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Competing processes determine the long-term impact of basal friction parameterizations for Antarctic mass loss

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Rewriting of the basal friction parameterizations

For Eq 1.2, the rewriting is straight-forward:

$$C_p = \frac{C_c N \left(\frac{u_b}{u_b + u_0}\right)^m}{u_b^m} \tag{S1}$$

For the Pseudoplastic law, a similar method is used:

$$C_{cp} = \frac{C_c \left(\frac{u_b}{u_b + u_0}\right)^m}{\left(\frac{u_b}{u_0}\right)^{\frac{1}{pp}}} \tag{S2}$$

Here, C_c is the inverted free parameter in Eq 1.4, and C_p the constant needed in Eq 1.2, C_{cp} the constant needed in Eq. 1.3. Rewriting Eq. 1.3 is less straight-forward, first we define:

$$\beta = \frac{\tau_{b,Reg}}{u_b} = C_c N \frac{u_b^{m-1}}{(u_b + u_0)^m} \tag{S3}$$

Eq 1.6 has two free parameters, C_p and C_c. The former controls the strength of the Weertman style friction, the latter controls the strength of the coulomb style friction. Since we inverted for C_c in Eq 1.12 in the initialization procedures, we use the inferred value to compute a C_p so that Eq 1.2. gives the same basal friction at the end of the initialization as Eq 1.1:

$$C_p = \left(\frac{\alpha}{1 - \gamma}\right)^{\frac{1}{m}} \tag{S4}$$

With:

$$\alpha = \left(\frac{\beta}{u_b^{m-1}}\right)^{\frac{1}{m}} \tag{S5}$$

And:

$$\gamma = \left(\frac{\beta}{C_c N u_b^{m-1}}\right)^{\frac{1}{m}} u_b \tag{S6}$$

