Supplement of “Simulating liquid water distribution at the pore scale in snow: water retention curves and effective transport properties”

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The Geodict software was used to estimate the 3D tensors of two key transport properties of wet snow at various water content: the unsaturated intrinsic water permeability $\mathbf{K}_w^{\mathrm{u}}$ in m^2, the unsaturated effective thermal conductivity $\mathbf{k}^{\mathrm{u}}$ in W m^{-1} K^{-1} and the unsaturated effective water vapor diffusivity $\mathbf{D}^{\mathrm{u}}$ in m^2 s^{-1}. For each property, a specific boundary value problem, arising from the homogenization technique is solved on the 3D images constituted of 3 phases (ice, air and water) and applying periodic boundary conditions on the external boundaries of each volume. Within the representative elementary volume (REV) of snow noted Ω, Ω_i, Ω_w and Ω_a are the domains occupied by the ice, the water and the air respectively, whereas $\Gamma_{\alpha\beta}$ is the interface between the phases α and β.

1 Intrinsic water permeability $\mathbf{K}_w^{\mathrm{u}}$

For a given volume fraction of water, the water permeability tensor is computed as

$$\mathbf{K}_w^{\mathrm{u}} = \langle \mathbf{k_w} \rangle \tag{S1}$$

where $\mathbf{k_w}$ is a second order tensor solution of the following boundary value problem over the water phase Ω_w.

$$\mu_w \triangle \mathbf{k}_w - \mathbf{grad}\ \mathbf{b} - \mathbf{I} = 0 \quad \text{in } \Omega_w \tag{S2}$$

$$\mathrm{div}\ \mathbf{k}_w = 0 \quad \text{in } \Omega_w \tag{S3}$$

$$\mathbf{k}_w = 0 \quad \text{on } \Gamma_{ia} \text{ and } \Gamma_{iw}. \tag{S4}$$

2 Unsaturated effective thermal conductivity $\mathbf{k}^{\mathrm{u}}$

The unsaturated thermal conductivity is defined as follows

$$\mathbf{k}^{\mathrm{u}} = \frac{1}{\Omega}\left(\int\limits_{\Omega_a} k_a(\mathbf{grad t}_a + \mathbf{I})\,\mathrm{d}\Omega + \int\limits_{\Omega_i} k_i(\mathbf{grad t}_i + \mathbf{I})\,\mathrm{d}\Omega + \int\limits_{\Omega_w} k_w(\mathbf{grad t}_w + \mathbf{I})\,\mathrm{d}\Omega\right) \tag{S5}$$

where k_i, k_a and k_w are the ice, air and water thermal conductivities taken as constants. The vectors $\mathbf{t}_i$, $\mathbf{t}_a$ and $\mathbf{t}_w$ are periodic vectors which characterize the fluctuation of temperature in both phases at the pore scale. These three vectors are solution of the following boundary value problem over the REV in a compact form:

$$\mathrm{div}(k_i(\mathbf{grad t}_i + \mathbf{I})) = 0 \quad \text{in } \Omega_i \tag{S6}$$

$$\mathrm{div}(k_a(\mathbf{grad t}_a + \mathbf{I})) = 0 \quad \text{in } \Omega_a \tag{S7}$$

$$\mathrm{div}(k_a(\mathbf{grad t}_a + \mathbf{I})) = 0 \quad \text{in } \Omega_w \tag{S8}$$

$$\mathbf{t}_i = \mathbf{t}_a \quad \text{on } \Gamma_{ai} \tag{S9}$$

$$\mathbf{t}_i = \mathbf{t}_w \quad \text{on } \Gamma_{iw} \tag{S10}$$

$$\mathbf{t}_w = \mathbf{t}_a \quad \text{on } \Gamma_{aw} \tag{S11}$$

$$(k_i(\mathbf{grad}\ \mathbf{t}_i + \mathbf{I}) - k_a(\mathbf{grad}\ \mathbf{t}_a + \mathbf{I}))\cdot\mathbf{n}_{ai} = 0 \quad \text{on } \Gamma_{ai} \tag{S12}$$

$$(k_i(\mathbf{grad}\ \mathbf{t}_i+\mathbf{I})-k_w(\mathbf{grad}\ \mathbf{t}_w+\mathbf{I}))\cdot\mathbf{n}_{iw}=0\quad\text{on }\Gamma_{iw}\tag{S13}$$

$$(k_w(\mathbf{grad}\ \mathbf{t}_w+\mathbf{I})-k_a(\mathbf{grad}\ \mathbf{t}_a+\mathbf{I}))\cdot\mathbf{n}_{aw}=0\quad\text{on }\Gamma_{aw}\tag{S14}$$

$$\frac{1}{\Omega}\int_{\Omega}(\mathbf{t}_{\mathrm{a}}+\mathbf{t}_{\mathrm{i}}+\mathbf{t}_{\mathrm{w}})\,\mathrm{d}\Omega=\mathbf{0}\tag{S15}$$

This latter equation is introduced to ensure the uniqueness of the solution.

3 Unsaturated effective diffusivity $\mathbf{D}^{\mathbf{u}}$

The effective unsaturated effective diffusivity is computed as:

$$\mathbf{D}^{\mathrm{u}}=\frac{1}{\Omega}\int_{\Omega_a}D_v(\mathbf{grad}\ \mathbf{g}_v+\mathbf{I})\,\mathrm{d}\Omega\tag{S16}$$

where D_v is the water vapor diffusion coefficient in air taken as constant. The vector $\mathbf{g}_v$ is a periodic vector which characterizes the fluctuation of water vapor density in the air phase at the pore scale. This vector is solution of the following boundary value problem over the REV in a compact form:

$$\mathrm{div}D_v^*(\mathbf{grad}\mathbf{g}_v+\mathbf{I})=0\quad\text{in }\Omega_a\tag{S17}$$

$$D_v(\mathbf{grad}\mathbf{g}_v+\mathbf{I})\cdot\mathbf{n}_{ai}=0\quad\text{on }\Gamma_{ai}\tag{S18}$$

$$D_v(\mathbf{grad}\mathbf{g}_v+\mathbf{I})\cdot\mathbf{n}_{aw}=0\quad\text{on }\Gamma_{aw}\tag{S19}$$

$$\frac{1}{\Omega}\int_{\Omega_a} \mathbf{g}_v \mathrm{d}\Omega = \mathbf{0} \tag{S20}$$

This latter equation is introduced to ensure the uniqueness of the solution.