Appendix: Calculation of the Compensation Terms

for the Paper *"Manufactured Solutions and the Verification of Three-dimensional Stokes Ice-Sheet Models"*

The compensation terms

$$
\begin{aligned}
F_1 &= \frac{\partial(2\mu\frac{\partial u}{\partial x} - p)}{\partial x} + \frac{\partial(\mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}))}{\partial y} + \frac{\partial(\mu(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}))}{\partial z}, && \text{(A1)}\\
F_2 &= \frac{\partial(\mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}))}{\partial x} + \frac{\partial(2\mu\frac{\partial v}{\partial y} - p)}{\partial y} + \frac{\partial(\mu(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}))}{\partial z}, && \text{(A2)}\\
F_3 &= \frac{\partial(\mu(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}))}{\partial x} + \frac{\partial(\mu(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}))}{\partial y} + \frac{\partial(2\mu\frac{\partial w}{\partial z} - p)}{\partial z} - \rho g && \text{(A3)}
\end{aligned}
$$

need to be added to the right-hand sides of the three components of the momentum equation (1)-(3), respectively. and the following terms, denoted as T_1, T_2, T_3, :

$$T_1 = \frac{1}{r_s}\left[-\frac{\partial s}{\partial x}\left(2\mu\frac{\partial u}{\partial x} - p\right) - \frac{\partial s}{\partial y}\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right], \quad \text{(A4)}$$

$$T_2 = \frac{1}{r_s}\left[-\frac{\partial s}{\partial x}\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{\partial s}{\partial y}\left(2\mu\frac{\partial v}{\partial y} - p\right) + \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right], \quad \text{(A5)}$$

$$T_3 = \frac{1}{r_s}\left[-\frac{\partial s}{\partial x}\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) - \frac{\partial s}{\partial y}\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) + \left(2\mu\frac{\partial w}{\partial z} - p\right)\right], \quad \text{(A6)}$$

where $r_s = \sqrt{1 + (\frac{\partial s}{\partial x})^2 + (\frac{\partial s}{\partial y})^2}$, need to be added into the right-hand sides of the three components of the top surface boundary equations (9)-(11), respectively.

Now let us present explicit formulas for calculating these compensation terms for the manufactured solution under the specific geometry derived in Section 3.2, i.e., the velocity solution (54)-(56) and the pressure solution (44) with the ice-sheet geometry having the surface elevation (48) and the bedrock elevation (46) on the domain, and a specific accumulation rate (50). In addition, we also fix $\lambda_2 = 4$ for the purpose of simplifying the formulas of the compensation terms except $\gamma_1 = 0$, $\lambda_1 = 4$, $c_{b1} = 0$, $c_{b2} = 0$ as chosen for the specific sample solution in Section 3.2. Thus the remaining free parameters are only c_1, c_2 and c_t.

Computation of the compensation terms in numerical codes requires the value of u, v, w, p, μ and their derivatives. Furthermore, u, v, w, p, μ all can be expressed as the functions of $s(x, y, t)$, $b(x, y)$, $h(x, y, t) = s - b$, $d(x, y, z, t) = \frac{s-z}{s-b}$, and the special term $q(x, y, t)$ (resulting from integration), and some of their derivatives.

First, let us write down explicit expressions for s, b, d, q, and some needed derivatives in the following. Note that $h = s - b$, thus all derivatives of h are directly the differences of corresponding derivatives of s and b and we will skip those. Recalled that $\xi(t) = 1 - \mathrm{e}^{-c_t\,t}$, then

$$s = -x\tan(\alpha) + \frac{1}{2}Z\sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)\xi(t), \tag{A7}$$

$$s_x = -\tan(\alpha) + Z\cos\left(\frac{2\pi x}{L}\right)\pi\sin\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L}, \tag{A8}$$

$$s_y = Z\sin\left(\frac{2\pi x}{L}\right)\pi\cos\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L}, \tag{A9}$$

$$s_{xx} = -2Z\sin\left(\frac{2\pi x}{L}\right)\pi^2\sin\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L^2}, \tag{A10}$$

$$s_{xy} = 2Z\cos\left(\frac{2\pi x}{L}\right)\pi^2\cos\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L^2}, \tag{A11}$$

