Supplementary material - Scatter of mass changes estimates at basin scale for Greenland and Antarctica

V. R. Barletta[1], L. S. Sørensen[1], and R. Forsberg[1]

[1]Geodynamics Department, DTU Space, Elektrovej 2800 Kgs. Lyngby, Denmark

1 Use of degree-1 sensitivity kernel

Degree-1 sensitivity kernel (main text table 1 and 2) is designed to be used with our basin definition (main text Fig.3) and for solely GRACE derived mass changes, i.e. mass changes that does not account for the degree-1. So, in principle if one uses the same basins definition as ours and a unit geocenter motion of 1 mm for each of the X, Y, Z axis, he should obtain a very similar Kernel for degree-1.

Given the time series of three components (X, Y, Z) of the geocenter motion:

$$GC_X(t), \quad GC_Y(t), \quad GC_Z(t)$$

and the three factor of our sensitivity kernel for the i-th basin $SK_X(i), SK_Y(i), SK_Y(i)$ the degree-1 correction $D1C(i,t)$ for the i-th basin is obtained in the following way:

$$D1C(i,t) = GC_X(t)SK_X(i) + GC_Y(t)SK_Y(i) + GC_Z(t)SK_Z(i)$$

This correction can be simply added to the time series obtained for the i-th basin from GRACE data not corrected for degree-1.

2 Monthly solution

Figures from SM.1 to SM.9 show the monthly solutions for each basin.

Correspondence to: V. R. Barletta
(vr.barletta@gmail.com)

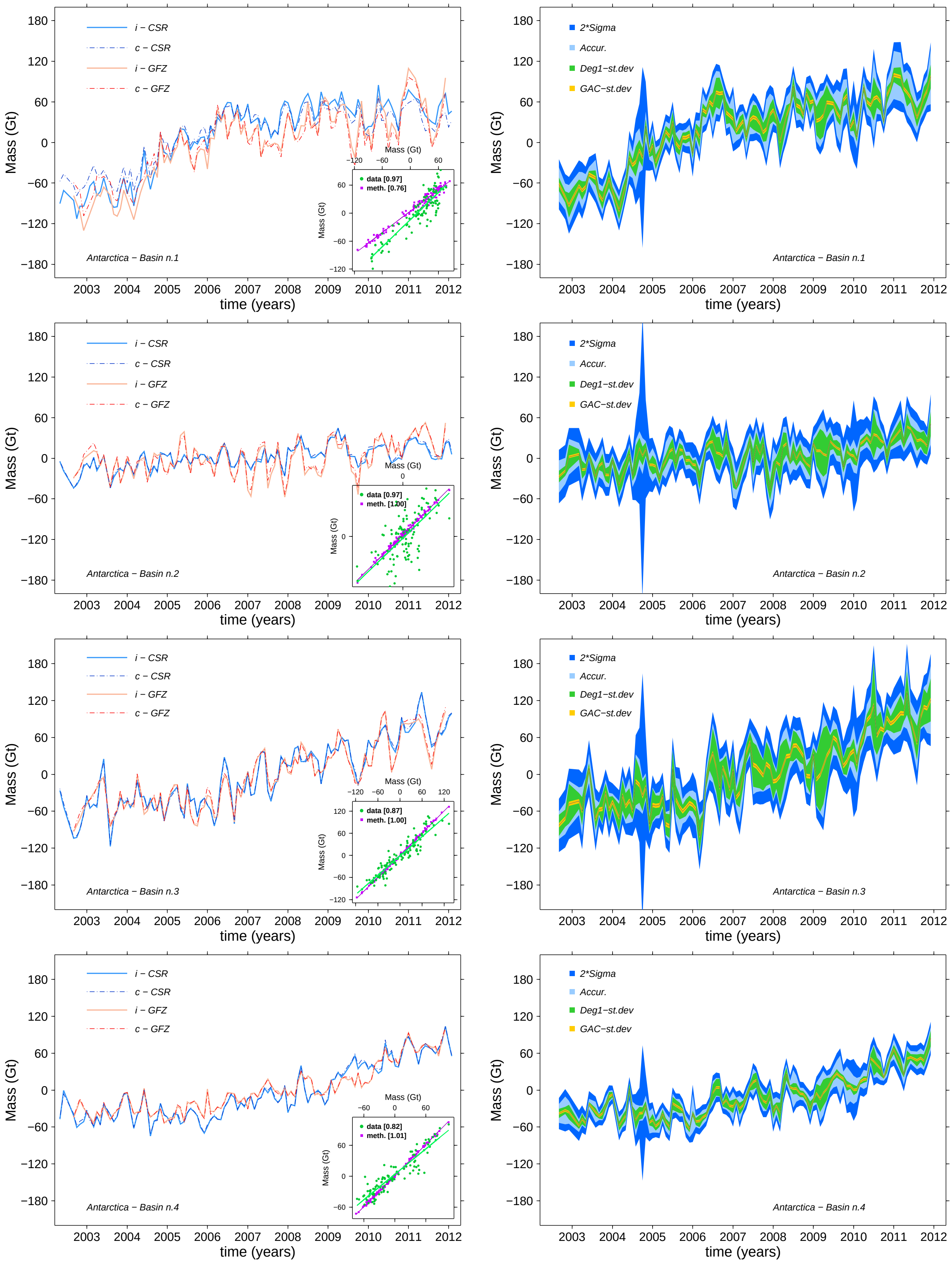

Fig. SM. 1. RL04 monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.3 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

