Supplement of The Cryosphere, 20, 4327–4344, 2026
https://doi.org/10.5194/tc-20-4327-2026-supplement

The Cryosphere
Open Access

Supplement of

A State-Space Model for Monitoring Greenland Ice Sheet Surface Elevation Change from CryoSat-2

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Supplementary Material S1: State-space model for gridded surface elevation change

Description

This section describes the workflow used to estimate gridded surface elevation change from point observations using a spatio-temporal state-space model. The approach combines spatial correlations and temporal dynamics within a unified statistical framework and is implemented using maximum likelihood estimation.

The analysis begins by reading the study-area boundary and the set of geolocated elevation observations. A regular grid is constructed over the spatial extent, and only grid cells located within the study polygon are retained. A neighborhood structure is then defined for all valid grid cells using a four-neighbour configuration, which forms the basis for representing spatial dependencies.

A spatial precision matrix is constructed from this neighborhood structure, allowing the model to represent spatial correlations through a Gaussian Markov random field. Each observation is subsequently assigned to a grid cell based on its coordinates, and observation times are re-indexed into consecutive time steps.

The elevation-change field is defined over all grid cells and time steps. Temporal evolution is modeled using a first-order autoregressive process, while spatial dependence is enforced through the neighborhood-based precision matrix. Together, this results in a separable spatio-temporal model that captures both spatial smoothness and temporal continuity.

Observations are linked to the field through a Gaussian observation model, in which measured elevations are represented as the sum of a spatially and temporally varying signal, a constant mean term, and observation noise.

Model parameters and the field are estimated simultaneously by minimizing the negative log-likelihood using an automatic differentiation framework. The field is treated as a random effect, enabling efficient estimation in high-dimensional settings. Uncertainty estimates are obtained from the inverse Hessian of the fitted model.

Finally, the estimated elevation-change field, associated uncertainties, grid coordinates, and observation-to-grid mappings are exported for further analysis and visualization.

Algorithm S1: Estimation of gridded surface elevation change

Input: Study-area boundary, observation coordinates, elevation measurements, observation times, grid spacing

Output: Gridded elevation-change field, uncertainty estimates, grid coordinates, and grid

Procedure: Procedure: A schematic overview of the procedure is shown in Figure S1.

1. **Read inputs and create analysis grid:** Load the study-area boundary and point observations, construct a regular grid over the spatial extent, and retain only cells located within the study polygon.

2. **Define neighborhood structure:** Establish a four-neighbour adjacency structure (up, down, left, right) between all valid grid cells.

3. **Construct spatial precision matrix:** Build a sparse precision matrix from the neighborhood structure to represent spatial connectivity.

4. **Assign observations to grid cells:** Map each observation to its corresponding grid cell based on spatial location.

5. **Define state-space model:** Represent the elevation-change field as a spatio-temporal process defined over all grid cells and time steps.

6. **Model spatial dependencies:** Represent spatial correlations using a Gaussian Markov random field defined by the precision matrix.

7. **Model temporal evolution:** Describe temporal dynamics using a first-order autoregressive process.

8. **Link observations to the field:** Relate measured elevations to the underlying spatio-temporal field through a Gaussian observation model with additive noise.

9. **Estimate model and uncertainties:** Estimate model parameters and the spatio-temporal field using maximum likelihood, and derive associated uncertainty estimates.