$$s_{yy} = -2Z\sin\left(\frac{2\pi x}{L}\right)\pi^2\sin\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L^2}, \tag{A12}$$

$$s_{xxx} = -4Z\cos\left(\frac{2\pi x}{L}\right)\pi^3\sin\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L^3}, \tag{A13}$$

$$s_{xxy} = -4Z\sin\left(\frac{2\pi x}{L}\right)\pi^3\cos\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L^3}, \tag{A14}$$

$$s_{xyy} = -4Z\cos\left(\frac{2\pi x}{L}\right)\pi^3\sin\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L^3}, \tag{A15}$$

$$s_{yyy} = -4Z\sin\left(\frac{2\pi x}{L}\right)\pi^3\cos\left(\frac{2\pi y}{L}\right)\xi(t)\frac{1}{L^3}, \tag{A16}$$

$$b = -x\tan(\alpha) + \frac{1}{2}Z\sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right) - Z, \tag{A17}$$

$$b_x = -\tan(\alpha) + Z\cos\left(\frac{2\pi x}{L}\right)\pi\sin\left(\frac{2\pi y}{L}\right)\frac{1}{L}, \tag{A18}$$

$$b_y = Z\sin\left(\frac{2\pi x}{L}\right)\pi\cos\left(\frac{2\pi y}{L}\right)\frac{1}{L}, \tag{A19}$$

$$b_{xx} = -2Z\sin\left(\frac{2\pi x}{L}\right)\pi^2\sin\left(\frac{2\pi y}{L}\right)\frac{1}{L^2}, \tag{A20}$$

$$b_{xy} = 2Z\cos\left(\frac{2\pi x}{L}\right)\pi^2\cos\left(\frac{2\pi y}{L}\right)\frac{1}{L^2}, \tag{A21}$$

$$b_{yy} = -2Z\sin\left(\frac{2\pi x}{L}\right)\pi^2\sin\left(\frac{2\pi y}{L}\right)\frac{1}{L^2}, \tag{A22}$$

$$b_{xxx} = -4Z\cos\left(\frac{2\pi x}{L}\right)\pi^3\sin\left(\frac{2\pi y}{L}\right)\frac{1}{L^3}, \tag{A23}$$

$$b_{xxy} = -4Z\sin\left(\frac{2\pi x}{L}\right)\pi^3\cos\left(\frac{2\pi y}{L}\right)\frac{1}{L^3}, \tag{A24}$$

$$b_{xyy} = -4Z\cos\left(\frac{2\pi x}{L}\right)\pi^3\sin\left(\frac{2\pi y}{L}\right)\frac{1}{L^3}, \tag{A25}$$

$$b_{yyy} = -4Z\sin\left(\frac{2\pi x}{L}\right)\pi^3\cos\left(\frac{2\pi y}{L}\right)\frac{1}{L^3}, \tag{A26}$$

$$q = \frac{1}{2} c_1 \, Z \cos\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi y}{L}\right) \mathrm{e}^{-c_t \, t}, \tag{A27}$$

$$q_x = -c_1 \, Z \sin\left(\frac{2\pi x}{L}\right) \pi \cos\left(\frac{2\pi y}{L}\right) \mathrm{e}^{-c_t \, t} \frac{1}{L}, \tag{A28}$$

$$q_y = -c_1 \, Z \cos\left(\frac{2\pi x}{L}\right) \pi \sin\left(\frac{2\pi y}{L}\right) \mathrm{e}^{-c_t \, t} \frac{1}{L}, \tag{A29}$$

$$q_{xx} = -2c_1 \, Z \cos\left(\frac{2\pi x}{L}\right) \pi^2 \cos\left(\frac{2\pi y}{L}\right) \mathrm{e}^{-c_t \, t} \frac{1}{L^2}, \tag{A30}$$

$$q_{xy} = 2c_1 \, Z \sin\left(\frac{2\pi x}{L}\right) \pi^2 \sin\left(\frac{2\pi y}{L}\right) \mathrm{e}^{-c_t \, t} \frac{1}{L^2}, \tag{A31}$$

$$q_{yy} = -2c_1 \, Z \cos\left(\frac{2\pi x}{L}\right) \pi^2 \cos\left(\frac{2\pi y}{L}\right) \mathrm{e}^{-c_t \, t} \frac{1}{L^2}, \tag{A32}$$