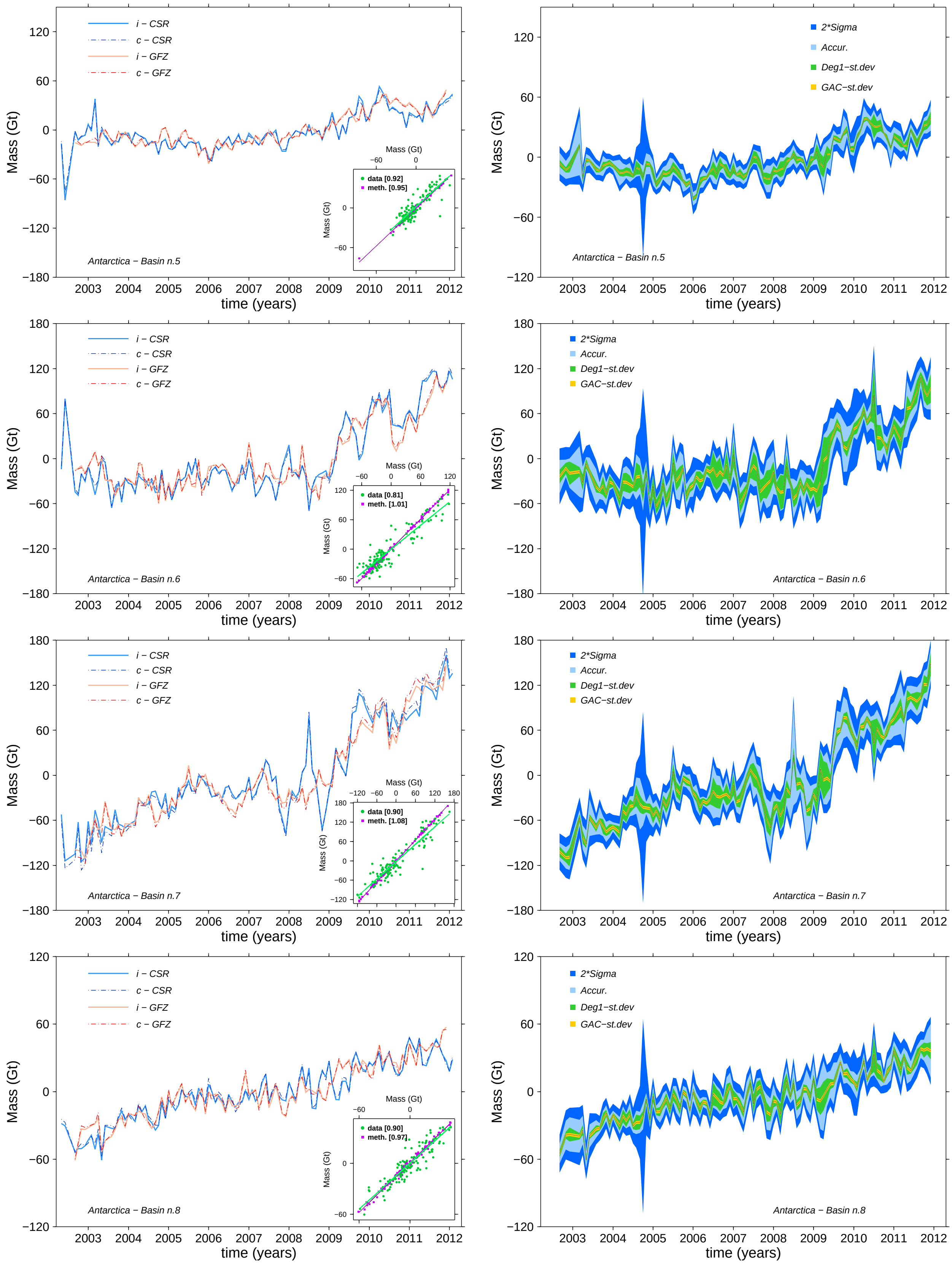

Fig. SM. 2. RL04 monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

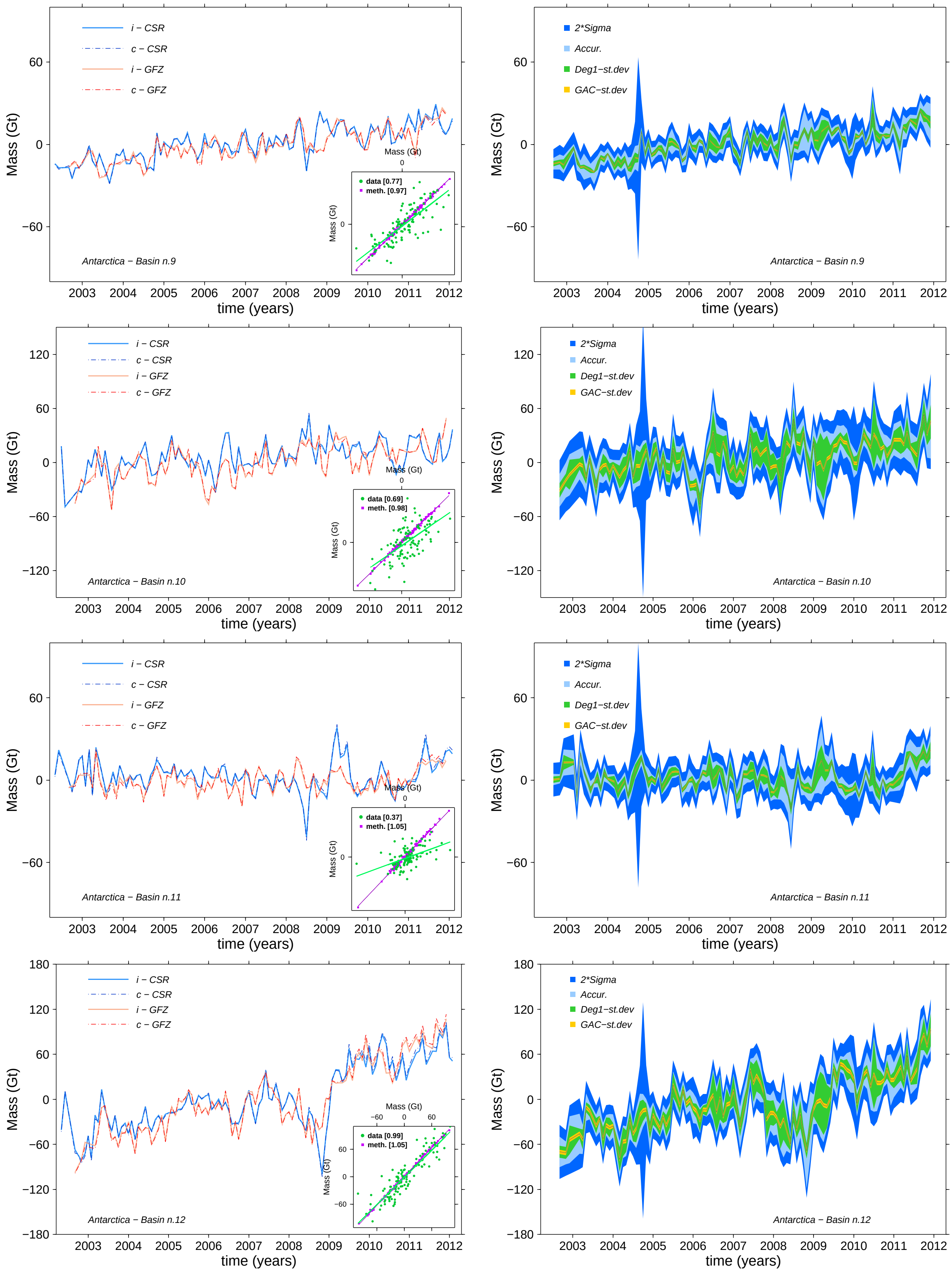

Fig. SM. 3. RL04 monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

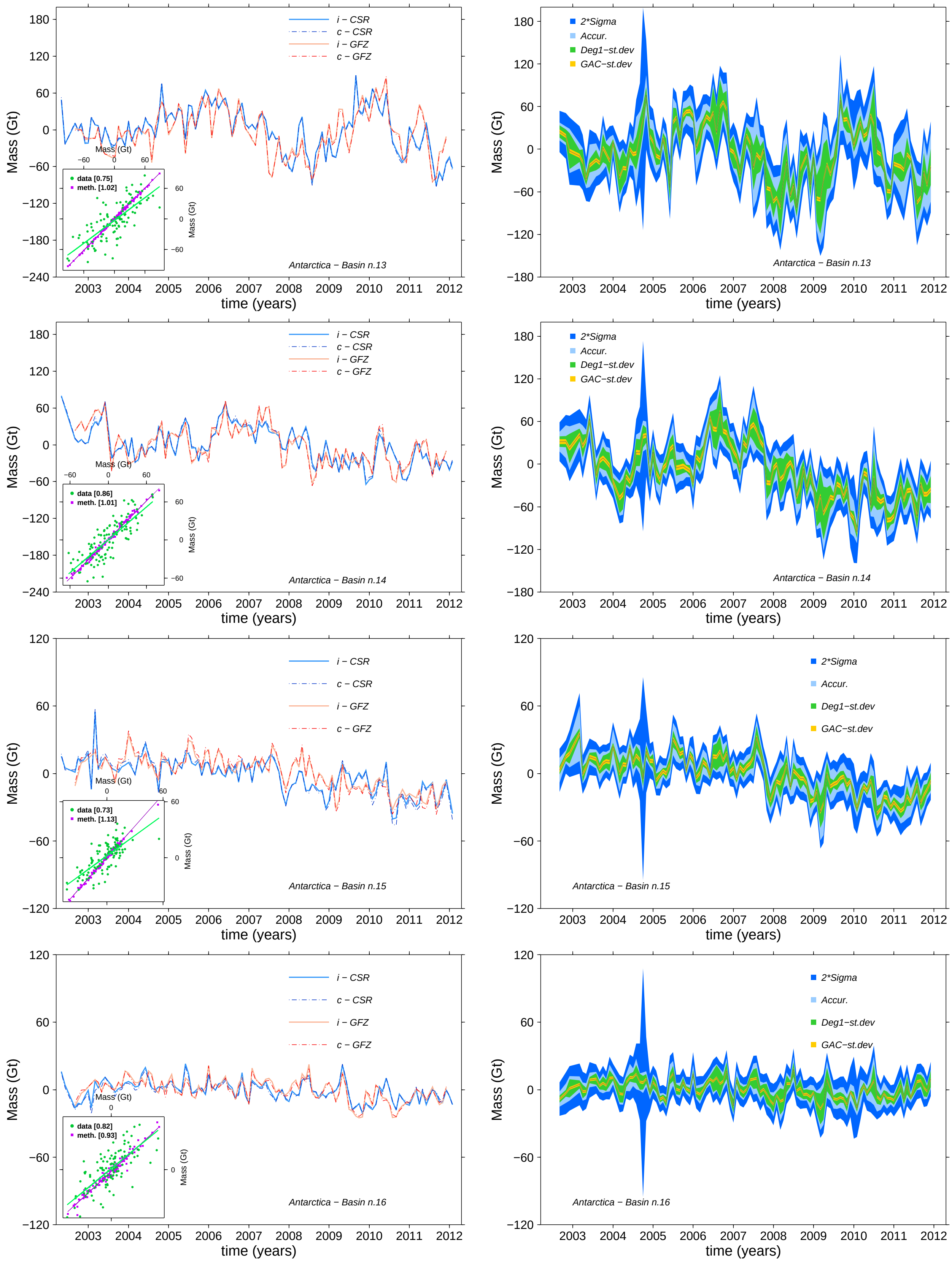

Fig. SM. 4. RL04 monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

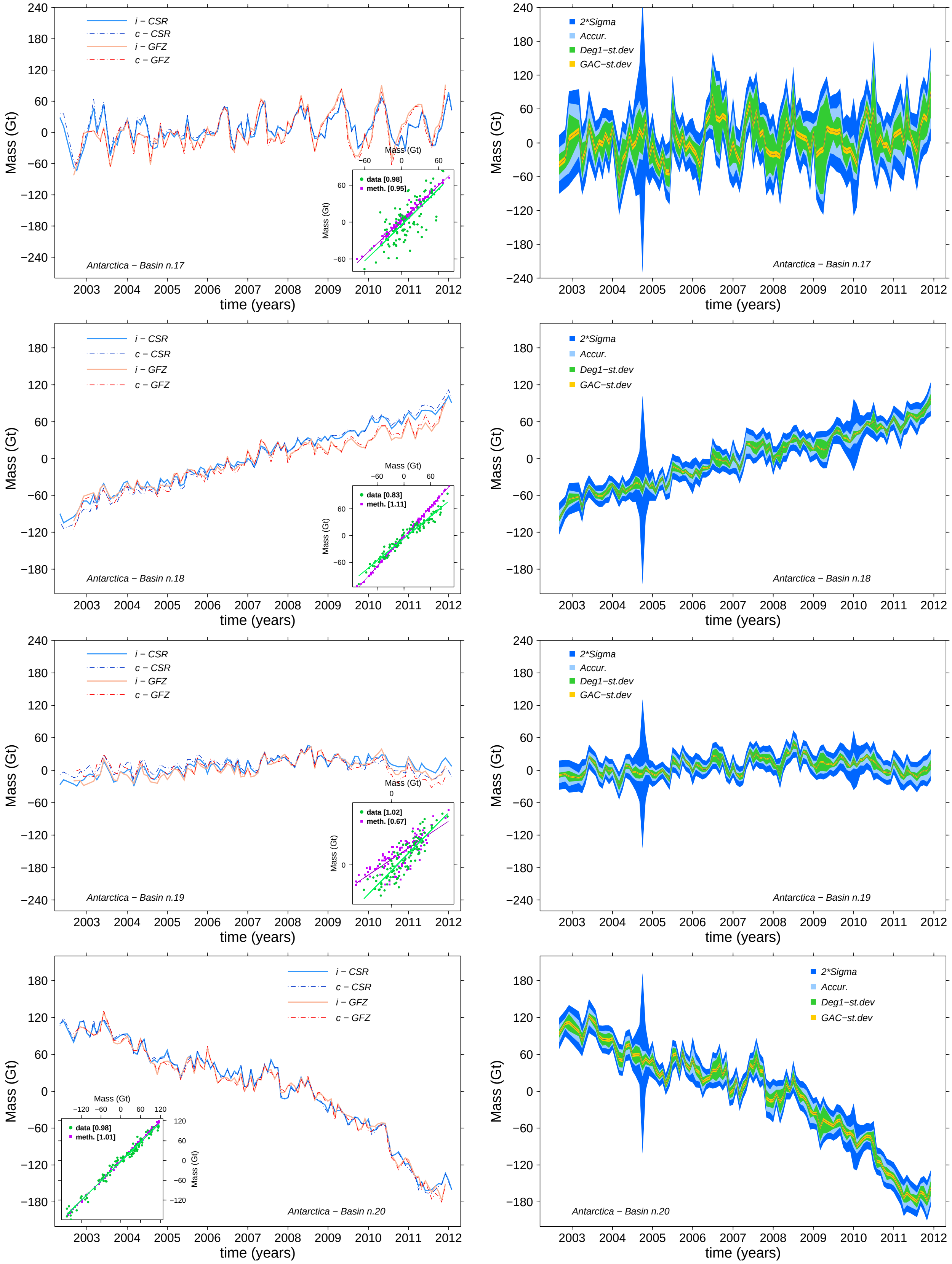

Fig. SM. 5. RL04 monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

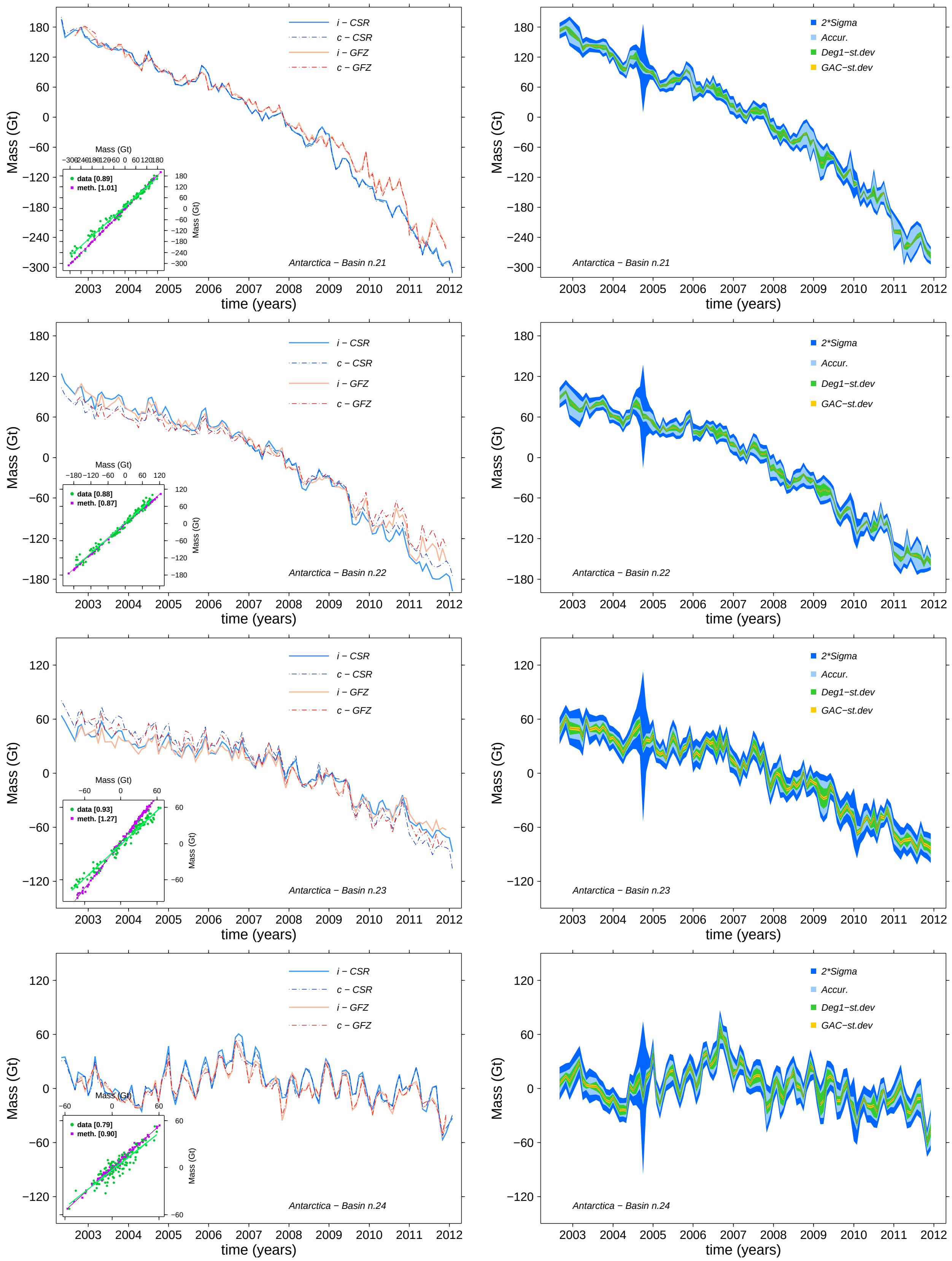

Fig. SM. 6. RL04 monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

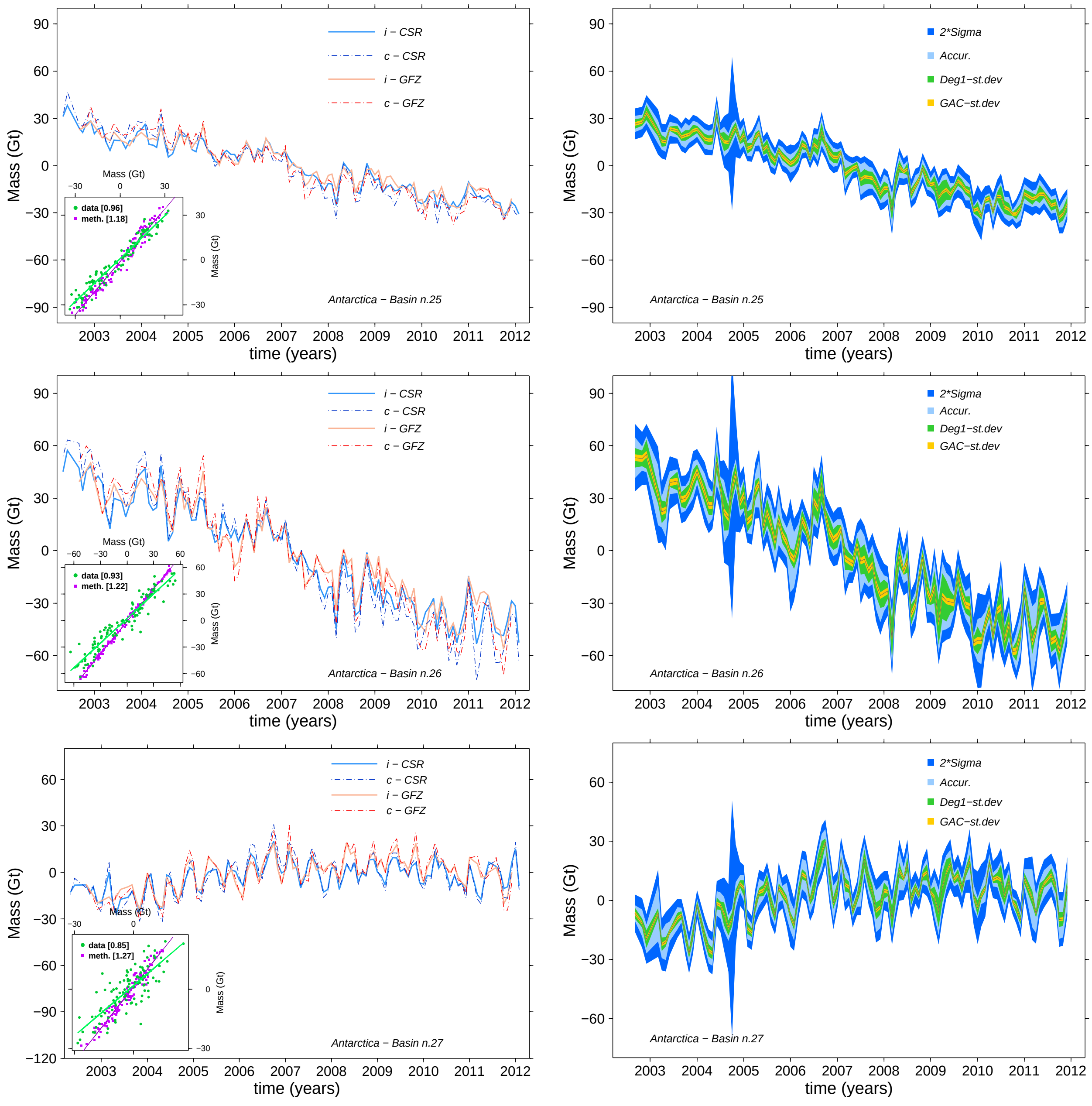

Fig. SM. 7. RL04 monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

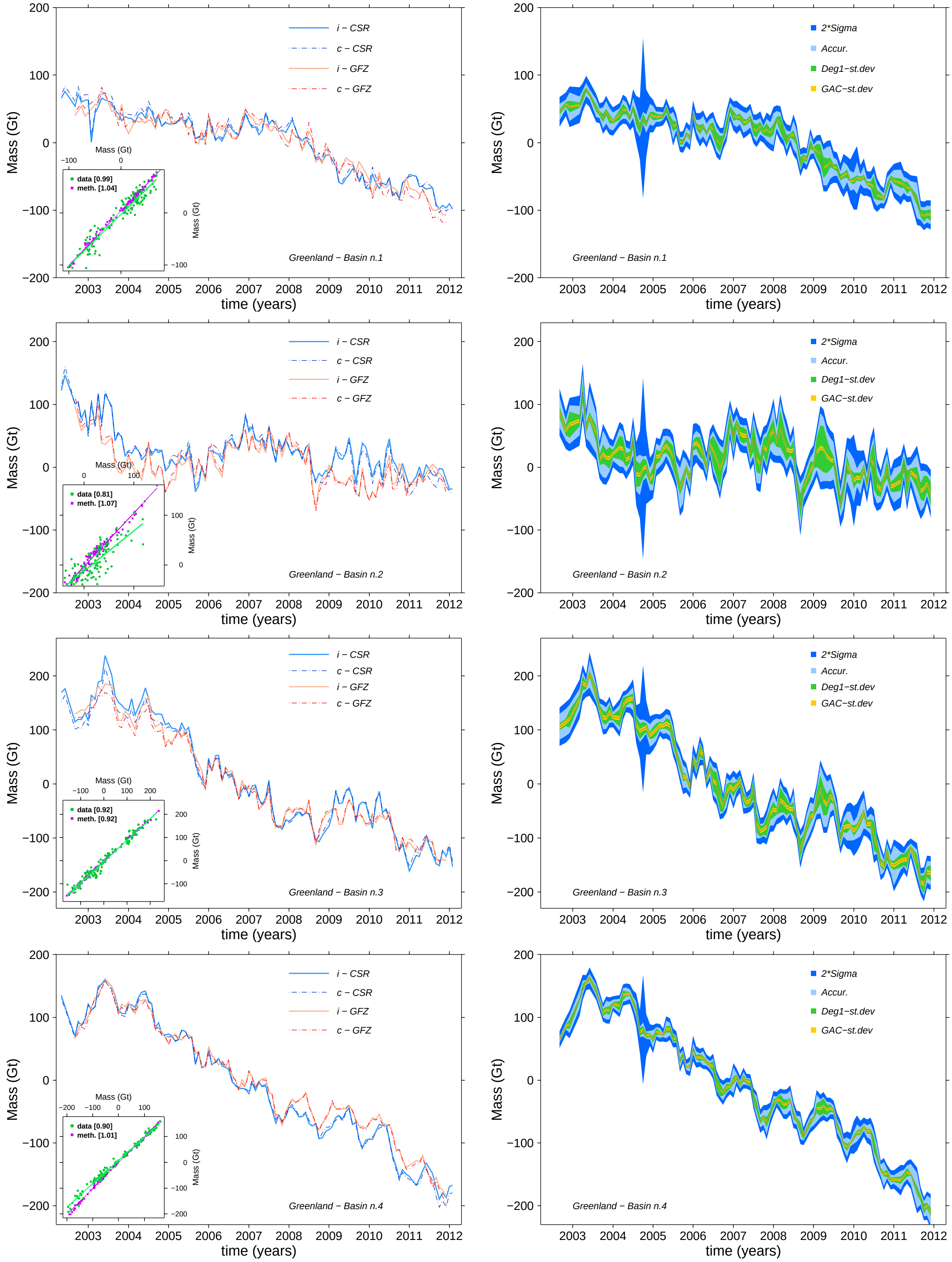

Fig. SM. 8. RL04 monthly solution for Greenland. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

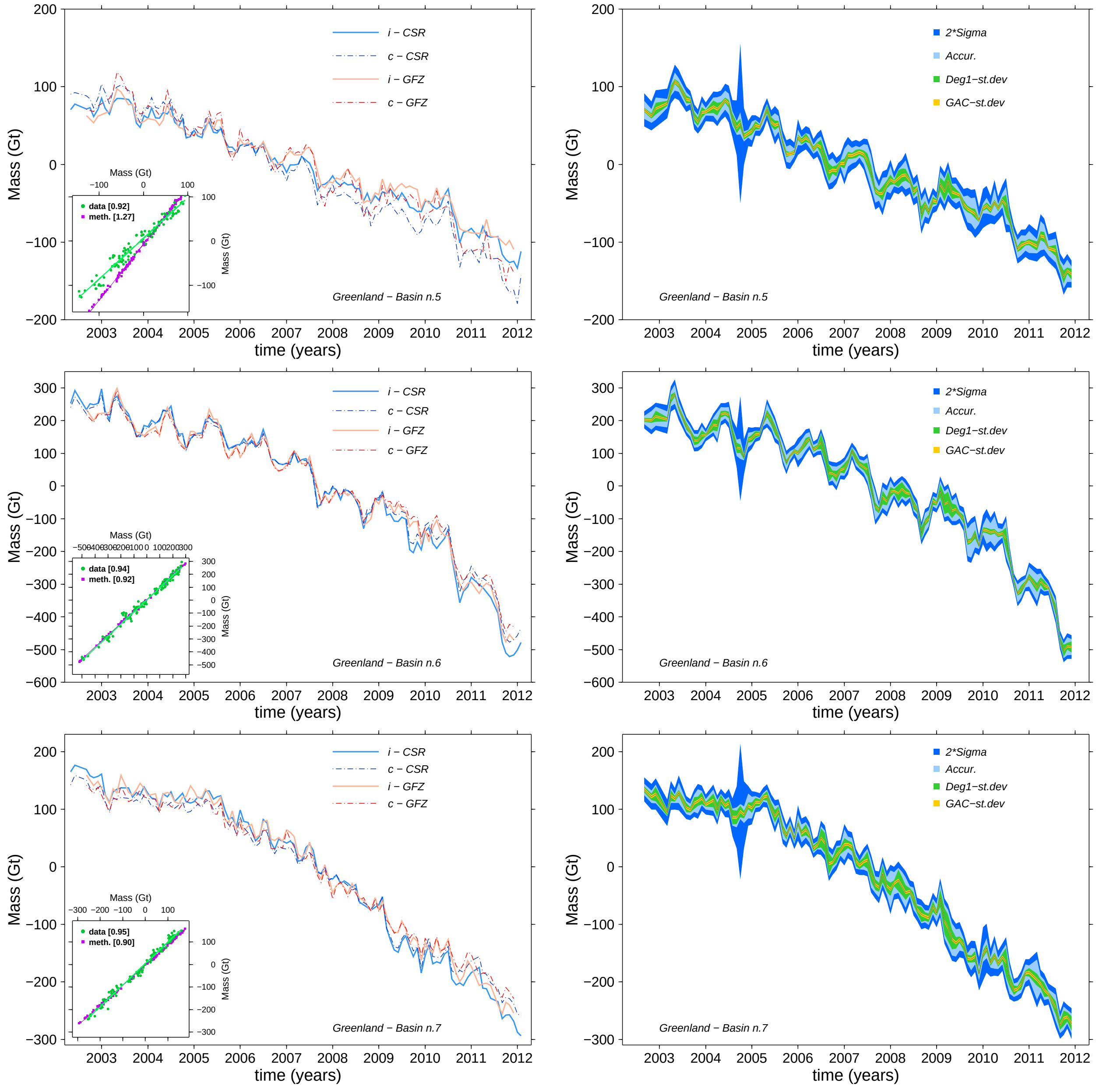

Fig. SM. 9. RL04 monthly solution for Greenland. Basin number is indicated in each plot. Left column represents the comparison between the two methods and two data sets (CSR