**The Cryosphere
Discussions**

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Comment

Interactive comment on "Estimation of thermal properties of saturated soils using in-situ temperature measurements" *by* D. J. Nicolsky et al.

D. J. Nicolsky et al.

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We are thankful to this reviewer for close reading, and thoughtful suggestions to improve quality of our manuscript. We revised the manuscript according to all issues raised by the referee in the following comments (marked by bold). In a new version of the manuscript, suggestions to specific comments are marked with bold font style.

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Response to the general comments:

Despite the interesting content of the paper, the reader is not sufficiently supported in understanding and distinguishing different model approaches and parameter estimations. Minor revisions in description and structure are needed to make the presented work accessible for a larger community. Compared to title and abstract, the development of the new algorithm takes a bigger weight in the main part and the conclusions. If this algorithm is not yet presented in other publications, its visibility here should increase. To consolidate the results of the final model evaluation, the described difference between measurement and simulation could be compared with the accuracy of preexisting models for the same situation.

We adopted reviewer's suggestion and modified the title, please see below.
This paper was not intended to be a review of different model approaches and parameter estimations. However, in the modified introduction, we provide a brief review of several well known methods with citations. An interested reader could follow these citations and understand those methods in depth.

Response to the specific comments:

1. **Title: In consideration of the comment above, I suggest to integrate the aspect of model development also to the title (e.g. "estimation of thermal properties of saturated soils minimizing differences between a new heat conduction model and in-situ temperature measurements").**

   We followed the suggestion and integrate the term "optimization problem" into

manuscript's title: "Using in-situ temperature measurements to estimate saturated soil thermal properties by solving a sequence of optimization problems."

2. **Section 2: The definitions of some parameters repeat and are sometimes inaccurate (see technical corrections). Formula (3) could be supported with a graph (or refer to figure 2 with an additional b value)**

   We modified the definition of ϕ according to reviewer's suggestions in order to remove inaccuracy. Additionally we refer to Figure 2 (in the previous version of the manuscript), that now incorporates the value of b for which the graph was computed.

3. **Section 3: Subtitles would help for internal structure of the paragraph as the change from review to description of approach is not very sharp: e.g. 3.1 geophysical techniques; 3.2 inverse modeling techniques; 3.3 inversion by cost function minimization; 3.4 example**

   Section 4: From p. 226, l. 11 the description of the developed algorithm starts but it's still under the subtitle: ";4.1 A review of numerical methods";. A new subtitle (4.2) would emphasize this change.

   We agree that the manuscript could be better organized. We reorganized the presentation in several places to ease following its logic. For example, we redistributed material in Section "Review of existing methods" that contained parts which typically belong into the introduction and the inverse problem. Section "Solution of the heat equation... " now directly follow Section "Modeling of soil freezing and thawing". Section "Selection of an initial approximation" follows description of the optimization process, because the importance of the initial values for a gradient-type method. Extra special care was taken in Sections 5 and 7.2 of the original manuscript in order to make them more clear to understand.

4. **p. 226, l. 14:The term between formula (13) and (14) would better be ex-**

pressed as formula with the mentioned left hand side. In the finite element formulation the latent heat term reappears (16) and does not follow the earlier introduced C_{app} expression.

We revised the part of manuscript between formulae (13) and (14) according to suggested improvements. The modified version has the latent heat of fusion and is easier to understand.

5. **I don't understand the system used for the indices (i) and (j) in this paragraph (as they both refer to soil horizons and not to the matrix dimensions?). How exactly the porosity η; is estimated (p. 234, l. 3) without influencing the later approximation of thermal conductivity? On p. 235, l. 6 it is mentioned that C_t was approximated before but it is nowhere mentioned. Refer to formula (6) and the earlier approximation of C_f and η**

   - Both indices i and j refer to different subsets of parameters in C. We removed ambiguity by substituting k for i in the description of the proposed algorithm in subsection "General methodology".
   - In the model the thermal conductivity λ is parameterized by

   $$\lambda = \lambda_f^{1-\phi} \lambda_t^{\phi}, \qquad (1)$$

   where ϕ is the liquid pore water fraction, λ_f and λ_t are thermal conductivities of the "completely" frozen and thawed ground, respectively.

   First we find thermal conductivity λ_f for the "completely" frozen ground during the "winter" interval. At this time interval phase change processes are negligibly small, and hence temperature does not depend on variations in the soil porosity η. Therefore, the found value of λ_f that is independent on the soil porosity.

   At the second step, we consider the "summer and fall" interval. For the sake of simplicity we assume that pores are always fully saturated, and all liquid

water in them can freeze. During "summer and fall" interval the temperature dynamics "primarily" depends on the porosity η and thermal conductivity λ. However, since λ_t and λ_f are related by

$$\lambda_t = \lambda_f \left[\frac{\lambda_l}{\lambda_i}\right]^{\eta}, \tag{2}$$

the value of λ in (1) can be computed if the porosity η is given (the thermal conductivity of liquid water λ_l and ice λ_i are well known). Hence during the "summer and fall" interval, the temperature "primarily" depends on the soil porosity η if model assumptions (1-2) hold. Therefore, we can find the soil porosity η.

- During active soil thawing a contribution of the heat capacity C into the apparent heat capacity C_{app} is negligibly small comparing to the contribution of the latent heat term $Ld\theta_l/dT$. Thus we approximate $\{C_t^{(i)}\}$ using published data by analyzing the soil texture and moisture content. The corresponding correction is made in text in the paragraph describing Δ_2 interval, subsection "Subproblems".