and

$$d = \frac{s-z}{h}, \qquad d_x = \frac{s_x}{h} - \frac{(s-z)\, h_x}{h^2}, \qquad d_y = \frac{s_y}{h} - \frac{(s-z)\, h_y}{h^2}, \qquad d_z = -\frac{1}{h}, \tag{A33}$$

$$d_{xx} = \frac{s_{xx}}{h} - 2\frac{s_x h_x}{h^2} + 2\frac{(s-z)\, {h_x}^2}{h^3} - \frac{(s-z)\, h_{xx}}{h^2}, \tag{A34}$$

$$d_{xy} = \frac{s_{xy}}{h} - \frac{s_x h_y}{h^2} - \frac{s_y h_x}{h^2} + 2\frac{(s-z)\, h_x h_y}{h^3} - \frac{(s-z)\, h_{xy}}{h^2}, \qquad d_{xz} = \frac{h_x}{h^2}, \tag{A35}$$

$$d_{yy} = \frac{s_{yy}}{h} - 2\frac{s_y h_y}{h^2} + 2\frac{(s-z)\, {h_y}^2}{h^3} - \frac{(s-z)\, h_{yy}}{h^2}, \qquad d_{yz} = \frac{h_y}{h^2}, \qquad d_{zz} = 0, \tag{A36}$$

$$\begin{aligned} d_{xxx} = {} & \frac{s_{xxx}}{h} - 3\frac{s_{xx} h_x}{h^2} + 6\frac{s_x {h_x}^2}{h^3} - 3\frac{s_x h_{xx}}{h^2} - 6\frac{(s-z)\, {h_x}^3}{h^4} + 6\frac{(s-z)\, h_x h_{xx}}{h^3} \\ & - \frac{(s-z)\, h_{xxx}}{h^2}, \end{aligned} \tag{A37}$$

$$\begin{aligned} d_{xxy} = {} & \frac{s_{xxy}}{h} - \frac{s_{xx} h_y}{h^2} - 2\frac{s_{xy} h_x}{h^2} + 4\frac{s_x h_x h_y}{h^3} - 2\frac{s_x h_{xy}}{h^2} + 2\frac{s_y {h_x}^2}{h^3} - 6\frac{(s-z)\, {h_x}^2 h_y}{h^4} \\ & + 4\frac{(s-z)\, h_x h_{xy}}{h^3} - \frac{s_y h_{xx}}{h^2} + 2\frac{(s-z)\, h_{xx} h_y}{h^3} - \frac{(s-z)\, h_{xxy}}{h^2}, \end{aligned} \tag{A38}$$

$$d_{xxz} = -2\frac{{h_x}^2}{h^3} + \frac{h_{xx}}{h^2}, \tag{A39}$$

$$\begin{aligned} d_{xyy} = {} & \frac{s_{xyy}}{h} - 2\frac{s_{xy} h_y}{h^2} + 2\frac{s_x {h_y}^2}{h^3} - \frac{s_x h_{yy}}{h^2} - \frac{s_{yy} h_x}{h^2} + 4\frac{s_y h_x h_y}{h^3} - 2\frac{s_y h_{xy}}{h^2} \\ & - 6\frac{(s-z)\, h_x {h_y}^2}{h^4} + 4\frac{(s-z)\, h_{xy} h_y}{h^3} + 2\frac{(s-z)\, h_x h_{yy}}{h^3} - \frac{(s-z)\, h_{xyy}}{h^2}, \end{aligned} \tag{A40}$$

$$d_{xyz} = -2\frac{h_x h_y}{h^3} + \frac{h_{xy}}{h^2}, \qquad d_{xzz} = 0, \tag{A41}$$

$$d_{yyy} = \frac{s_{yyy}}{h} - 3\frac{s_{yy} h_y}{h^2} + 6\frac{s_y {h_y}^2}{h^3} - 3\frac{s_y h_{yy}}{h^2} - 6\frac{(s-z)\, {h_y}^3}{h^4} + 6\frac{(s-z)\, h_y h_{yy}}{h^3} - \frac{(s-z)\, h_{yyy}}{h^2}, \tag{A42}$$

$$d_{yyz} = -2\frac{{h_y}^2}{h^3} + \frac{h_{yy}}{h^2}, \qquad d_{yzz} = 0, \qquad d_{zzz} = 0. \tag{A43}$$