and GFZ) as in Fig.4 in the main text. Right column represents the monthly average solution where each color in the band around the average represents a contribution to the error estimate as in Fig. 7 of the main text.

NOTE.

After the first paer revision (in Jannuary 2013) we updated the release 05 data, which has become complete and consolidated since our first processing (Jannuary 2012). So the figures about GRACE RL05 data in this supplementary material (form fig SM.11 to SM.21) are related to the new processing. However in the main text we only use updated data for comparison between RL05 and RL04 (fig. 5).

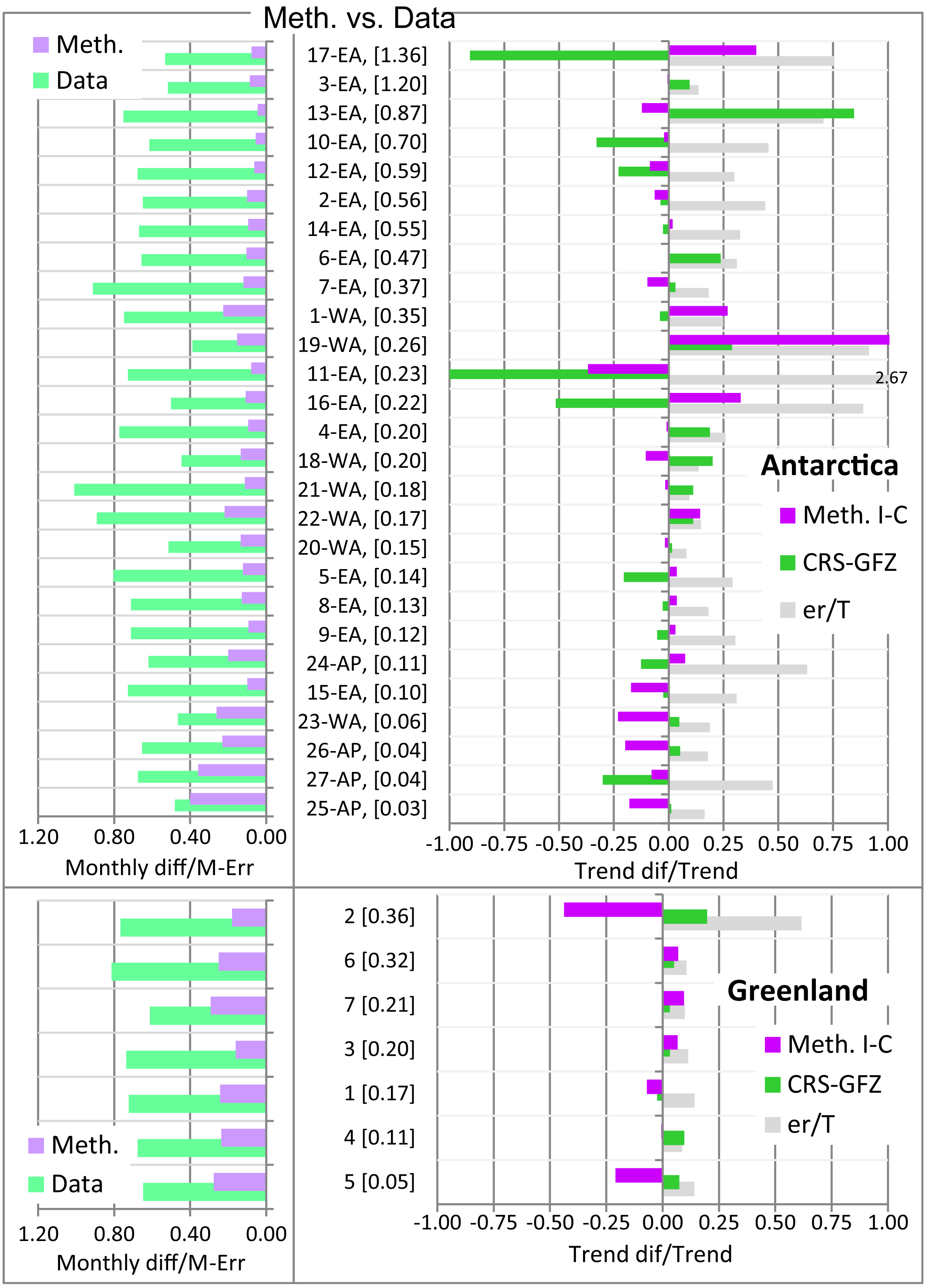

Fig. SM. 10. For each basin, differences are plotted for the two methods (purple) and for the two datasets (green). The light colors represent the (quadratic sum of) the monthly difference with respect to (the average of) the monthly errors. The normal colors represent the difference in trends and the gray bar is the error on the trend. All these quantity are normalized with respect to the trend.