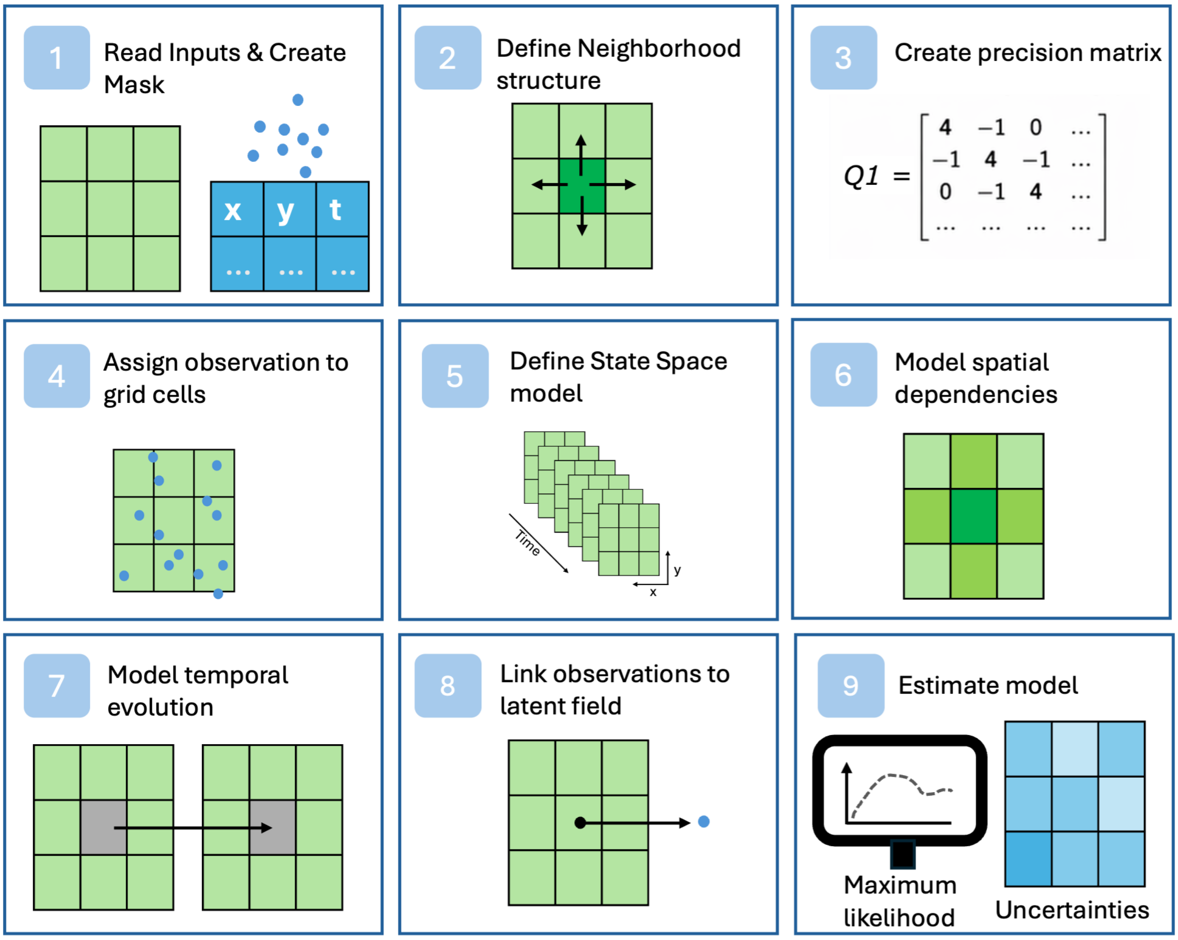

Figure S1: Schematic overview of the state-space modeling framework used to estimate gridded surface elevation change. Observations are mapped onto a regular grid, where spatial dependencies are represented through a neighborhood-based Gaussian Markov random field and temporal evolution is modeled using an autoregressive process. The combined spatio-temporal model is fitted using maximum likelihood, producing continuous fields of elevation change and associated uncertainty.

Supplementary Material S2: Product intercomparison

This section provides additional spatial and temporal comparisons between the dSEC product and independent surface elevation change products from ATL15, Khan et al. (2025), and Zhang et al. (2022). These analyses complement the product intercomparison presented in the main manuscript and provide further insight into product agreement across Greenland.

S2.1 Trend comparison

Figure S2 presents maps of mean elevation-change rates derived from dSEC, ATL15, Khan et al. (2025), and Zhang et al. (2022), together with maps showing pairwise differences relative to dSEC. All products were compared over their overlapping time periods.