Then, the velocity components u, v, w can be computed by using

$$u = c_1 \left(1 - d^4\right), \tag{A44}$$

$$v = \frac{c_2}{h}(1 - d^4) - \frac{q}{h}(1 - d^4), \tag{A45}$$

$$w = uhd_x + vhd_y, \tag{A46}$$

and their derivatives can be expressed as

$$u_x = -4c_1\, d^3 d_x, \qquad u_y = -4c_1\, d^3 d_y, \qquad u_z = -4c_1\, d^3 d_z, \tag{A47}$$
$$u_{xx} = -12c_1\, d^2 {d_x}^2 - 4c_1\, d^3 d_{xx}, \qquad u_{xy} = -12c_1\, d^2 d_x d_y - 4c_1\, d^3 d_{xy}, \tag{A48}$$
$$u_{xz} = -12c_1\, d^2 d_x d_z - 4c_1\, d^3 d_{xz}, \qquad u_{yy} = -12c_1\, d^2 {d_y}^2 - 4c_1\, d^3 d_{yy}, \tag{A49}$$
$$u_{yz} = -12c_1\, d^2 d_y d_z - 4c_1\, d^3 d_{yz}, \qquad u_{zz} = -12c_1\, d^2 {d_z}^2 - 4c_1\, d^3 d_{zz}, \tag{A50}$$

$$v_x = \frac{-4c_2\, d^3 d_x - q_x + q_x d^4 + 4qd^3 d_x}{h} - \frac{\left(c_2 - c_2\, d^4 - q + qd^4\right) h_x}{h^2}, \tag{A51}$$
$$v_y = \frac{-4c_2\, d^3 d_y - q_y + q_y d^4 + 4qd^3 d_y}{h} - \frac{\left(c_2 - c_2\, d^4 - q + qd^4\right) h_y}{h^2}, \tag{A52}$$
$$v_z = \frac{-4c_2\, d^3 d_z + 4qd^3 d_z}{h}, \tag{A53}$$
$$\begin{aligned} v_{xx} = &\frac{1}{h}\Big[- 12c_2\, d^2 {d_x}^2 - 4c_2\, d^3 d_{xx} - q_{xx} + q_{xx} d^4 + 8q_x d^3 d_x + 12qd^2 {d_x}^2 + 4qd^3 d_{xx}\Big] \\ &+\frac{1}{h^2}\Big[- 2\left(-4c_2\, d^3 d_x - q_x + q_x d^4 + 4qd^3 d_x\right) h_x - \left(c_2 - c_2\, d^4 - q + qd^4\right) h_{xx}\Big] \\ &+\frac{2}{h^3}\Big[\left(c_2 - c_2\, d^4 - q + qd^4\right) {h_x}^2\Big], \end{aligned} \tag{A54}$$
$$\begin{aligned} v_{xy} = &\frac{1}{h}\Big[- 12c_2\, d^2 d_x d_y - 4c_2\, d^3 d_{xy} - q_{xy} + q_{xy} d^4 + 4q_x d^3 d_y + 4q_y d^3 d_x + 12qd^2 d_x d_y + 4qd^3 d_{xy}\Big] \\ &+\frac{1}{h^2}\Big[- \left(c_2 - c_2\, d^4 + -q + qd^4\right) h_{xy} - \left(-4c_2\, d^3 d_x - q_x + q_x d^4 + 4qd^3 d_x\right) h_y \\ &\qquad - \left(-4c_2\, d^3 d_y - q_y + q_y d^4 + 4qd^3 d_y\right) h_x\Big] \\ &+\frac{2}{h^3}\Big[\left(c_2 - c_2\, d^4 - q + qd^4\right) h_x h_y\Big], \end{aligned} \tag{A55}$$
$$\begin{aligned} v_{xz} = &\frac{1}{h}\Big[- 12c_2\, d^2 d_x d_z - 4c_2\, d^3 d_{xz} + 4q_x d^3 d_z + 12qd^2 d_x d_z + 4qd^3 d_{xz}\Big] \\ &-\frac{1}{h^2}\Big[\left(-4c_2\, d^3 d_z + 4qd^3 d_z\right) h_x\Big], \end{aligned} \tag{A56}$$
$$\begin{aligned} v_{yy} = &\frac{1}{h}\Big[- 12c_2\, d^2 {d_y}^2 - 4c_2\, d^3 d_{yy} - q_{yy} + q_{yy} d^4 + 8q_y d^3 d_y + 12qd^2 {d_y}^2 + 4qd^3 d_{yy}\Big] \\ &+\frac{1}{h^2}\Big[- 2\left(-4c_2\, d^3 d_y - q_y + q_y d^4 + 4qd^3 d_y\right) h_y - \left(c_2 - c_2\, d^4 - q + qd^4\right) h_{yy}\Big] \\ &+\frac{2}{h^3}\Big[\left(c_2 - c_2\, d^4 - q + qd^4\right) {h_y}^2\Big], \end{aligned} \tag{A57}$$
$$\begin{aligned} v_{yz} = &\frac{1}{h}\Big[-12c_2\, d^2 d_y d_z - 4c_2\, d^3 d_{yz} + 4q_y d^3 d_z + 12qd^2 d_y d_z + 4qd^3 d_{yz}\Big] \\ &-\frac{1}{h^2}\Big[\left(-4c_2\, d^3 d_z + 4qd^3 d_z\right) h_y\Big], \end{aligned} \tag{A58}$$
$$v_{zz} = \frac{1}{h}\Big[-12c_2\, d^2 {d_z}^2 - 4c_2\, d^3 d_{zz} + 12qd^2 {d_z}^2 + 4qd^3 d_{zz}\Big], \tag{A59}$$