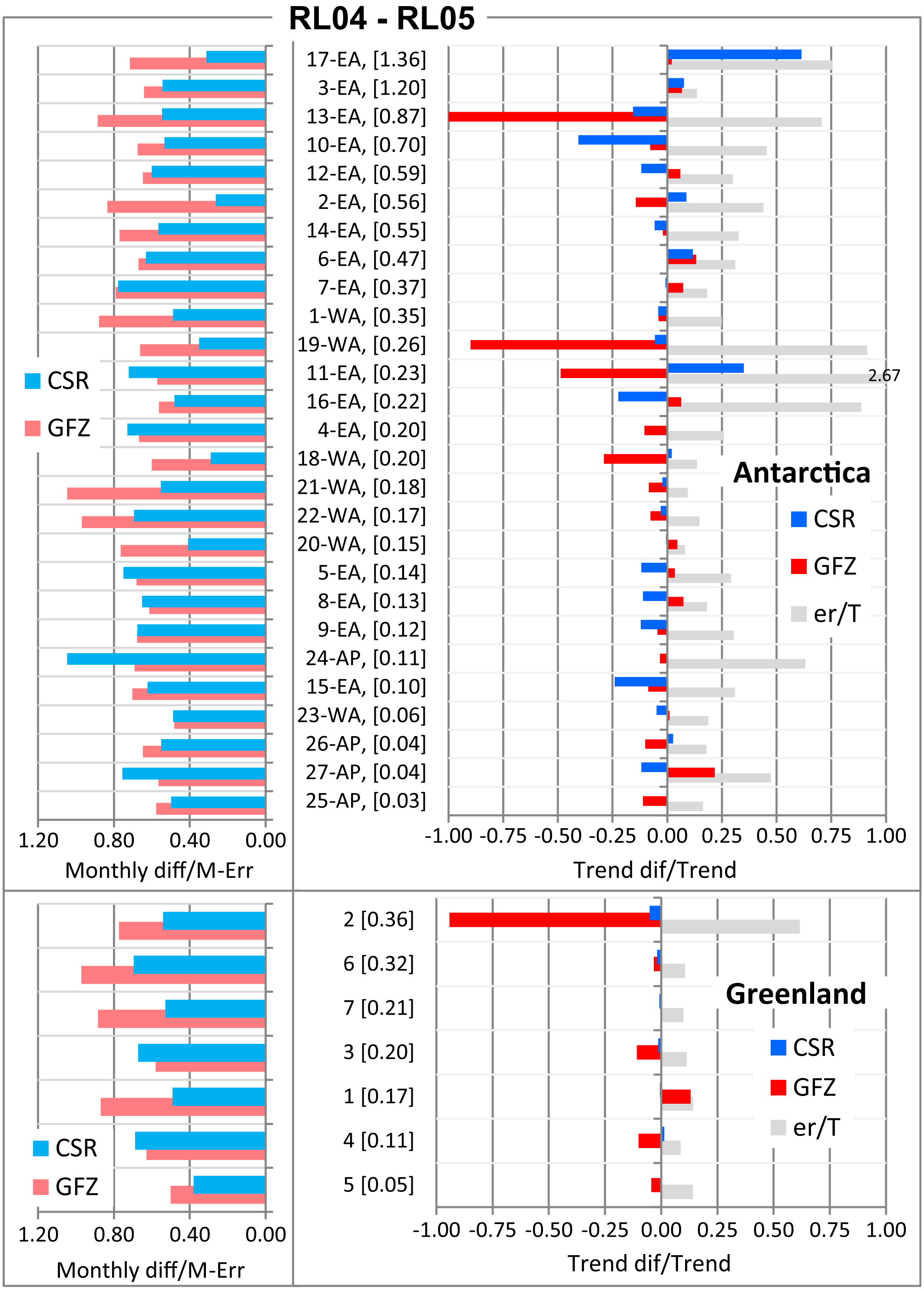

Fig. SM. 11. Trend differences between use of release 04 and 05. For each basin differences between RL04 and RL05 are plotted for the use of CSR (blue) and GFZ (red). The light colors represent the (quadratic sum of) the monthly difference with respect to (the average of) the monthly errors. The normal colors represent the difference in trends and the gray bar is the error on the trend. All these quantity are normalized with respect to the trend.

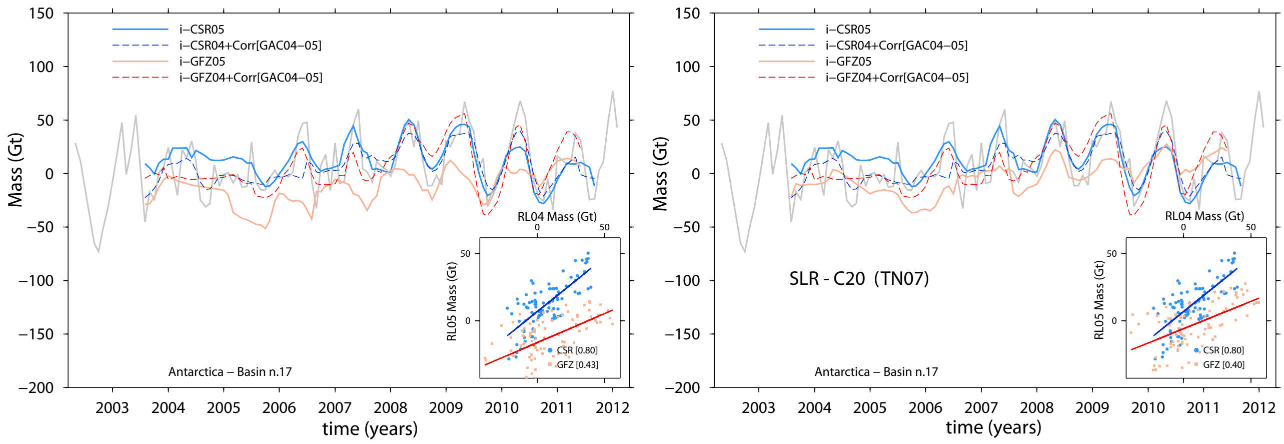

Fig. SM. 12. Smoothed monthly solutions for basin 17 for Antarctica with original C20 in GFZ-RL05 solution (left) and with replaced C20 from TN07 (right). Comparison between the two releases RL04 and RL05 (CSR and GFZ) with inversion method: the use of CSR is indicated by light blue and blue lines, and the GFZ by light red and red lines. The release RL05 is the solid line, while the RL04, with GAC[04–05] correction, is the dashed line. The grey line represent the original CSR time series before the GAC correction. Each of the small dispersion graphics shows the time series obtained with RL04 versus RL05 with the use of CSR (blue) and GFZ (red).