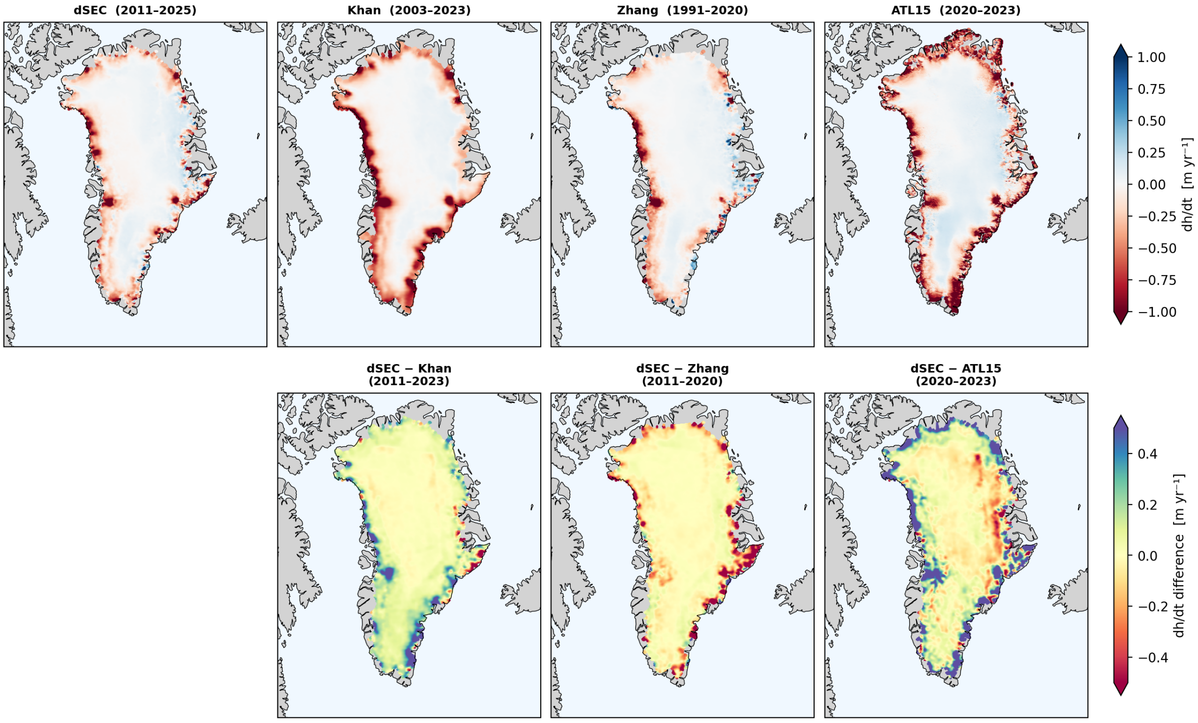

Figure S2: Comparison of mean elevation-change rates and corresponding difference maps.

S2.2 Seasonal amplitude comparison

Figure S3 compares the annual seasonal amplitude of the monthly elevation-change time series.

