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The Cryosphere Discussions

Interactive comment on "Reformulating the full-Stokes ice sheet model for a more efficient computational solution" *by* J. K. Dukowicz

R. C. A. Hindmarsh

rcah@bas.ac.uk

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1 Summary

Dukowicz (2011) has shown a derivation of the integro-differential forms of the Stokes equations using a variational method. The objectives of this commentary are to

1. Reiterate point made by Brown (2011) that these forms are known in glaciological literature, and in any case are conceptually similar to streamfunction formulation.
2. Point out the difficulties in implementing stress boundary conditions.

3. Mention numerical implementations which perform as well as primitive variable (velocity-pressure) formulations.
4. Point out that the method exacerbates difficulties with singularities on the basal boundary.

2 Discussion

1. The fact that these integro-differential forms (IDF) can arise from a variational principle is of interest. As Brown (2011) points out, these forms are well known in glaciology (e.g. Van der Veen and Whillans, 1989; Hindmarsh 1993, eqn. 4; Hindmarsh, 2006, eqn. 3.3; numerous other authors), and the integro-differential *term* is known in glaciology as the "T-term" (e.g. Van der Veen and Whillans, 1989). IDFs are in fact generalisations of streamfunction forms. In plane flow, with a linear rheology and uniform viscosity, they are integrated forms of the bi-harmonic equation, where two of the boundary conditions, from the bed and the surface, have been included.
2. In common with streamfunction forms, IDFs are somewhat complicated when incorporating stress boundary conditions. In glaciological applications, one often needs to know the stress at the base, which in IDFs is obtained by integrating the vertical momentum balance from the surface, and similar problems arise from applying stresses at the calving front. These can be quite awkward to program.
3. I included a numerical implementation of an IDF (2d and 3d) in the ISMIP-HOM experiments (Model RHI3, Pattyn and others, 2008), though this is not clear from the write-up, as well as a primitive variables formulation (Model RHI1). These were solved using pseudo-spectral methods, which, owing to their ability to chain discrete differential and integral operators, allow IDFs to be programmed easily.

The models gave very similar results, which allays some of the concerns of Brown (2011). However, the matrix equations were solved directly, so it is not clear whether there exists a practical implementation for large-scale ice-sheet simulators. Stress conditions were not required as ISMIP-HOM experiments have no floating ice and are horizontally periodic.

4. Transitions in the basal boundary condition, from grounded to cavitated/floating usually involve singularities (Fowler, 1988; Gagliardini and others, 2007; Nowicki and Wingham, 2008; Durand and others, 2009). Singularities are best dealt with by solving weak forms of the equations, and with the primitive variables formulation, this requires continuity of velocity and pressure fields across element boundaries. Weak formulations of the fourth order streamfunction and IDF equations require in addition continuity of the first horizontal derivative, which would seem to create an additional set of difficulties.

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