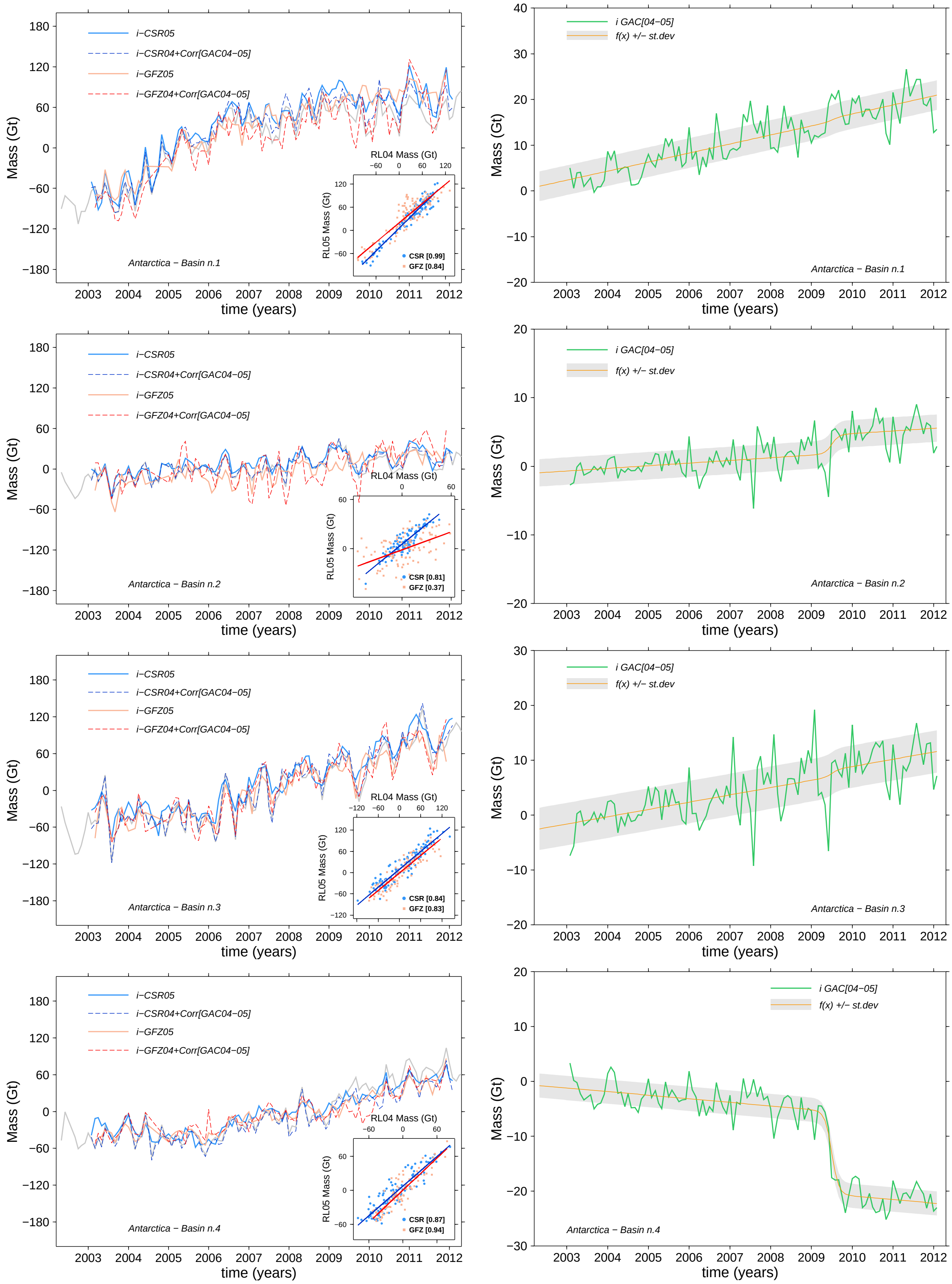

Fig. SM. 13. Monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

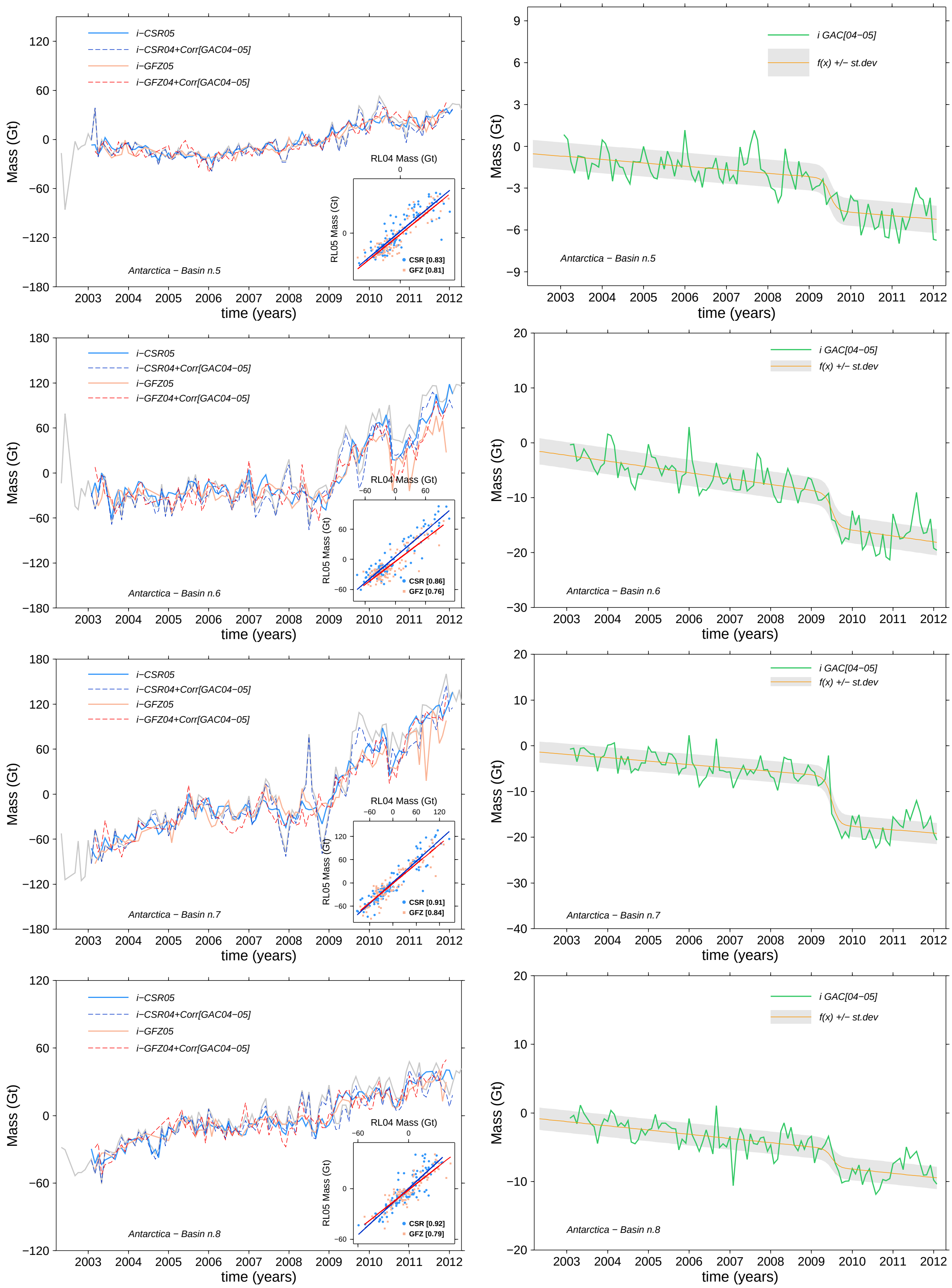

Fig. SM. 14. Monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

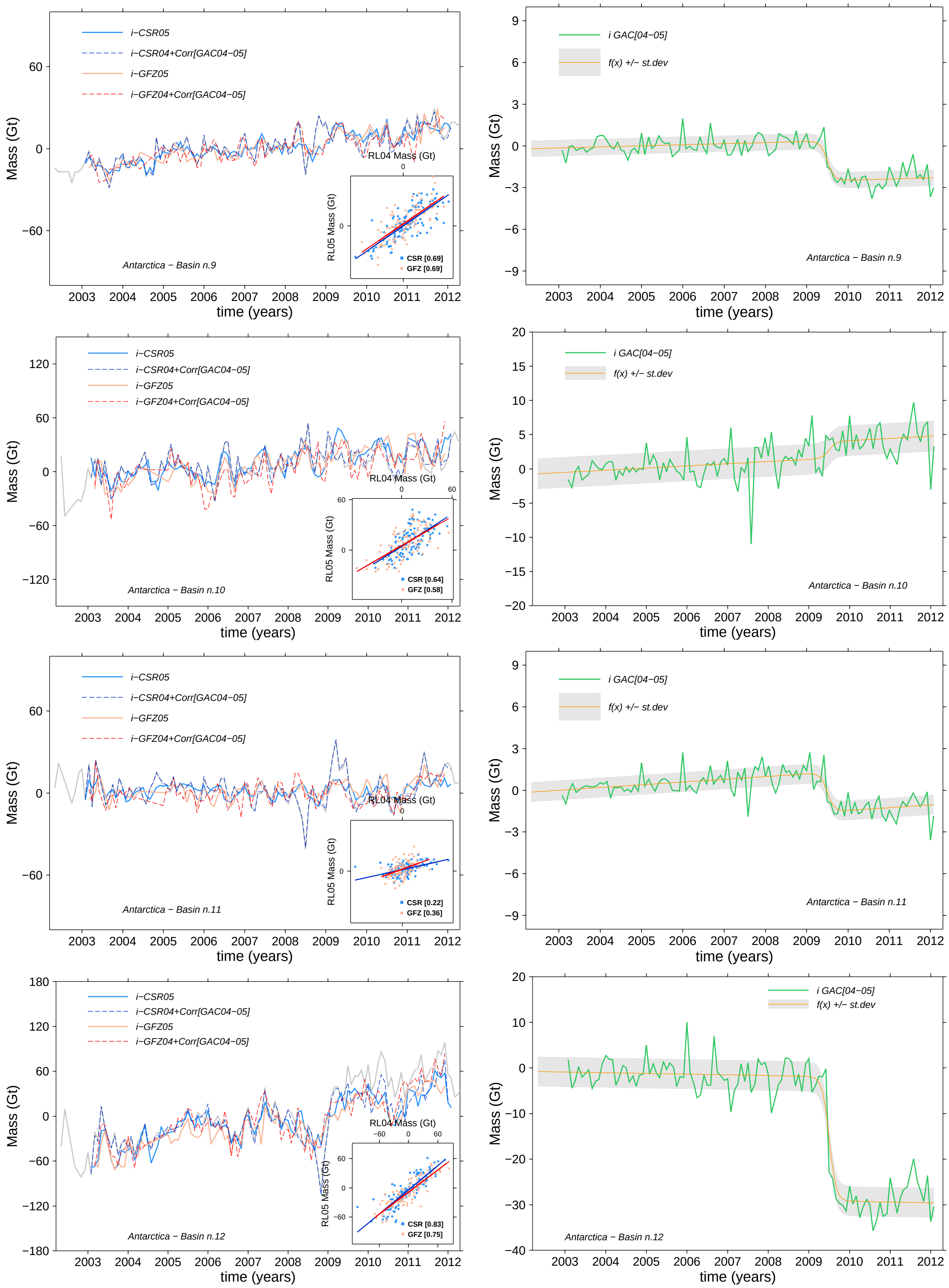

Fig. SM. 15. Monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

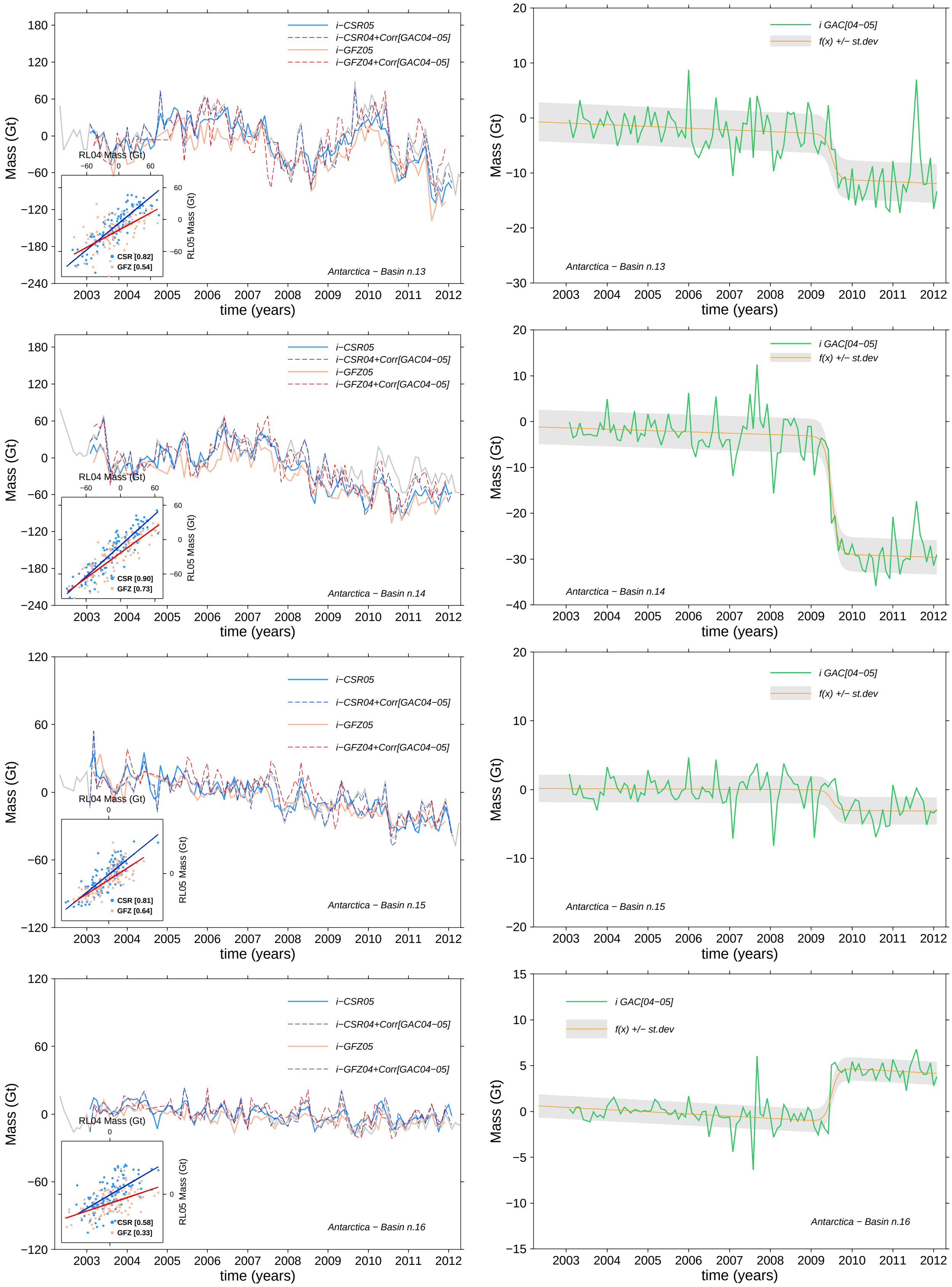

Fig. SM. 16. Monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

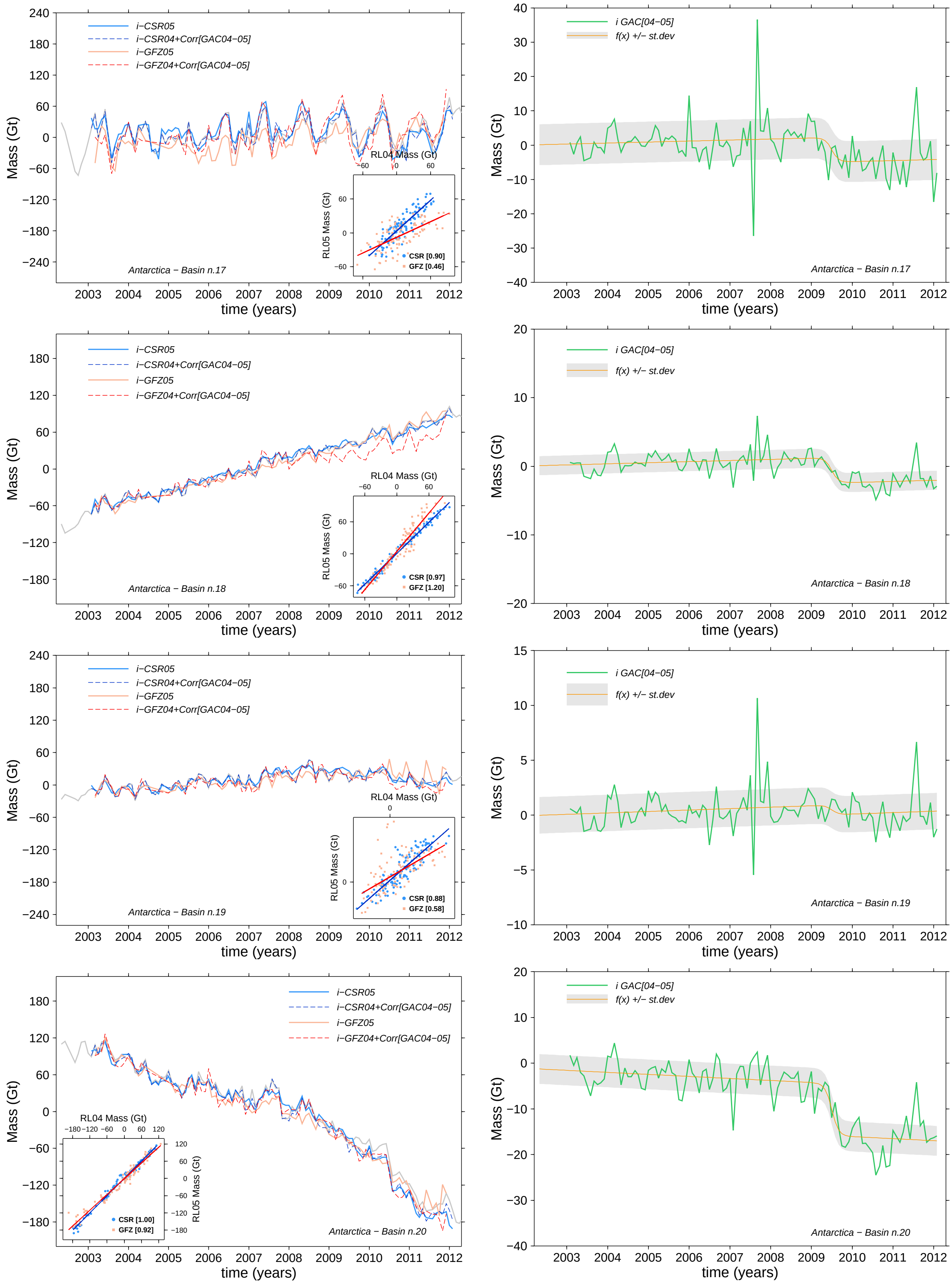

