The Cryosphere Discuss., 4, C1773–C1780, 2011
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The Cryosphere
Discussions

# *Interactive comment on* "An energy-conserving model of freezing variably-saturated soil" *by* M. Dall'Amico et al.

**M. Dall'Amico et al.**

matteo.dallamico@ing.unitn.it

Received and published: 28 March 2011

We are grateful for the detailed and constructive comments of Referee #1 that have helped to improve the strongly revised manuscript. Below, we have listed the essence of each issue raised together with a short reply and the revised text or its location where appropriate.

Comment: The derivation of equation (6) is not clear. How do you integrate equation (4) or (8) and obtain the Heaviside function?

Reply: The derivation of equations (4) to (8) was cumbersome and we have now fully revised the text, adding all the necessary passages to ease the comprehension.

Changes: Section 3 (governing equations) has been split in two sections: (i) Mass and



energy in the soil, where the mass and energy conservation equations are explained; (ii) Pressure and temperature under freezing condition, where the equations on the freezing point depression and the freezing pressure are reported. Furthermore, the Appendix A helps to understand the derivation and the boundaries of each term.

Comment: The governing equations (12) and (16) are not the equations that are being solved. It would be great to see the final system of equations with the boundary conditions.

Reply: It is true that it was not clear in the text which one, between the equations of the mass conservation (12) and energy conservation (16), was the equation really solved. Furthermore it was difficult to understand at which equation the test case (analytical solution, section 6 and experimental data, section 7) was referring to.

Changes: Section 4 (The decoupled solution) and 5 (A conservative discretization for the energy equation) have been changed to: (i) The decoupled solution: splitting method, in which we explain better how the coupled system of equations may be solved decoupled (the Appendix B has also been added to ease the comprehension of each term); (ii) The numerical implementation, in which we fully discretize the energy conservation equation according to the Newton method. In this section we have changed the Eq. (35) as we considered that J (the advected heat) is solved during the first step of the splitting method. Furthermore, we have added a subsection called "The boundary conditions" where we explicitly explain the type of boundary conditions that can be used.

Comment: P. 1247, Line 10, "...to our knowledge this equation has never been fully derived from a thermodynamical point of view leaving some doubt on its limitations." An attempt to derive the generalized Clayperon equation from the point of view non-equilibrium thermodynamics was done by M. Mikkola & J. Hartikainen, 2001, Int. J. Numer. Meth. Engng; 52:543–557.

Reply: Thanks for highlighting this article, that was previously unknown to us. Actually the closure required by our system of equation derives from the Clausius-Clapeyron formula (written according to the Gibbs-Duhem formulation), and then the soil water retention curve relationship (e.g. Van Genucthetn) is used to calculate the volumetric liquid water fraction. In fact this is one of the major innovation in our paper, which does not deal with fully saturated soil but with unsaturated soil, thus required further constitutive relations with respect to that reference. The paper by Mikkola and Harikainen (MH) has been cited. MH base their work (and notation) on the previous contribution by Fremond and Nicolas, 1990, which they extend (theoretically) to freezing soil. However, the derivation of the CC equation (formula 28 and 51 of their paper), is not made there, but in Hartikainen and Mikkola, 1997. MH paper refers to completely saturated soil, while our approach tries a parameterization of freezing of unsaturated soil. In any case, the derivation of the Clausius-Clapeyron formulation, and its discussion, is not the focus of this paper, which indeed tries to give new insights in the formulation of the freezing characteristic curve of a non-saturated soil, based on the freezing=drying assumption, and on the numerical implementation.

Changes: For this reason we decided to remove the sentence referring to the Clausius-Clapeyron formulation and therefore no reference to previous work on thermodynamics is needed, but we cited MH elsewhere.

Comment: Equation (12). Reference is missing. Equation can be found e.g. in Harlan, R.L., (1973). "Analysis of Coupled Heat-Fluid Transport in Partially Frozen Soil", Water Resource Res., 9:1314 - 1323.

Reply: Yes, you are right.

Changes: We added the reference.

Comment: Equation (16) Reference is missing. Reply: Yes, you are right. Changes: We added the reference to Fuchs et al. (1978)

Comment: P 1258, Line 10, "It is important to notice that if the Newton method is



solved exactly, energy is preserved. This differentiates the new method from previous work." Run a test and show that mass and energy are actually. Provide references to "the previous work"

Reply: Actually the energy is preserved, according to the tolerance of the Newton method. We have therefore revised the text and inserted the equation that states that the method converges when the norm of the residual is smaller than a tolerance, which dictates the resolution of the method. Furthermore, we have run a simulation in order to verify the energy loss during the test with an analytical solution to the Stefan problem (see new Fig. 6).

Changes: The sentence “this differentiates the new method from the previous work” has been removed.

Comment: P 1259, Line 15, "A considerable improvement was obtained changing to the so-called globally convergent Newton scheme". A reference is missing.

Reply: You are right.

Changes: Added the following reference, which is a comprehensive text on Newton’s methods. Kelley, C. (2003), Solving nonlinear equations with Newton’s method, Society for Industrial Mathematics.

Comment: Derivation of well-known analytical solution in Appendix not needed, just provide reference.

Reply: Actually it is true that also Nakano and Brown (1971) and Carlaw and Jaeger (1959) show the solution, but just for the freezing case. We thought that, reporting the full equation with the hypothesis, both for the freezing and for the thawing case, helps the reader to follow the text and, in case, repeat the results.

Comment: Figures 3 and 4: plot solutions by lines with symbols to enhance b/w printing.

Reply: Figure changed accordingly.

