**The Cryosphere
Discussions**

Interactive comment on "Reformulating the full-Stokes ice sheet model for a more efficient computational solution" *by* J. K. Dukowicz

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Reply to Comment C781 by Jed Brown

The commenter raises several interesting points. The following are replies to his comments in sequence:

Points 1 & 2: A practical discretization would be based on minimizing a discrete version of the action, Eq. (20), with respect to discrete values of the horizontal velocity specified on a suitable grid, and not on the integro-differential equations and boundary conditions given by Eqs. (29)-(31). In other words, the discrete conservation statement applied to the functional given by Eq. (20) is the discrete version of Eq. (8). A natural discretization of this problem is by the use of the Galerkin method. However, the choice

of the approximation space for the Galerkin discretization is beyond the scope of this paper.

The integro-differential equations associated with this reformulation of the Stokes problem have been presented primarily as a matter of interest for comparison with the full-Stokes equations and the Blatter-Pattyn equations. As such, they may suggest other, perhaps more accurate, approximations to the Stokes model.

Point 3: As pointed out by the commenter, conditioning may be difficult. However, this is common for this type of problem and is typically addressed by preconditioning. Fortunately, there exist various simple and inexpensive approximate ice sheet models that may serve as physically-based preconditioners.

Point 4: The commenter is presumably well aware that a practical way to solve this discrete problem is by use of the JFNK method, as pointed out in the paper.

Point 5: It is not the purpose of the present paper to provide a new "continuum" formulation, given the boundary condition difficulties alluded to by the commenter (See Points 1 & 2 above). Rather, the purpose is to provide the basis for a new discrete formulation based on the existence of a minimization principle associated with the discretized action, Eq. (20). The existence of this principle bypasses the regularity considerations mentioned by the commenter. Again, a practical implementation is beyond the scope of this paper.

Reply to Comment C832 by R. C. A. Hindmarsh

The following are replies to points raised by this commenter, in sequence:

Point 1: As the commenter is well aware, not all integro-differential formulations are the same or equivalent. Thus, the present formulation is entirely different from the formulations referred to by the commenter (i.e., Van der Veen and Whillans, 1989; Hindmarsh, 1993, 2006). An important point to note in this connection is that the primary objective of the paper is not to present a competing integro-differential formulation but to outline

a reformulated variational principle, Eq. (20), that may form the basis for improved discrete models.

Point 2: As the commenter points out, stress boundary conditions are complicated in integro-differential formulations, and indeed also in plain differential formulations. Thus a second important point is that implementation of boundary conditions is considerably simplified by use of variational principles, such as the reformulated variational principle of this paper (see also reference DPL2 in the paper).

Point 3: In view to the response to Point 1, above, this comment is not relevant to this paper.

Point 4: As pointed out in the response to Point 2, the reformulated variational principle of this paper amounts to a weak formulation (in Galerkin form) and therefore ameliorates difficulties associated with boundary conditions. Nevertheless, it is true that the reformulated functional contains higher order derivatives than standard, at least in the horizontal direction, and this may impose additional continuity requirements on the basis functions in the approximating space. However, as pointed out earlier, a discussion of the requirements for the approximation space in the Galerkin discretization is beyond the scope of this paper.

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