Results – Buttressing number

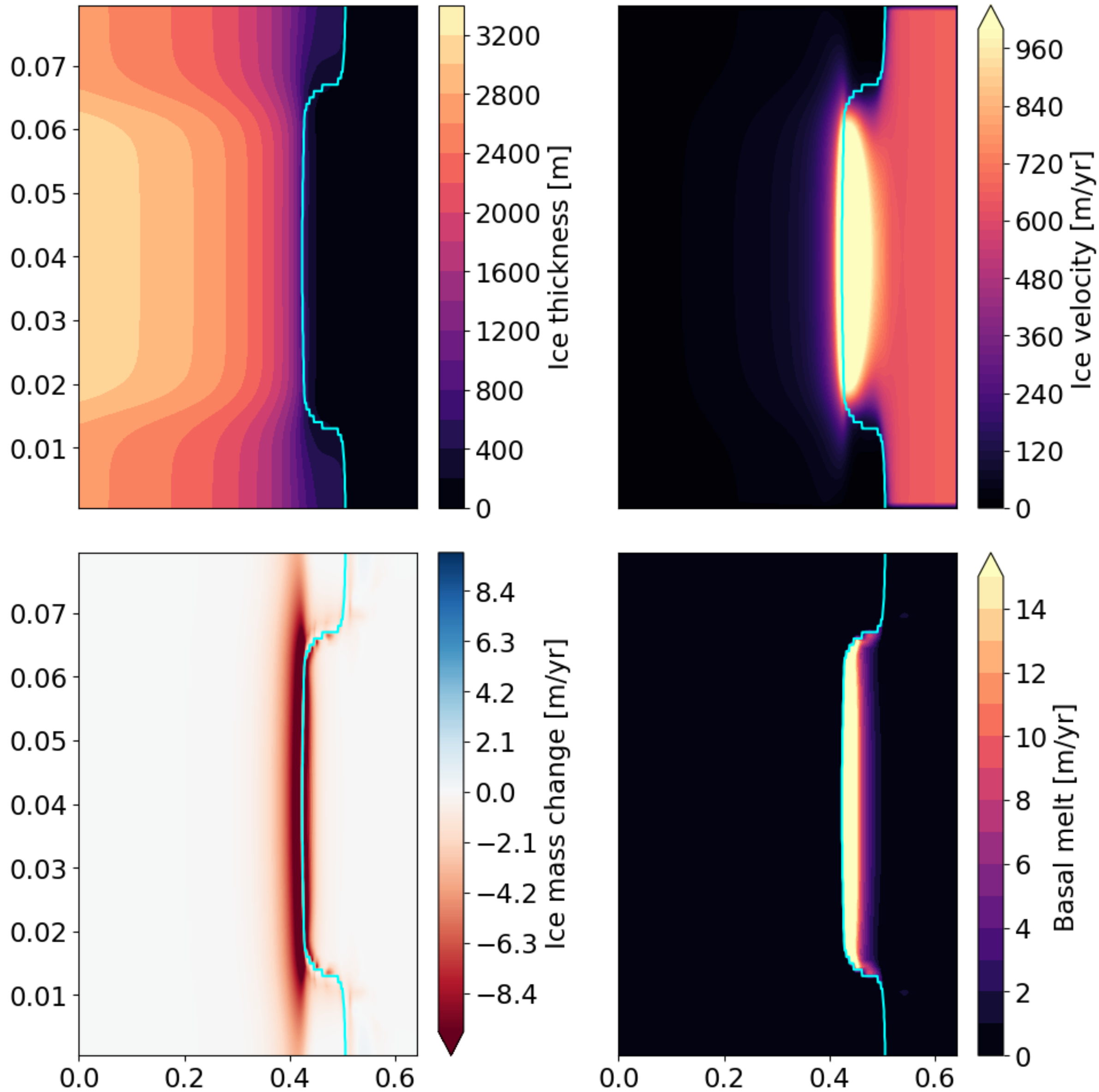


Fig S1 Characteristics of the modelled state of the MISMIP+ Ice 1r experiment after 50 years. (Top left) Modelled ice thickness, (top right) modelled ice surface velocity, (bottom left) ice mass change and (bottom right) basal melt rates. The modelled grounding line is shown in cyan. The x and y coordinates are fictive.