Fig. SM. 17. Monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

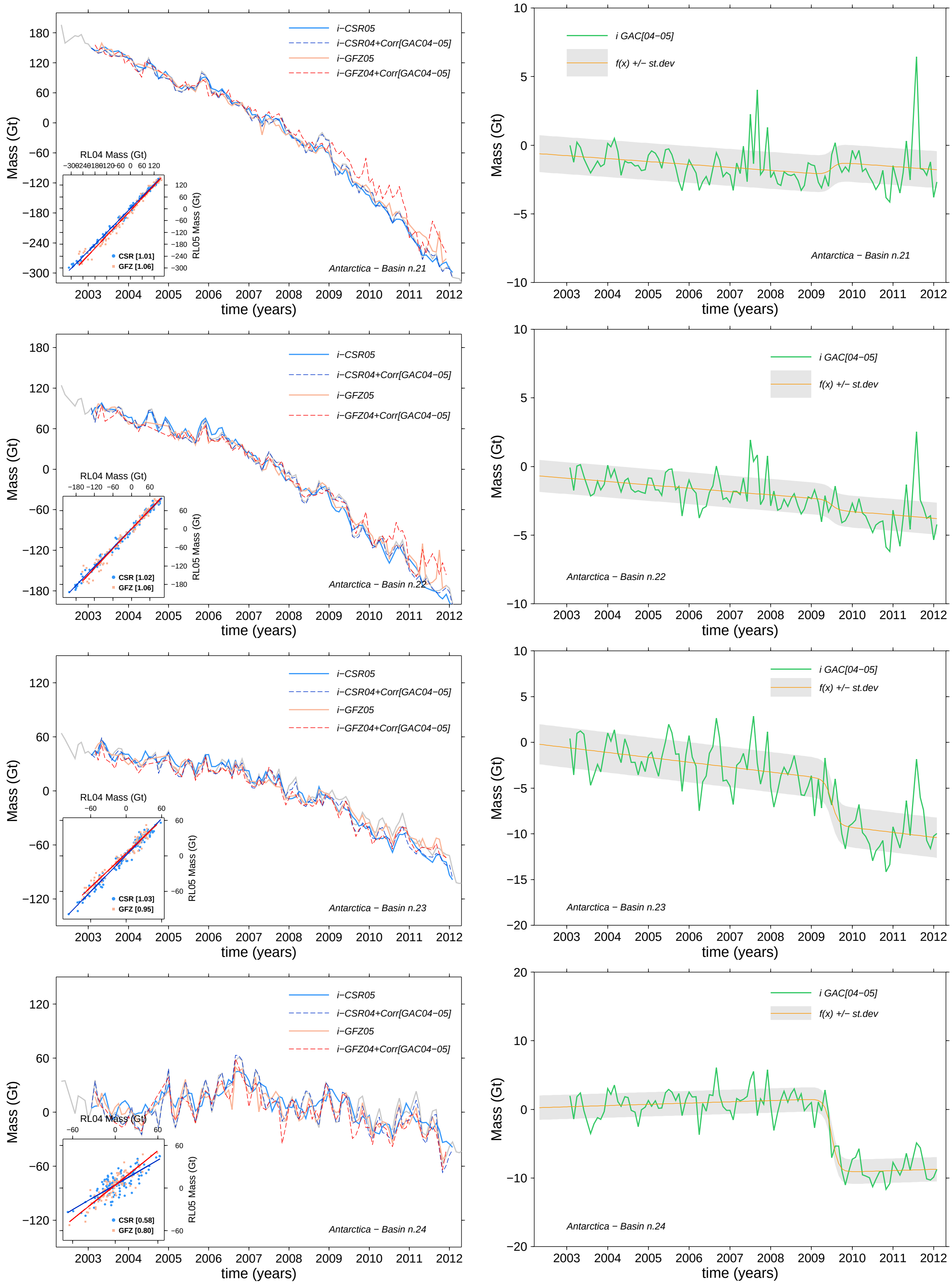

Fig. SM. 18. Monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

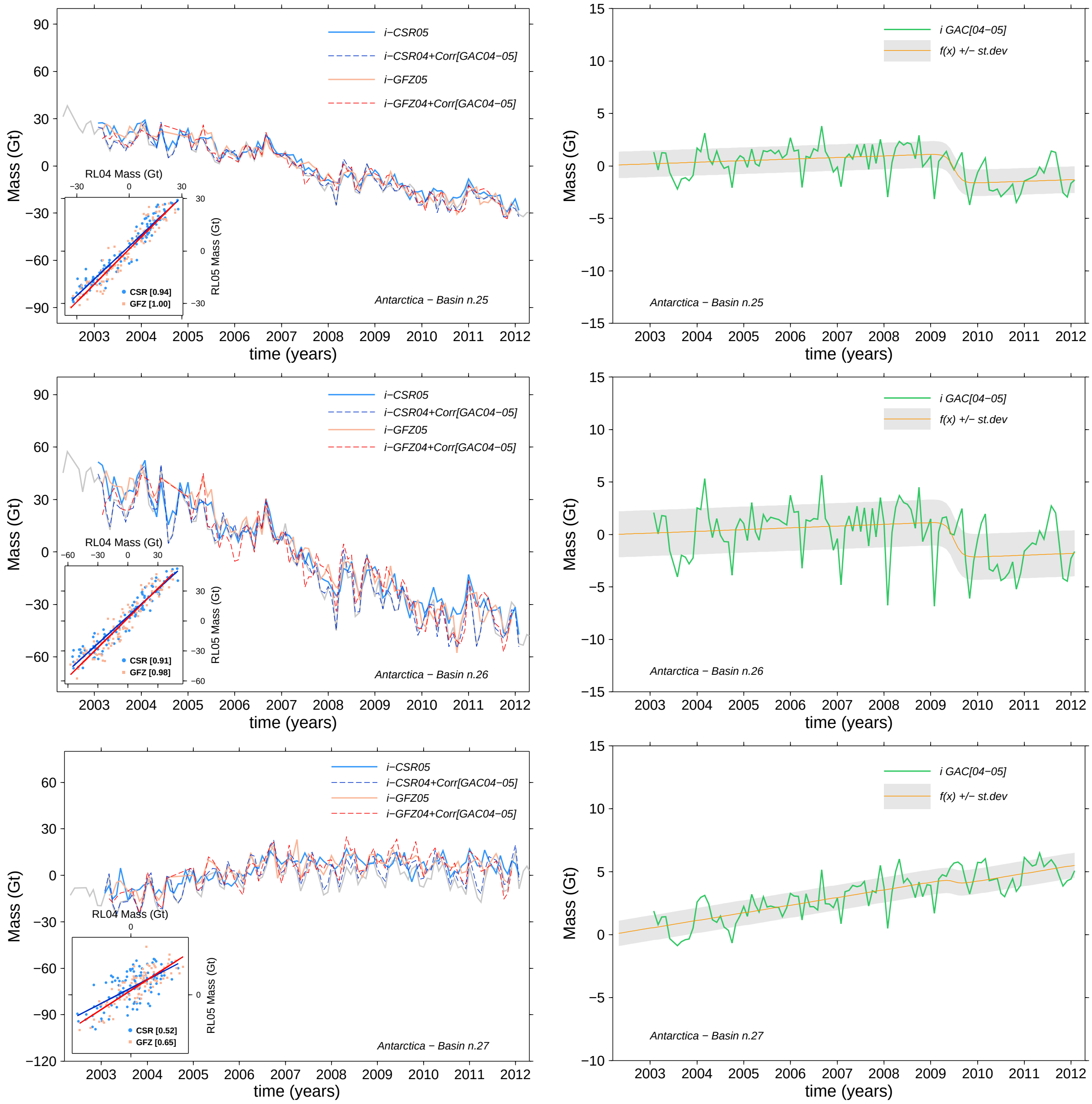

Fig. SM. 19. Monthly solution for Antarctica. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

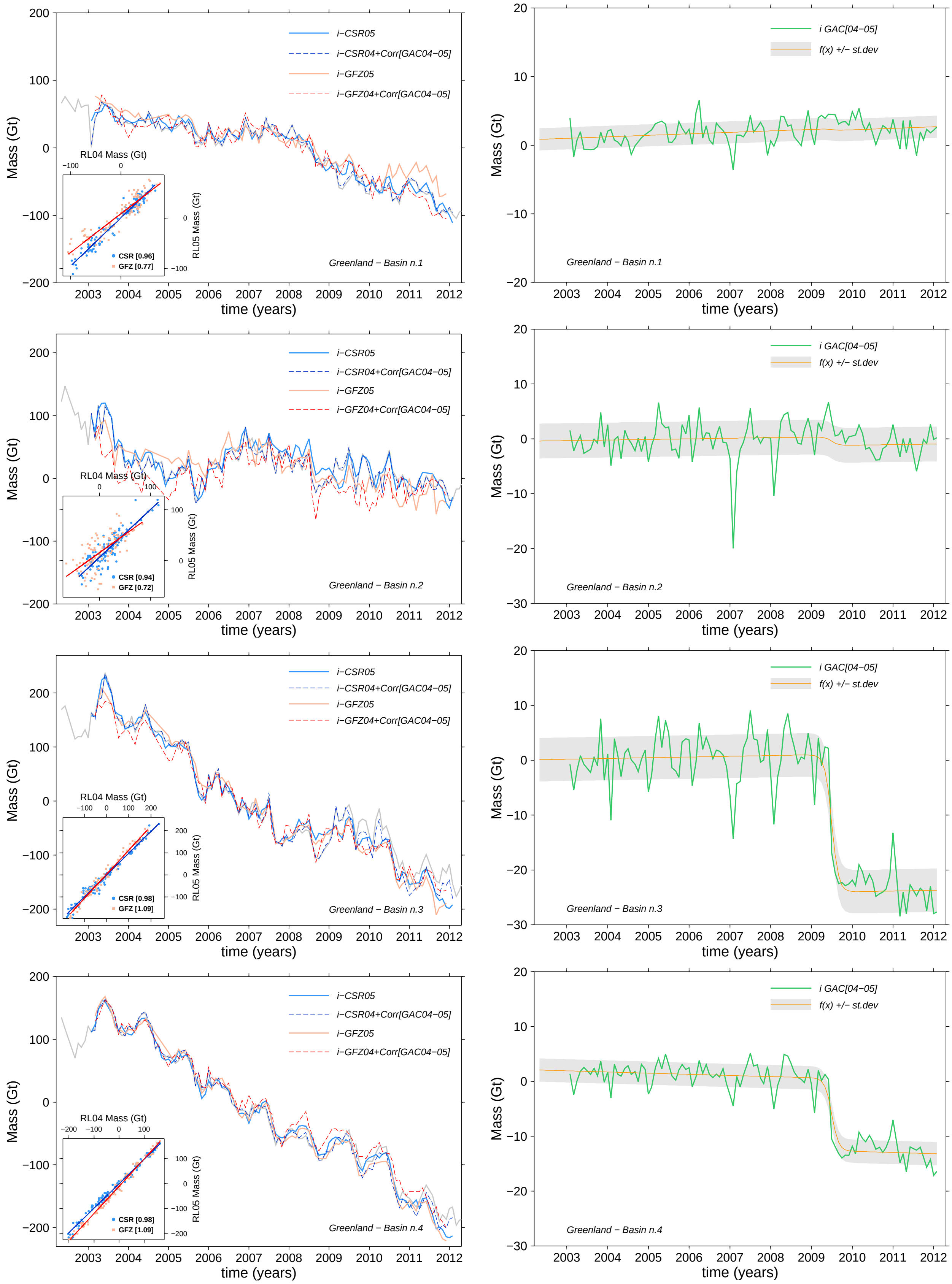

Fig. SM. 20. Monthly solution for Greenland. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