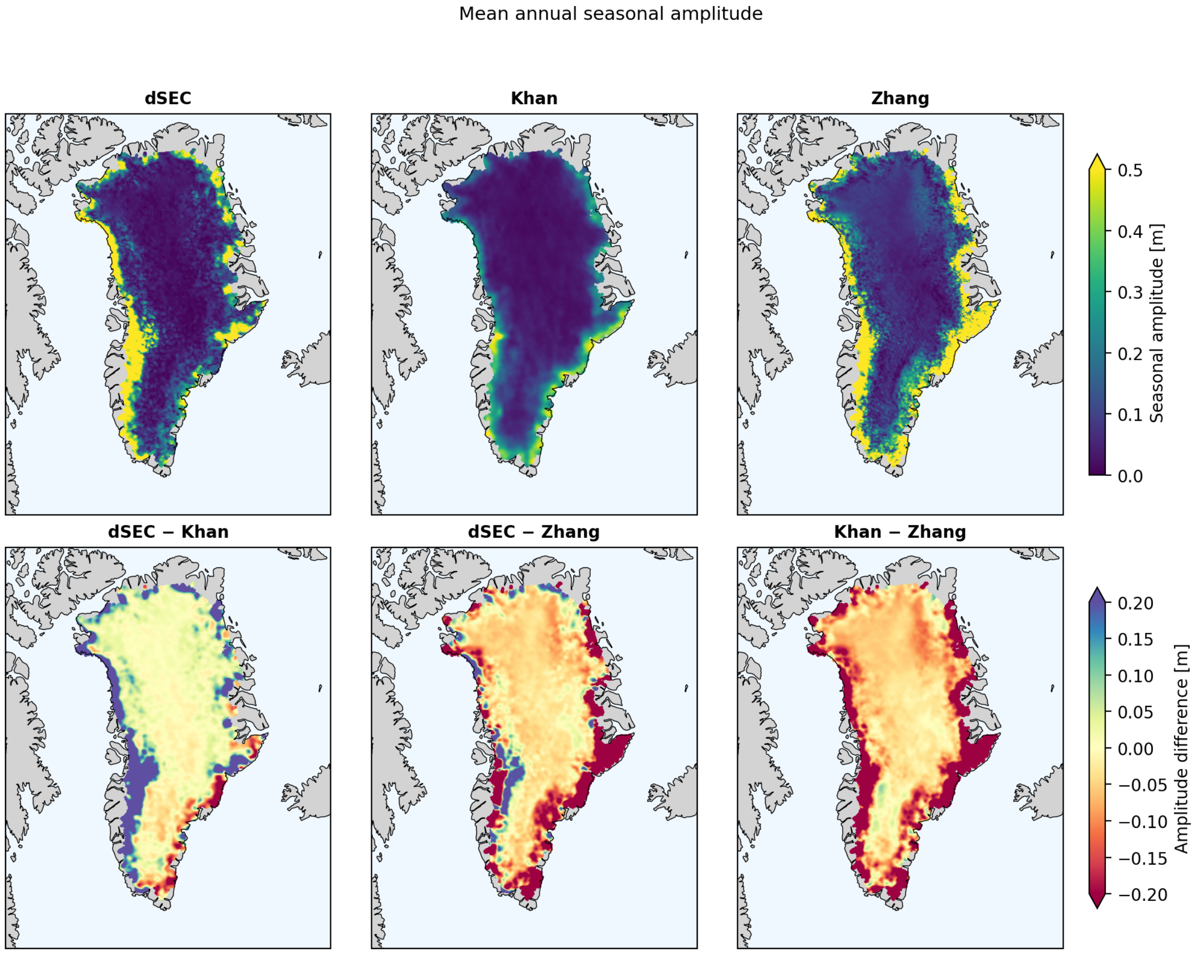

Figure S3: Comparison of annual seasonal amplitudes and pairwise differences between products.

S2.3 Representative time series

Figure S4 shows elevation-change time series at six selected locations across Greenland, including major outlet glaciers and an interior ice-sheet site. These examples provide a direct comparison of temporal behaviour between products.

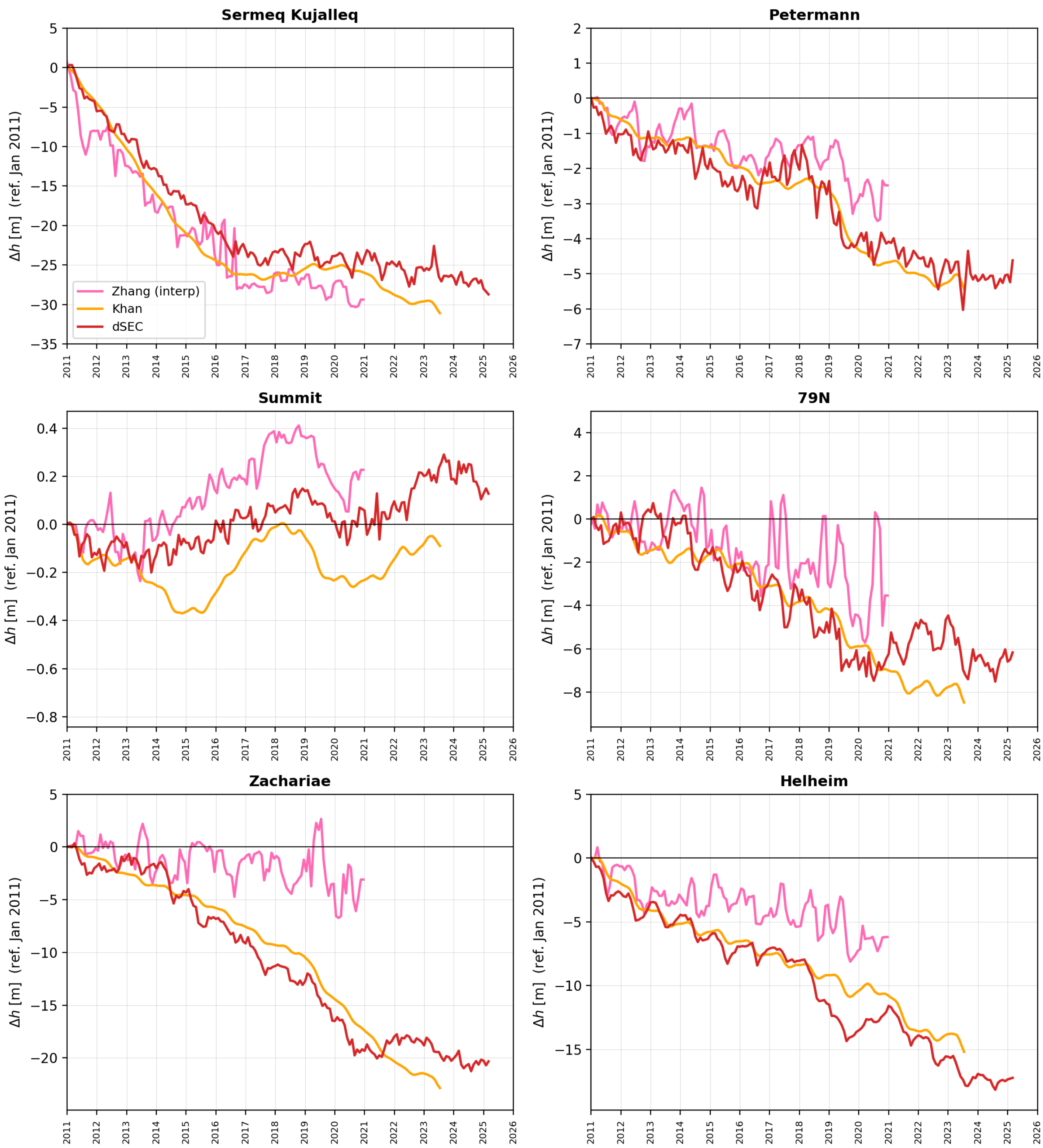

Figure S4: Elevation-change time series at six representative locations across Greenland.