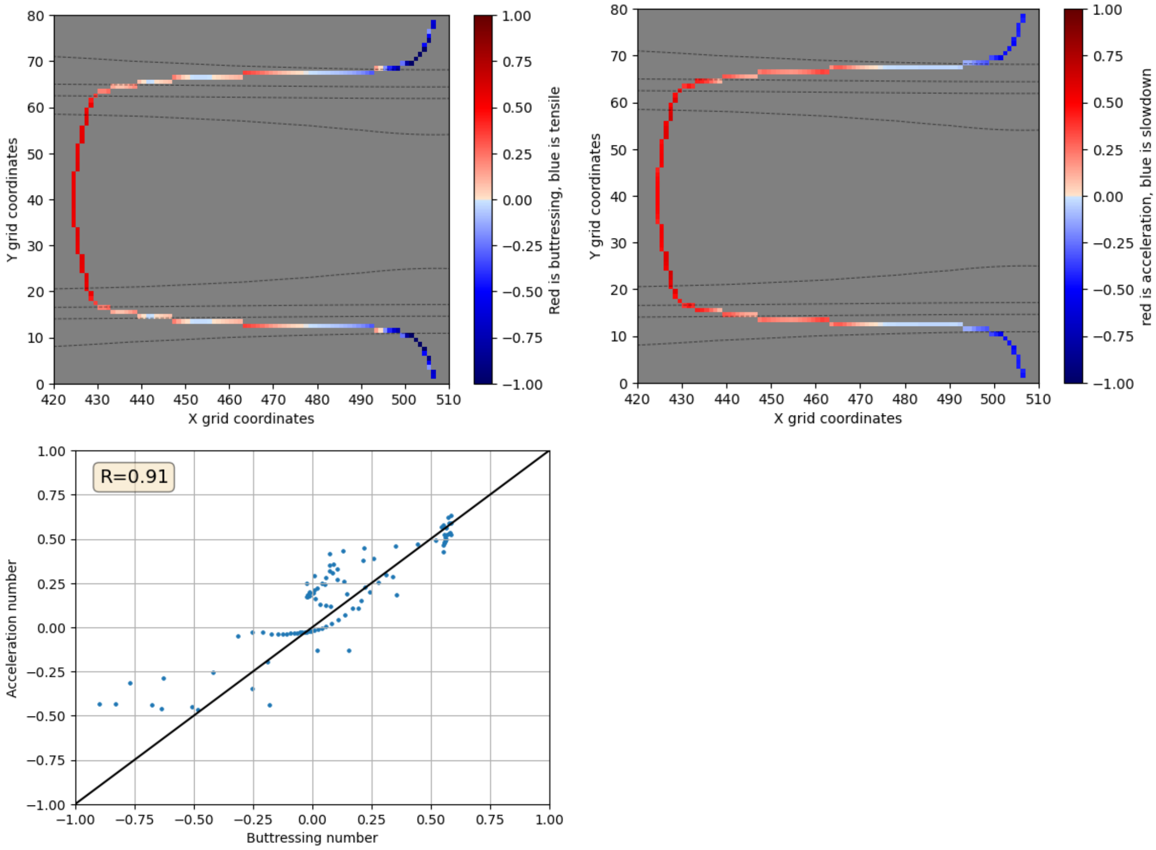
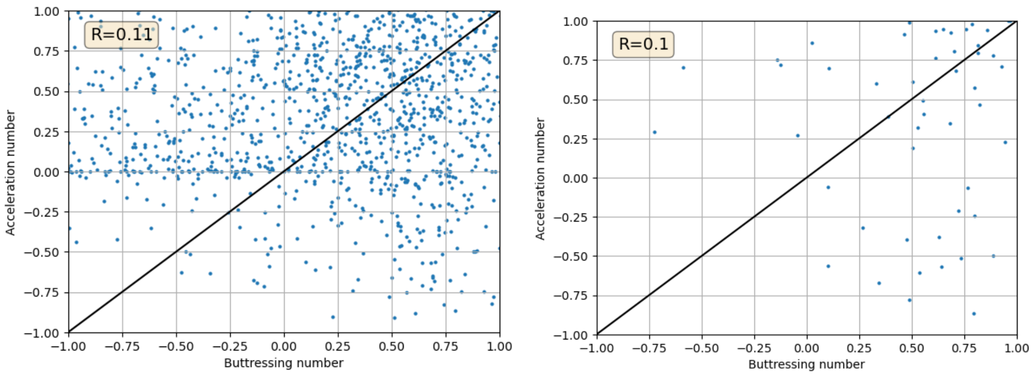


Fig S2 buttressing number and the acceleration number of the MISMIP+ Ice1r experiment. (Top row, left) the buttressing number at the grounding line after 40 years of the MISMIP+ Ice1r experiment. (Top row, right) the acceleration at the grounding line when the shelf is instantly removed. Black dashed lines represent the bedrock topography, from -750 meters in the centre of the plot with increments of 150 meters in the directions of the lateral margins. (Bottom row) correlation plot between the acceleration and buttressing number, 208 data points are used.



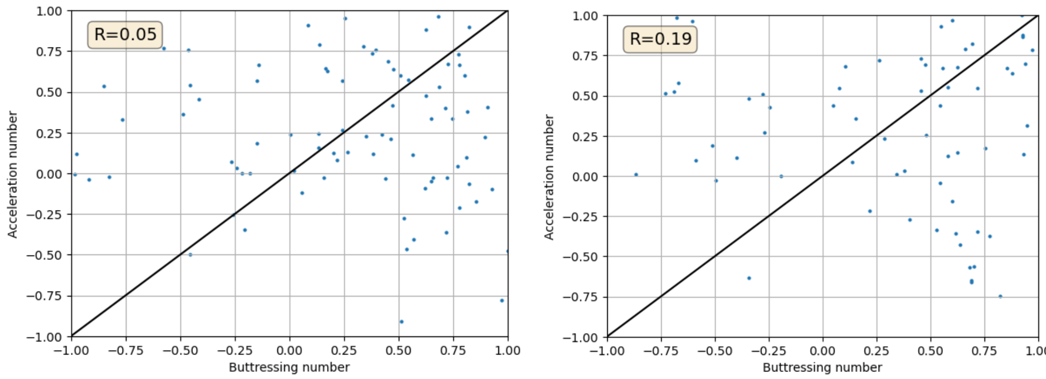


Fig S3. Acceleration number and buttressing number for (top,left) the whole Antarctic ice sheet (top, right) ASE region (bottom, left) the Ross shelf and (bottom, right) the Filchner-Ronne.