and

$$w_x = u_x h d_x + u h_x d_x + u h d_{xx} + v_x h d_y + v h_x d_y + v h d_{xy}, \tag{A60}$$
$$w_y = u_y h d_x + u h_y d_x + u h d_{xy} + v_y h d_y + v h_y d_y + v h d_{yy}, \tag{A61}$$
$$w_z = u_z h d_x + u h d_{xz} + v_z h d_y + v h d_{yz}, \tag{A62}$$

$$
\begin{aligned}
w_{xx} = & u_{xx}hd_x + 2u_xh_xd_x + 2u_xhd_{xx} + uh_{xx}d_x + 2uh_xd_{xx} + uhd_{xxx} \\
& +v_{xx}hd_y + 2v_xh_xd_y + 2v_xhd_{xy} + vh_{xx}d_y + 2vh_xd_{xy} + vhd_{xxy}, \quad (A63) \\
w_{xy} = & u_{xy}hd_x + u_xh_yd_x + u_xhd_{xy} + u_yh_xd_x + uh_{xy}d_x + uh_xd_{xy} + u_yhd_{xx} + uh_yd_{xx} + uhd_{xxy} \\
& +v_{xy}hd_y + v_xh_yd_y + v_xhd_{yy} + v_yh_xd_y + vh_{xy}d_y + vh_xd_{yy} + v_yhd_{xy} + vh_yd_{xy} + vhd_{xyy}, \quad (A64) \\
w_{xz} = & u_{xz}hd_x + u_xhd_{xz} + u_zh_xd_x + uh_xd_{xz} + u_zhd_{xx} + uhd_{xxz} \\
& +v_{xz}hd_y + v_xhd_{yz} + v_zh_xd_y + vh_xd_{yz} + v_zhd_{xy} + vhd_{xyz}, \quad (A65) \\
w_{yy} = & u_{yy}hd_x + 2u_yh_yd_x + 2u_yhd_{xy} + uh_{yy}d_x + 2uh_yd_{xy} + uhd_{xyy} \\
& +v_{yy}hd_y + 2v_yh_yd_y + 2v_yhd_{yy} + vh_{yy}d_y + 2vh_yd_{yy} + vhd_{yyy}, \quad (A66) \\
w_{yz} = & u_{yz}hd_x + u_yhd_{xz} + u_zh_yd_x + uh_yd_{xz} + u_zhd_{xy} + uhd_{xyz} \\
& +v_{yz}hd_y + v_yhd_{yz} + v_zh_yd_y + vh_yd_{yz} + v_zhd_{yy} + vhd_{yyz}, \quad (A67) \\
w_{zz} = & u_{zz}hd_x + 2u_zhd_{xz} + uhd_{xzz} + v_{zz}hd_y + 2v_zhd_{yz} + vhd_{yzz}. \quad (A68)
\end{aligned}
$$