Interactive comment on The Cryosphere Discuss., 4, 1243, 2010.



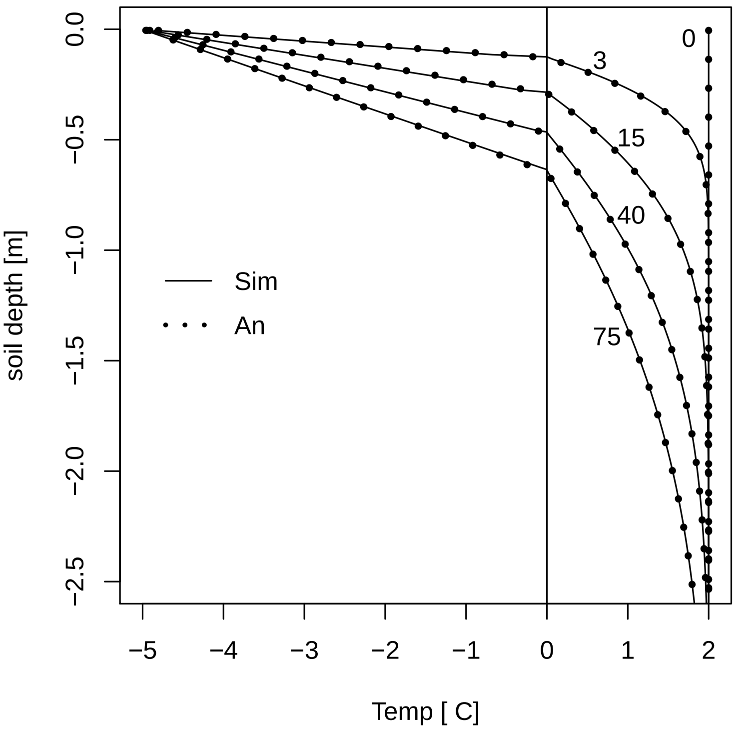


**Fig. 1.** enhanced b/w printing capabilities of Fig. 3 (Comparison between the simulated numerical and the an- alytical solution. Soil profile temperature at different days. Grid size=10 mm, N=500 cells)

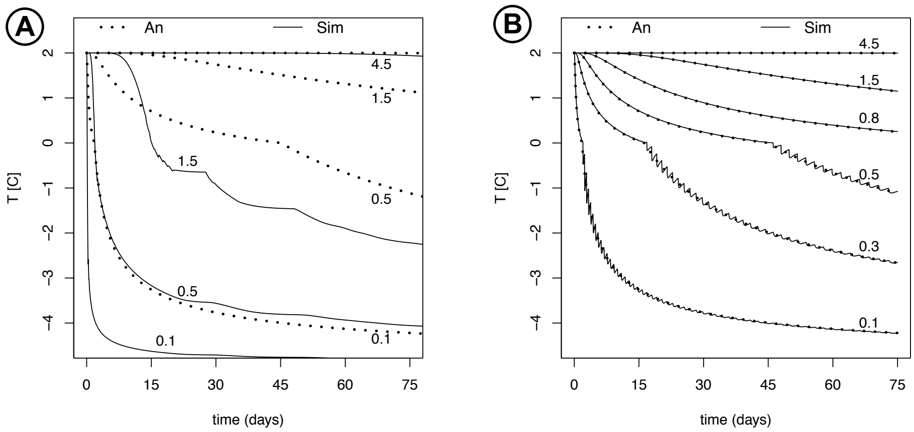


**Fig. 2.** enhanced b/w printing capabilities of Fig. 3 (bis). X-axis represents the time. Left: comparison against non-globally convergent Newton scheme. Right: globally convergent scheme



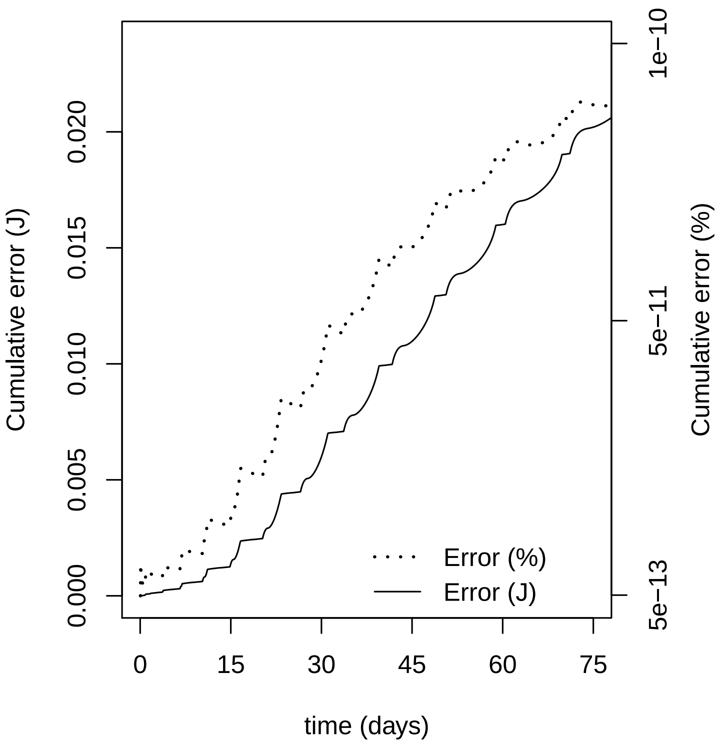


**Fig. 3.** Cumulative error associated with the the glob. conv. Newton scheme. Plain: cumulative error (J), dotted: cumulative error (%) as the ratio between the error and the total energy of the soil