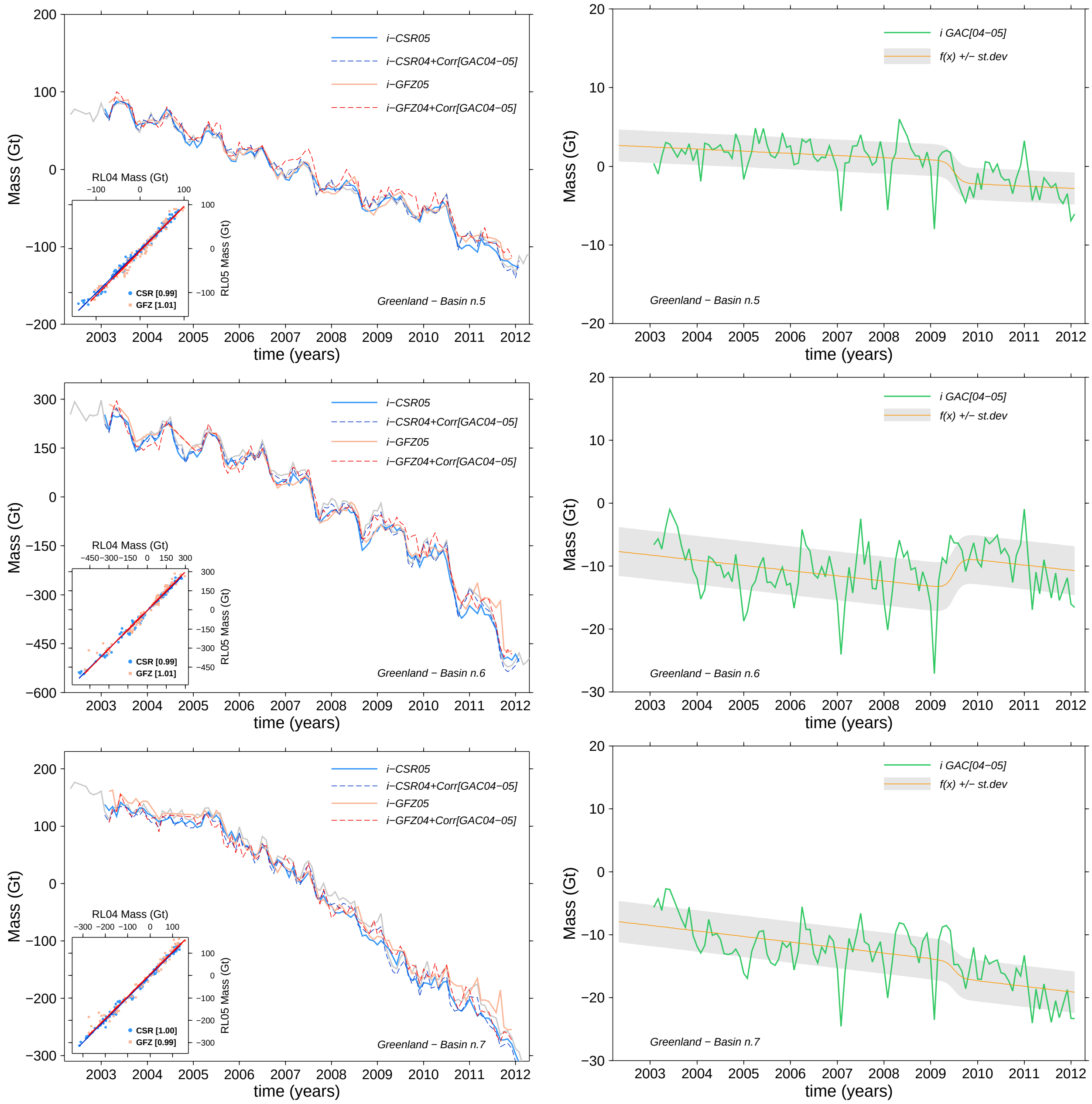

Fig. SM. 21. Monthly solution for Greenland. Basin number is indicated in each plot. Left column represents the comparison between the two release RL04 and RL05 as in Fig.5 in the main text. Right column represents the monthly difference between the GAC-RL04 and the GAC-RL05 (green line), and the fitted function (orange line) with its standard deviation (grey band) as in Fig. 1 of the main text.

Refinement of Inversion Method and Calibration

VALENTINA R. BARLETTA

1. The inversion Method

The inversion is performed on a set of N_y observations $\underline{y} = \{\delta g_k\}$, k = 1. . .$N_y$, i.e. gravity disturbances at the altitude of the satellites, and solved for a point-like mass ensemble $\underline{x} = \{m_j\}$, j = 1. . .N_x located at coordinates (θ_j ,ϕ_j) which define the solution area. The linear problem $\underline{y} = \mathbf{A}\,\underline{x}$ is solved using a generalized Tychonov inverse approach $\underline{x} = (\mathbf{A}^T\mathbf{A}+\lambda\mathbf{I})^{-1}\mathbf{A}^T\underline{y}$, where λ is a smoothing parameter and the observation matrix **A** is built upon the attraction of a point mass of the sphere to the measured gravitational attraction at the orbit level by

$$\delta g_k = G\, m_j\, a^2\, (\,(h+a) - a \cos \psi_{kj}\,) \,/\, r^3_{kj}$$

Here G is the gravity Newton constant, a is the mean radius of the Earth, h is the height of the observation, r_{kj} and ψ_{kj} are the distance and the angle, respectively, between the observation δg_k and the solution point m_j . The expression for the gravity disturbance is $\delta g = dV/dr$ where V is the gravity potential given in terms of normalized Stokes coefficients in GRACE handbook.

We use a icosahedron-based grids (Tegmark, 1996) with disk elements of almost equal area. The number of disks does not change the result appreciably, if they are smaller than the input data resolution, but it does affect the computational costs. Moreover the disks are only a convenient representation, in fact point-like mass are actually inverted to find the mass distribution. The spacing of the point-like mass (which corresponds to the disk radius) reflects the distribution and it affect the results only if the points are separated by more than 300 km.

2. Trend on GRACE data

Figure 1 shows the trend (Jan 2003 – Dec 2010) of the heights in water equivalent (w.e.) extracted by calculating the trend on each harmonic coefficient (with the GIA Paulson removed and Swenson et al. 2008 degree-1 added) and then simply converted (on the left), and inverted (on the right) using a very wide solution area. The trends are calculated by fitting a function with linear and seasonal (annual and semiannual) components.

The conversion from SH in WE (tuned from J. Wahr 2002) uses the assumption that the mass variation is originating from a thin layer on the surface. After that assumption the resulting mass distribution is localized as much as the low resolution allows it, so each pixel obtained converting the gravity field in the mass distribution, accounts for a region which is as wide as the GRACE resolution, i.e. 300 – 400 km.

The geoid or the gravity field is sensitive to mass changes in the far field, but the converted mass distribution is not. And we are relying on this property for the pre-processing of our inversion method.

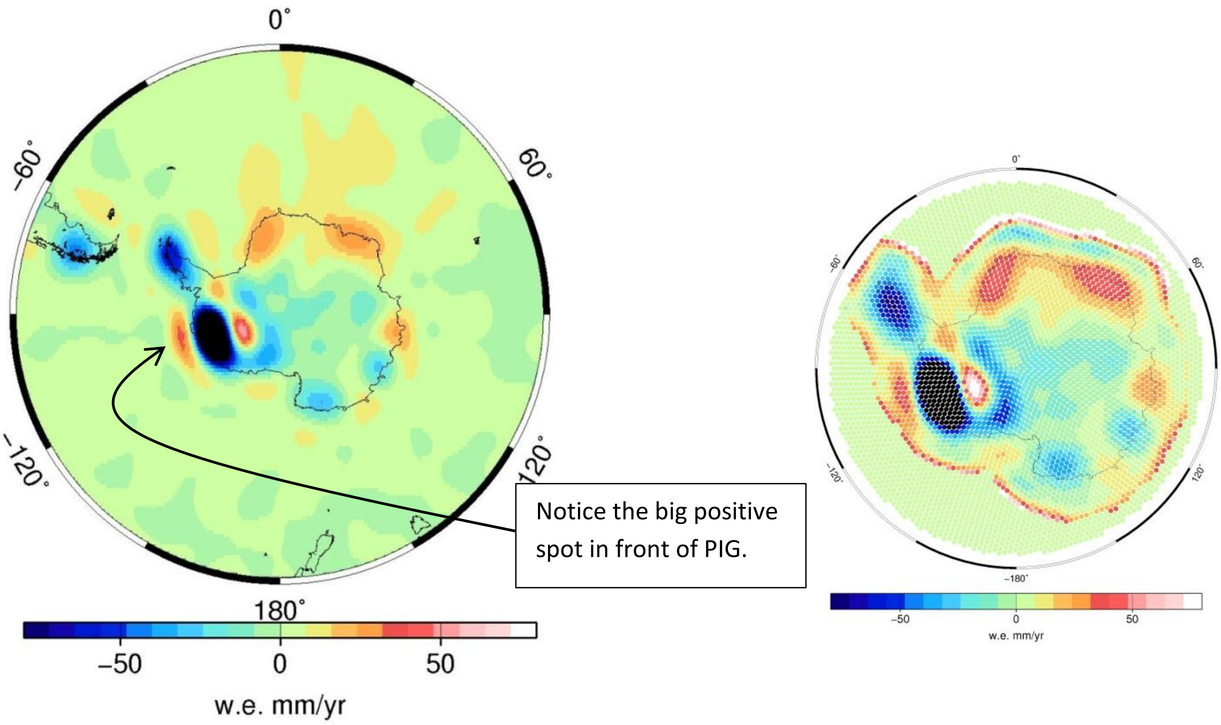

Figure 1: Left: Trend on harmonic coefficients of CSR L2 – RL04 GRACE (GIA Paulson removed) in the period Jan 2003 – Dec 2010. Right: the inversion method performed with a very wide solution area (including 800 km of sea surrounding the continent).

3. The synthetic data

Part of the following synthetic data has been also used in the IMBIE project. Three different kinds of synthetic data are designed to test how much mass the method is able to recover. The following masses are located only in the interior of Antarctica and they are weighted to give a total mass loss of 100 Gt/yr and transformed in Stokes harmonic coefficients. Note that we include degree 1 in all our synthetic data because it does have an effect.

A) Negative masses with pattern similar to the observed one (extracted from GRACE)
B) Negative GIA signal (only over Antarctica).
C) Uniform negative mass over Antarctica.

Another kind of synthetic data is set to zero over Antarctica, and over the ocean it reproduces the observed signal. A test using these synthetic data should give information on how much signal the method recover from outside the region of interest. If the method is performing correctly the result should be zero. The total mass (integral over the globe) is of +135 Gt/yr.

At last, there is a synthetic data set aimed at understanding the role of the C_00 coefficient. It is similar to the data for A) but with an equivalent amount of water added to the ocean (with SLE solved consistently), in this way the C_00 is zero, and there is a tiny signal over the sea surrounding Antarctica. Also in this case if the method performs correctly it should give a mass loss of 100 Gt/yr. Note that the extra signal around Antarctica is negative, so also in this case the test on this data should give an idea on how much is the leakage from the sea.

D) Negative masses + consistent sea signal
E) Synthetic over the ocean.

4. First test with synthetic data

The inversion method (with smoothing parameter 60,000) applied on synthetic data gives in Gt/yr:

on A) -99.04 on B) -99.86 on C) -99.38

Using the synthetic data to test the leakage from the sea, the method gives in Gt/yr:

on D) -97.79 on E) **+54.49**

This confirms the fact that the method requires the strong assumption that the signal over the ocean has to be negligible. However, when this assumption is verified, it is clear that the method is able to recover up to 99% of the mass, and for this reason it is reasonable to use this characteristic to perform some calibrations.

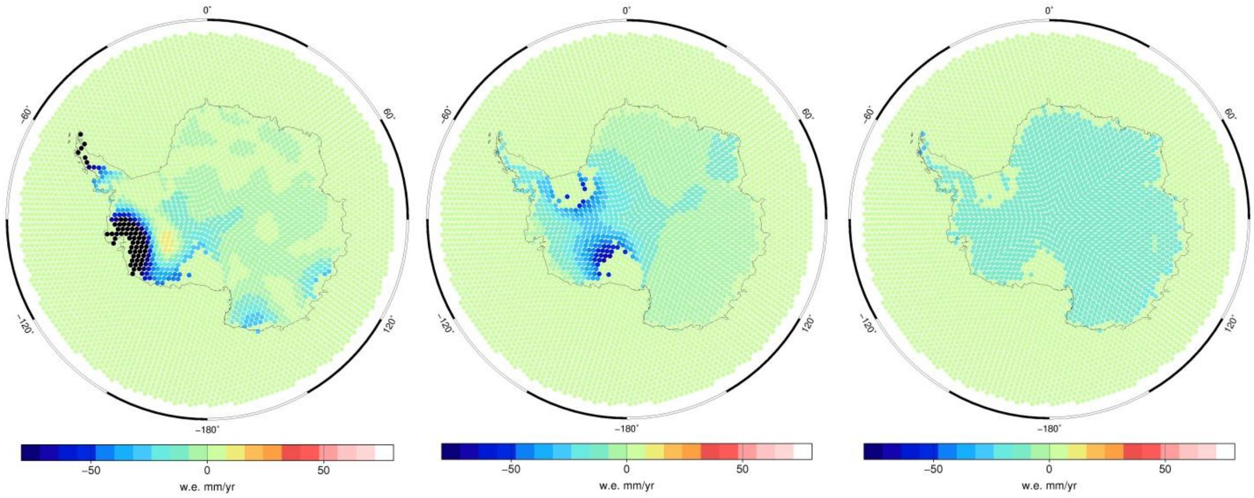

Figure 2: Mass distribution: left for case A, center for case B and right for case C. The C case is made with the 80,000 smoothig param, the others with the 60,000.