Supplementary Material S3: AWS validation

The AWS comparison provides an independent point-scale evaluation of temporal variability at selected locations. The stations are not intended to be representative of the spatial variability of the Greenland Ice Sheet as a whole.

S3.1 Correlation analysis

Figure S5 presents the correlation analysis between dSEC and AWS observations for the three validation sites discussed in the main manuscript.

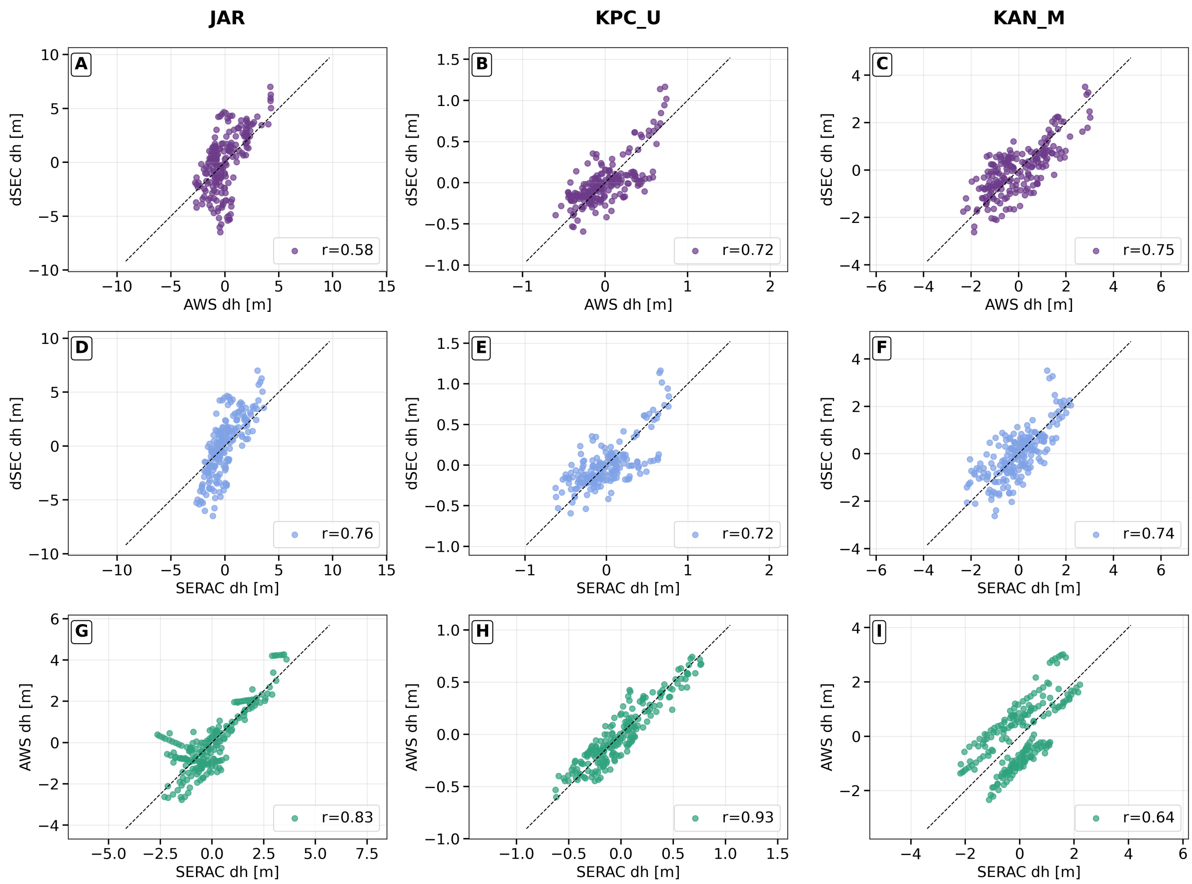

Figure S5: Correlation between AWS observations and dSEC-derived elevation change.