The viscosity μ and its derivatives are given by

$$
\begin{aligned}
\mu = & \frac{1}{2}A^{-1/3}\Big[\frac{1}{4}\left(u_y+v_x\right)^2 + \frac{1}{4}\left(u_z+w_x\right)^2 + \frac{1}{4}\left(v_z+w_y\right)^2 - u_xv_y - u_xw_z - v_yw_z\Big]^{-1/3}, \quad (A69) \\
\mu_x = & -\frac{8}{3}A\mu^4\Big[\frac{1}{2}\left(u_y+v_x\right)\left(u_{xy}+v_{xx}\right) + \frac{1}{2}\left(u_z+w_x\right)\left(u_{xz}+w_{xx}\right) \\
& +\frac{1}{2}\left(v_z+w_y\right)\left(v_{xz}+w_{xy}\right) - u_{xx}v_y - u_xv_{xy} - u_{xx}w_z - u_xw_{xz} - v_{xy}w_z - v_yw_{xz}\Big], \quad (A70) \\
\mu_y = & -\frac{8}{3}A\mu^4\Big[\frac{1}{2}\left(u_y+v_x\right)\left(u_{yy}+v_{xy}\right) + \frac{1}{2}\left(u_z+w_x\right)\left(u_{yz}+w_{xy}\right) \\
& +\frac{1}{2}\left(v_z+w_y\right)\left(v_{yz}+w_{yy}\right) - u_{xy}v_y - u_xv_{yy} - u_{xy}w_z - u_xw_{yz} - v_{yy}w_z - v_yw_{yz}\Big], \quad (A71) \\
\mu_z = & -\frac{8}{3}A\mu^4\Big[\frac{1}{2}\left(u_y+v_x\right)\left(u_{yz}+v_{xz}\right) + \frac{1}{2}\left(u_z+w_x\right)\left(u_{zz}+w_{xz}\right) \\
& +\frac{1}{2}\left(v_z+w_y\right)\left(v_{zz}+w_{yz}\right) - u_{xz}v_y - u_xv_{yz} - u_{xz}w_z - u_xw_{zz} - v_{yz}w_z - v_yw_{zz}\Big]. \quad (A72)
\end{aligned}
$$

The pressure and its derivatives are given by

$$
\begin{aligned}
p &= -2\mu u_x - 2\mu v_y + \rho g(s-z), \quad (A73) \\
p_x &= -2\mu_x u_x - 2\mu u_{xx} - 2\mu_x v_y - 2\mu v_{xy} + \rho g s_x, \quad (A74) \\
p_y &= -2\mu_y u_x - 2\mu u_{xy} - 2\mu_y v_y - 2\mu v_{yy} + \rho g s_y, \quad (A75) \\
p_z &= -2\mu_z u_x - 2\mu u_{xz} - 2\mu_z v_y - 2\mu v_{yz} - \rho g. \quad (A76)
\end{aligned}
$$

Finally the compensation terms F_1, F_2, F_3 are given by

$$
\begin{aligned}
F_1 &= 2\mu_x u_x + 2\mu u_{xx} - p_x + \mu_y\left(u_y+v_x\right) + \mu\left(u_{yy}+v_{xy}\right) + \mu_z\left(u_z+w_x\right) + \mu\left(u_{zz}+w_{xz}\right), \quad (A77) \\
F_2 &= \mu_x\left(u_y+v_x\right) + \mu\left(u_{xy}+v_{xx}\right) + 2\mu_y v_y + 2\mu v_{yy} - p_y + \mu_z\left(v_z+w_y\right) + \mu\left(v_{zz}+w_{yz}\right), \quad (A78) \\
F_3 &= \mu_x\left(u_z+w_x\right) + \mu\left(u_{xz}+w_{xx}\right) + \mu_y\left(v_z+w_y\right) + \mu\left(v_{yz}+w_{yy}\right) + 2\mu_z w_z + 2\mu w_{zz} - p_z, \quad (A79)
\end{aligned}
$$

and the terms T_1, T_2, T_3 can be calculated using A4-A6.