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The Cryosphere Discussions

Interactive comment on "Reformulating the full-Stokes ice sheet model for a more efficient computational solution" *by* J. K. Dukowicz

J. Brown

jedbrown@mcs.anl.gov

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It's well known that this transformation is possible, but it produces an integro-differential equation which has dense coupling in columns (making it much less practical). This paper does not:

1. Discuss stable approximation spaces. Getting rid of the saddle point does not automatically make "natural" discretization spaces for the integro-differential equation stable. The inf-sup compatibility associated with Stokes approximation spaces becomes a compatibility between the discretization space and the integration method.

2. State any discrete conservation statement that can be enforced by a stable discretization. PDEs are never solved strongly. To be useful for numerical simulation, a

continuum formulation must make certain computable (weak) statements have useful properties.

3. Discuss conditioning of the system. This integro-differential equation may have much worse conditioning than the differential operator. A related transformation to eliminate pressure and vertical velocity is used by codes at LANL that solve the hydrostatic primitive equations (e.g. POP and variants, with which the author is very familiar). The ocean equations solved in POP have relatively low diffusion and, in particular, are not steady state (elliptic) problems like the Stokes problem here. Even so, when long time steps are used with implicit time integration methods (producing systems that are similar to the steady state systems solved by Stokes) as done in the implicit POP project, the systems become extremely ill-conditioned. In particular, the conditioning is worse than the usual behavior for stable discretizations, and is worse than the conditioning produced if pressure and vertical velocity are not eliminated. The author should discuss these issues with Wilbert Weijer (LANL) and especially Chris Siefert (SNL) who have been working in implicit methods for POP.

4. Suggest a practical way to solve the integro-differential equation. I have ideas, but it's extremely unclear that any would be competitive with what I can do for Stokes. In any case, if you had a solver for one, a stable discretization for the integro-differential equation, and a compatible discrete way to transform between the two, then you would have a solver for the other. I suspect that the geometric interpretation for the compatible discrete transformation is likely to be problematic on nontrivial meshes. Note that essentially the same transformation can be performed at the discrete level for any stable discretization of the Stokes problem by constructing the Schur complement of pressure and vertical velocity in the Stokes problem, leaving a system operating only in the space of horizontal velocity. In general, this Schur complement is globally dense, further showing that the continuum formulation proposed in the present manuscript will actually compute different discrete solutions. There may be a way to choose a discretization such that the column-dense operator suggested here is spectrally equiv-

alent to the global operator, but that is not discussed here.

5. Provide any numerical implementation. After assuming sufficient regularity for the strong form of the continuum equations to be satisfied (not true in the presence of corners, which are present in all real-world ice sheets on realizable grids), an arbitrary number of equivalent formulations can be written. In practice, very few of these formulations are numerically practical. Without proofs of computable local conservation statements (or other desirable discrete properties) and without a numerical demonstration of practicality and robustness, it is impossible to objectively evaluate a "new" continuum formulation.

Interactive comment on The Cryosphere Discuss., 5, 1749, 2011.