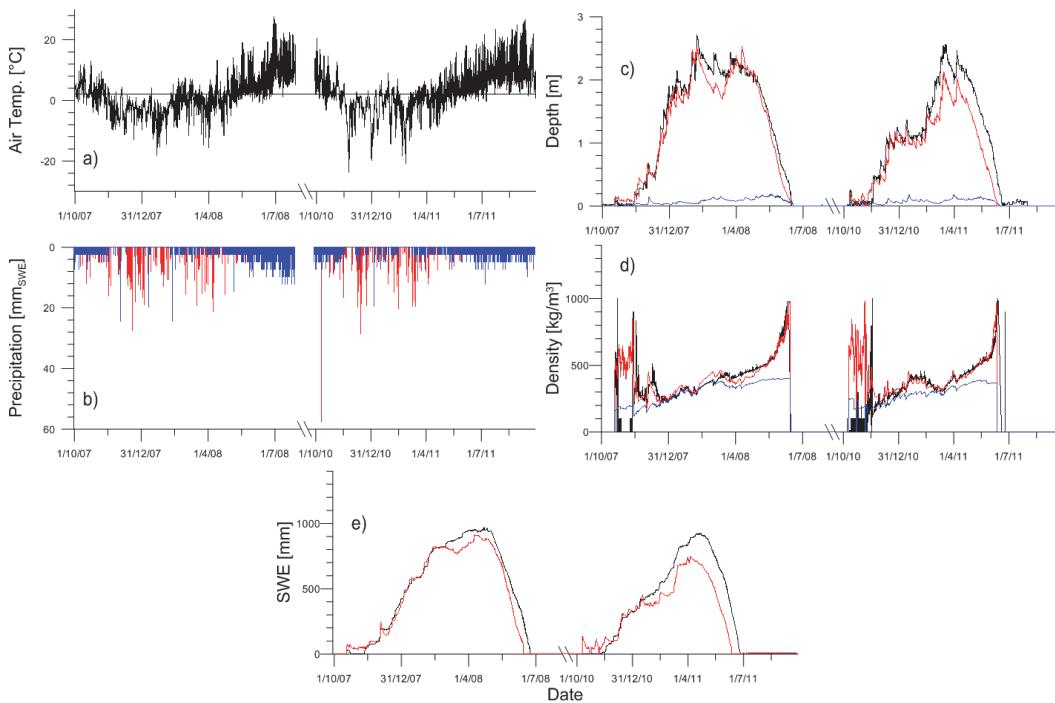


Fig. 2. Meteorological forcings, and dynamics of depth, density and SWE for S1 and two hydrologic years: 2007–2008 (calibration) and 2010–2011 (validation). Panel **(a)** Air temperature, panel **(b)** precipitation, liquid in blue and solid in red, panel **(c)** depth h in red modelled and in black observed, and h_w in blue, panel **(d)** density ρ in red modelled and in black observed and ρ_D in blue, and panel **(e)** SWE in red modelled and in black observed.

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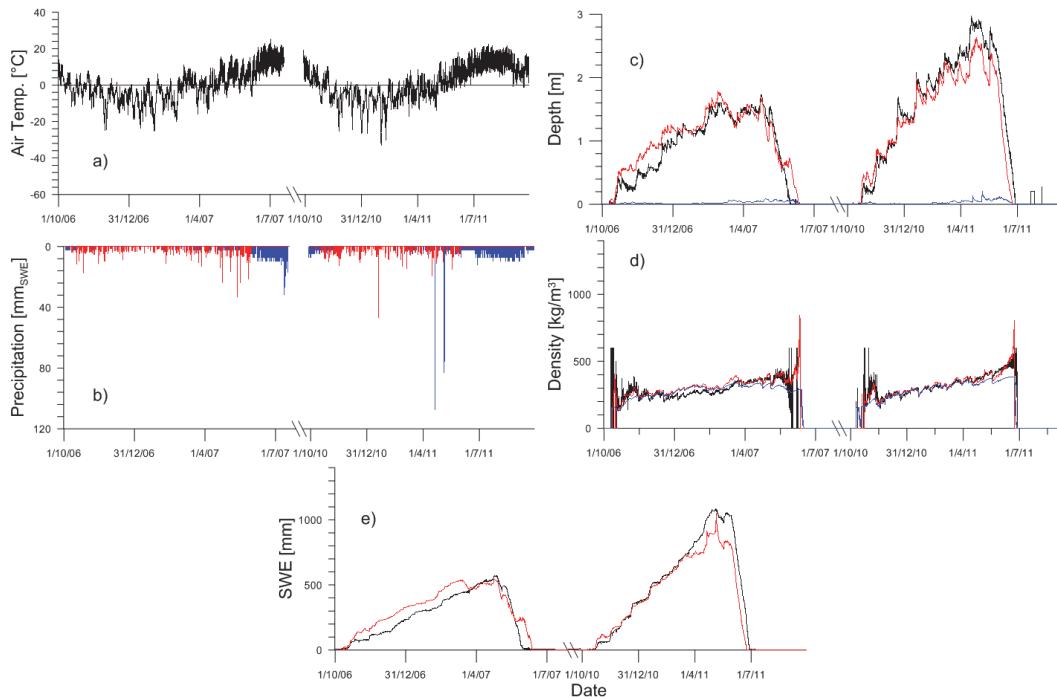


Fig. 3. Meteorological forcings, and dynamics of depth, density and SWE for S2 and two hydrologic years: 2006–2007 (calibration) and 2010–2011 (validation). Panel **(a)** Air temperature, panel **(b)** precipitation, liquid in blue and solid in red, panel **(c)** depth h in red modelled and in black observed, and h_w in blue, panel **(d)** density ρ in red modelled and in black observed and ρ_D in blue, and panel **(e)** SWE in red modelled and in black observed.

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