6. **Section 7: How do model errors influence the results of this approximation? Are they negligible? The difference between measurement levels and layers of the estimation should be stated clearer to avoid confusion (p. 236, l. 17ff).**

Despite that the presented heat equation with phase change is a basic approximation to the energy conservation principle, this equation is typically used in many engineering and science applications to compute the ground temperature dynamics. We stated limitations of applicability of the presented model in the begging of Section "Modeling of soil freezing and thawing". If these limitations are not applicable then it is hard to estimate model errors.

Other source of errors in the model is related to parametrization of coefficients in the heat equation. In this work, we assumed that the ground material has several horizons, within which the soil properties are constants. In nature, it is very often that the soil properties change smoothly with depth and do not have sharp boundaries. We investigated errors in approximation of the temperature dynamics (the synthetic data) computed with "smooth" soil properties by the temperature dynamics computed with soil properties "constants" in each horizon. In our numerical experiments we find soil properties within each horizon by using this synthetic data. The recovered soil properties are averaged values of their "smoothly" changing counterparts within each horizon. Another result is that the temperature dynamics computed for the recovered properties follows within of 5% the synthetic data. These work is prepared for publication and will be submitted soon to the Journal of Cold Regions Science and Technology.

7. **Section 8: p. 244, l. 11ff: But this depends on parameter b which is estimated in this model, doesn't it?**

While describing soil the freezing/thawing model, we considered parameterization of the unfrozen liquid water content θ_w:

$$\theta_l(T, x) = \eta(x)\phi(T, x), \quad \phi = \begin{cases} 1, & T \geq T_* \\ |T_*|^b |T|^{-b}, & T < T_* \end{cases}, \tag{3}$$

where $\phi = \phi(T, x)$ is the liquid pore water fraction. Note that if the temperature is sufficiently cold then almost all liquid water in soil pores is frozen. However, in some special cases only a fraction, $\alpha \in [0, 1]$, of water trapped in pores can change its phase. Therefore, for example, for liquid water and ice volume fraction we have

$$\theta_w = \alpha\phi\eta + (1 - \alpha)\eta, \qquad \theta_i = \eta - \theta_w = \alpha\eta(1 - \phi). \tag{4}$$

If $\alpha = 0$ then all water can change its phase, whereas if $\alpha = 0.5$ then only 50% of liquid water in pores can freeze. Substituting expressions for θ_w, θ_i, and $\theta_s = 1 - \eta$

into the formula for the thermal conductivity

$$\lambda = \lambda_s^{\theta_s} \lambda_i^{\theta_i} \lambda_w^{\theta_w},$$

we obtain the relation

$$\lambda = \tilde{\lambda}_f^{1-\phi} \tilde{\lambda}_t^{\phi}$$

which has exactly the same form as (1) (a similar expression can be obtained for the heat capacities). However, the quantities $\tilde{\lambda}_t$ and $\tilde{\lambda}_f$ now satisfy

$$\tilde{\lambda}_t = \tilde{\lambda}_f \left(\frac{\lambda_l}{\lambda_i} \right)^{\alpha\eta}. \tag{5}$$

We continue by considering the latent heat term in the heat equation. We note that

$$L\frac{d\theta_w}{dt} = L\alpha\eta\frac{d\phi}{dt}. \tag{6}$$

We emphasis that in (5), in (6) and in other places, *the soil porosity η occurs only in combination with the coefficient α, i.e. in the form $\alpha\eta$.* Therefore, two physical models (in which all liquid water or its some fraction freezes) can produce exactly the same temperature dynamics. Thus, if we apply the proposed algorithm under assumption that all liquid water freezes, then we compute η, under the other assumption we calculate $\alpha\eta$.

8. **Section 9: Better don't stat the algorithm development on the final position of the conclusions (p. 245, l. 5ff) as long as it is not one of the major topics of this paper (see also comment above).**

The major topic of this paper is development of the algorithm to find an initial approximation to the soil thermal properties, soil porosity and unfrozen water content. The development of a new FEM technique is not considered as a main focus, and hence it was stated in the last paragraph of conclusions.

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9. **Table 3: use the term initial approximation in the table description**

   Corrected

10. **Table 4: text not finished**

    Corrected

11. **Figure 3: mark area of right graph within left graph**

    Marked

12. **Figure 4: legend and description are repeating**

    Corrected

13. **Figure 8 to 11: Are all this figures necessary to follow the procedure of the initial approximation?**

    Many of the details in Figures 10, 11 and in the associated with them text are probably not important to the reader, while the idea of constructing subproblems is. Hence, we revised the text in Section 7.2 and consequently removed Figures 10 and 11.

Responses to technical corrections:

1. **p. 217, l. 4:** θ_l **is volumetric liquid water content For formula (3) one should emphasize that T is in** $^\circ C$

   Corrected

2. **p. 218, l. 1:** ϕ **is mentioned as soil saturation. But ice should not be included in** ϕ**;. Better: liquid pore water fraction**

   Corrected

3. **p. 221, formula (9): m is not introduced, shouldn't it be n?**

   p. 222, formula (10): same as (9), also in text
   We explicitly defined m. It is not necessary that $m=n$.

4. **p. 228, l. 5: reference for Picard iteration**
   Inserted

5. **p. 234, l. 1 (after (26)):** η **is porosity, not water content**
   Corrected

6. **p. 242, l. 17: global minimization instead of initial approximation**
   Corrected

7. **p. 243, l. 3: parameter estimation instead of initial approximation**
   Corrected

8. **p. 244, l. 13: clarify figure 2, not formula (2).**
   Corrected

Thanks you,
D. Nicolsky and V. Romanovsky

Interactive comment on The Cryosphere Discuss., 1, 213, 2007.

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