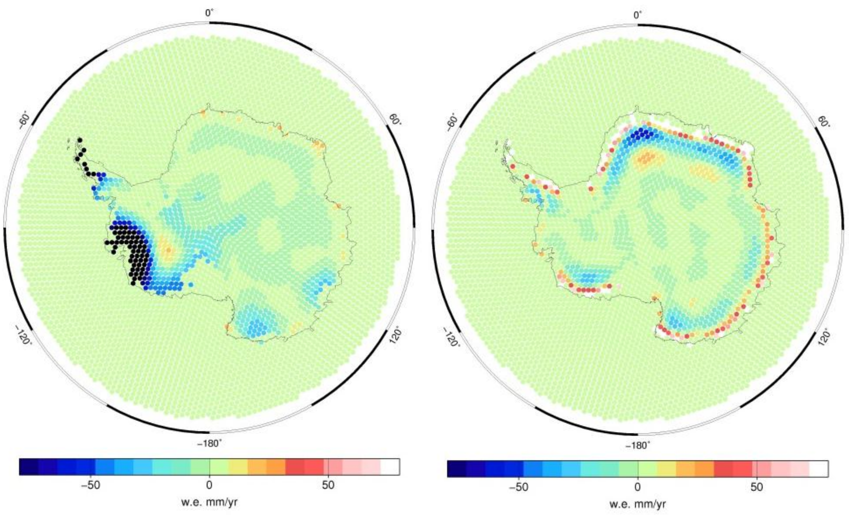

Figure 3: Mass distribution: left for the case D right for the case E and right for the GRACE data. These case use a smoothing param 80,000.

5. Calibration of the smoothing parameter

Since we understood that when using an inversion method a lot of signal can come from the sea, we need a series of calibrations, including one on the smoothing parameter (SM param).

By performing some experiment with the new configurations (mass over the ocean and removing signal over the ocean) we realized (with visual inspection) that changing the smoothing parameter (up to 180,000) does not affect so much the resulting pattern and we think this is because the algorithm do not have to deal with the signal from the sea.

However changing the smoothing parameter affects (of some percent) the total mass balance recovered (usually increasing the total mass loss), so we need to calibrate also this parameter.

For the calibration of the smoothing parameter we used the original method on the synthetic data which has fixed mass loss of -99.8 Gt/yr.

The effect of increasing the smoothing parameter from 50,000 to 180,000 is to increase the total mass calculated from 98.5 to 100.1 Gt/yr. A visual inspection does not show significant differences.

The 80,000 gives best compromise with different synthetic data, but also the 60,000 could be used. The visual inspection is not useful to discriminate, and a better test should be designed to calibrate and choose this parameter. For the moment we stick to the principle of better recovering the total mass.

6. Dealing with the signal over the ocean

There are two main ways to deal with the signal over the ocean:

1) using point-like mass over the sea far enough from the region of interest ,
2) erasing the whole signal outside the region of interest form a fixed distance onward.

Each solution has pros and cons, in any case it needs to be calibrated. One way is to use (again) the 5 sets of synthetic data.

6.1 Adding point-like masses over the sea

CSA (km)	A	B	C	D	E	GIA	GRACE
300	-95.2	-98.9	-96.8	-95.4	1.1	149.2	-146.2
350	-96.2	-99.3	-97.6	-96.6	1.4	151.3	-146.8
400	-97.2	-99.6	-98.0	-97.7	1.8	152.3	-148.5
450	-97.8	-99.8	-98.4	-98.5	2.4	152.5	-148.8
500	-98.2	-99.9	-98.6	-98.8	3.1	152.3	-147.1
550	-98.6	-100.0	-98.9	-99.4	4.1	152.0	-146.2
600	-98.8	-100.0	-99.0	-99.6	5.3	151.5	-144.2
650	-99.0	-100.0	-99.1	-99.8	6.7	150.8	-142.0
750	-99.3	-100.1	-99.3	-100.2	9.7	149.2	-137.3

Table 1: Values in Gt/yr for the synthetic data (A, B, C, D, E), for the GIA signal integrated over Antarctica, and for the GRACE trend. In the first column (DSA) represents the distance from the continent where the ocean point-like masses are placed.

The first way is described by a number indicating the distance (CSA) from the original solution area (the Antarctica grounded ice) in km, where the new Complementary Solution Area (point-like masses) over the sea is added. So we performed some tests using point-like masses over the ocean from 300 to 750 km. The results for the synth E (the leakage from the sea) immediately drop (from 54 Gt/yr) to less than 10 Gt/yr.

Form Table 1 is clear that adding a solution area over the sea makes the leakage from the sea drop and the closer are the point-like masses to the continent and the lower is the contamination from the sea (E). However the mass recovered over the continent become lower than expected if the point-like masses are too close to the continent. With the smoothing parameter at 80,000 the best compromises are 400, 450 and 500 km.

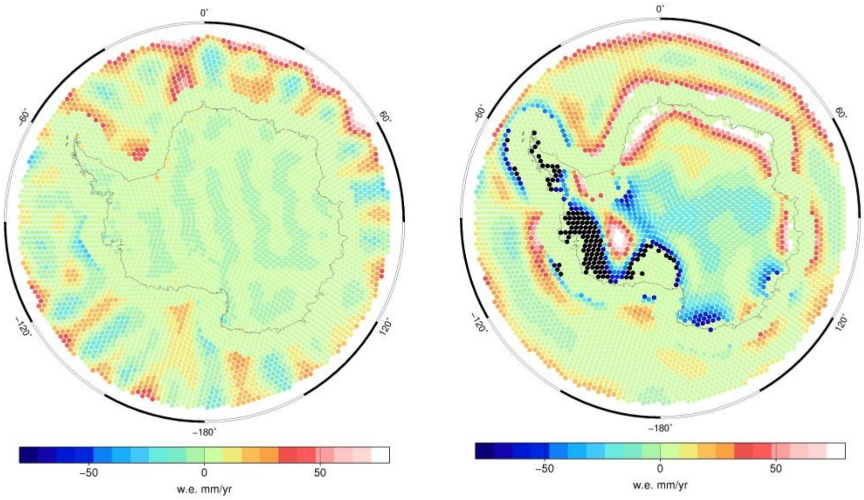

Figure 4: Left, the solution obtained with synth E using point-like masses also over the sea from 450 km. Right the solution obtained in the same way on GRACE trend.
Notice that, when using the observed data, the big positive spot in front of PIG is reproduced in the point-like masses over the sea.

6.2 Forcing to zero the signal over the sea

The second way is described by a number indicating the distance (MSK) from the continent from where a Zero Mask is used to set to zero the ocean signal. In detail the process consists in converting the harmonic coefficients (SH) in water equivalent representation (WE), apply the mask, and transform back in harmonic coefficients (new SH). These new SH are then used with the inversion method as before.

Also here, from Table 2 is clear that forcing to zero the signal over the sea make the leakage from the sea to drop and the closer is the mask to the continent and the lower is the contamination from the sea (E), which become even negative. This fact along with the other results for synth A, B, C, indicates an overestimate caused by masks to close to the continent (300 and 350 km).

Table 2: Values in Gt/yr for the synthetic data (A, B, C, D, E), for the GIA signal integrated over Antarctica, and for the GRACE trend. In the first column (MSK) represents the distance from the continent where the ocean signal is forced to be zero.

MSK (km)	A	B	C	D	E	GIA	GRACE
300	-100.4	-100.5	-100.1	-103.9	-1.1	158.2	-156.7
350	-100.4	-100.4	-100.4	-104.3	-0.3	157.9	-151.6
400	-99.9	-100.1	-100.1	-103.9	0.7	157.4	-149.4
500	-99.2	-99.8	-99.5	-103.4	4.0	153.3	-137.8

Probably, when using the observed data, the sudden drop of mass loss (from -149 to -138) could be due to the big positive spot in front of PIG that when using the mask from 400 km is erased, but it is not when using the mask from 500 km. So with the smoothing parameter at 80,000 the best compromise seems to be the mask at 400 km.

Note that in all this cases the synth D gives an overestimate of the mass loss and the reason is that the signal from the narrow band left around Antarctica (which is negative in this case) is still leaking into the continent.

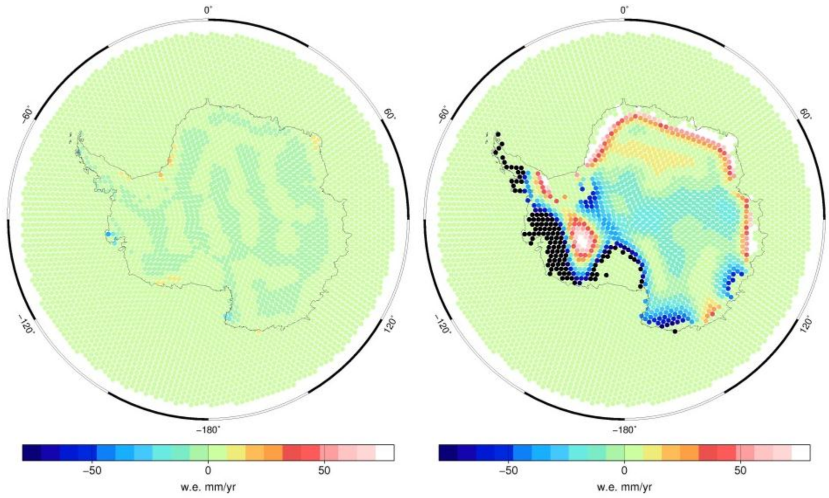

Figure 5: Left, the solution obtained with synth E erasing signal over the sea from 400 km. Right the solution obtained in the same way on GRACE trend.

7. Combining strategies.

So adding point-like masses over the sea makes difficult to choose the best compromise, and using a mask to force the ocean signal to zero depends too much on the features present in the observed data and not accounted for in the synthetic data. In fact the first strategy to recover all the mass over the continent (synth A, B, C) would need sea point-like masses far from the continent, while preventing leakage from the sea (synth E) would need sea point-like masses very close. Even if in theory can better control and reduce the leakage from the whole sea (synth E), the second strategy cannot account for sea closely surrounding the continent.

I tested also some combination of the two strategies to deal with the signal from the sea, to see if they could enforce their pros and cancel each other their cons.

From Table 3 it is clear that by using the combination of the two strategies above the leakage from the sea is much better controlled (synth D and E). The solution with DSA=500 and MSK between 400 and 500 looks particularly fine. From visual inspection (Figure 6) it is clear also that part of the signal of the Antarctic Peninsula and Pine Island is still also in the sea point-like masses. This could be also inferred from the results of synth A and C where a 1% is still missing.

Table 3: The meaning of the column are as in Table 1 and Table 2

CSA (km)	MSK (km)	A	B	C	D	E	GIA	GRACE
500	400	-98.5	-100.0	-98.8	-101.6	0.0	157.2	-152.2
500	450	-98.4	-99.9	-98.8	-101.5	0.2	157.2	-151.7
500	500	-98.2	-99.9	-98.7	-101.4	0.8	156.6	-149.8
550	400	-98.9	-100.1	-99.1	-102.2	0.1	157.7	-153.1
650	500	-99.0	-100.0	-99.2	-102.5	1.7	156.5	-148.2
700	500	-99.1	-100.0	-99.3	-102.8	2.0	156.4	-147.8
750	450	-99.4	-100.1	-99.6	-103.1	0.9	157.7	-151.0
750	500	-99.2	-100.0	-99.4	-102.9	2.3	156.1	-146.6

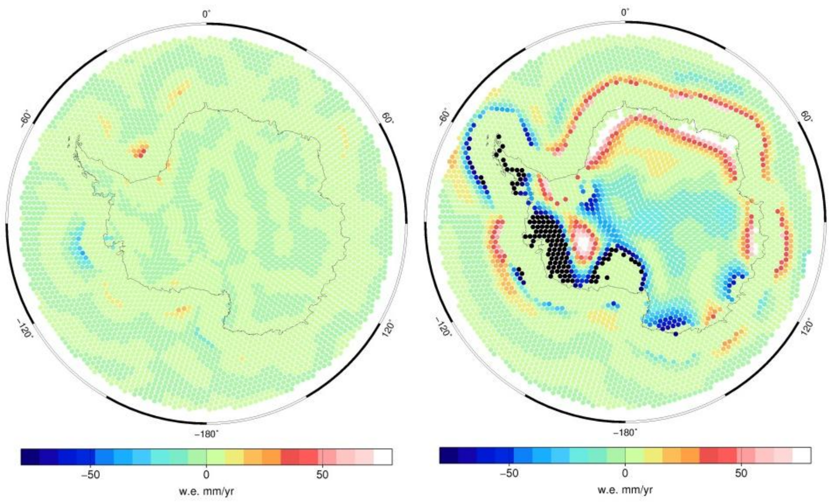

Figure 6: Left, the solution obtained with synth E with combined strategies (DSA=500 MSK=500). Right the solution obtained in the same way on GRACE trend.

8. Conclusions

The inversion method performs well especially when using suitable strategies to take into account the leakage from the sea. The calibration procedure shows how the variations of the masking (MSK) in the pre-processing and the distance of the Complementary Solution Area (CSA) affect the final results. However once one or both this leakage control are included in the processing (with an initial configuration reflecting the resolution of the data, i.e. from 250 to 400 km), the synthetic solution recover the correct amount within a range of less than 15\% error, i.e. variations in the parameters of the solution area have small effect on final results.

We have used different synthetic data sets, like uniform mass loss over the region of interest, mass loss with spatial distribution extracted from the real data, masked and scaled to have a fixed mass loss over the region of interest and zero in the rest of the world. We used the synthetic with zero mass loss over the region of interest and a small signal outside mimicking the ocean signal, and we also used pattern mimicking a known GIA correction.

However, the calibration could indeed change if it was optimized to deal only with GIA patterns of different amplitude (e.g. dependent from viscosity), but we find an optimal compromise in order to retrieve present day melting rather than the GIA correction.

After some tests and calibration we conclude that one of the best strategies is the combined one (CSA=500, MSK=500, SM=80,000) for Antarctica.