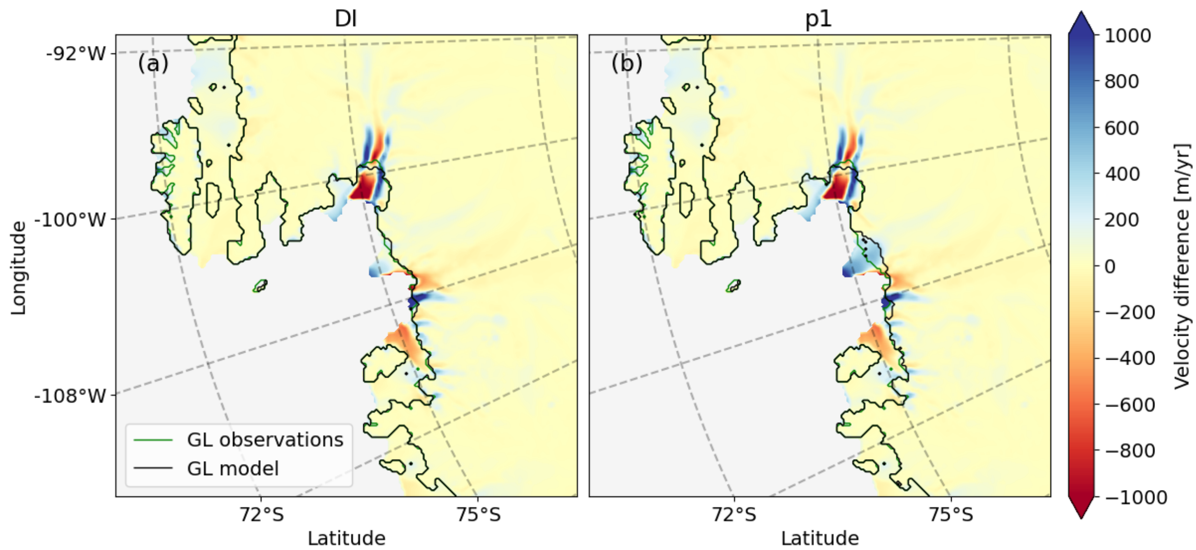


Figure S4. Ice surface velocity misfit for P05 (left) and P1 (right) at the end of initialization. Misfit is shown as model minus observations (of Rignot et al. (2011)). The modelled grounding line is shown in black, and the observed grounding line in green.

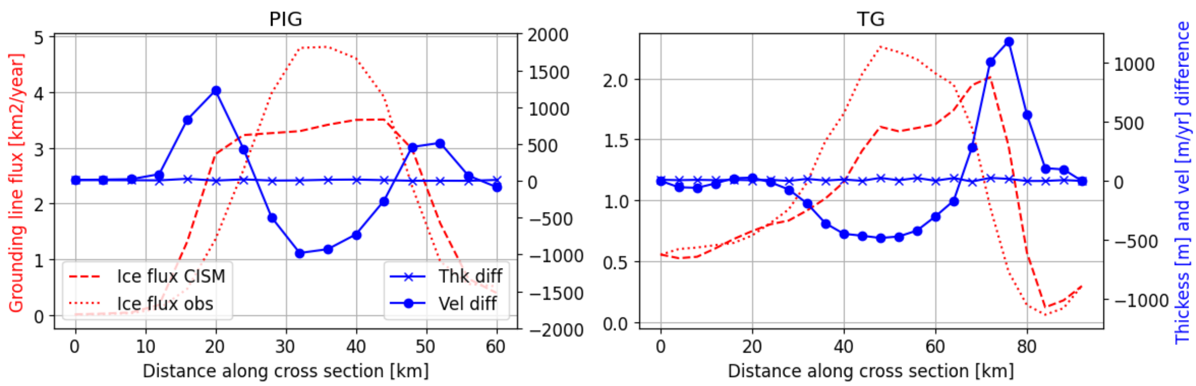


Figure S5. Grounding line fluxes (red, dashed: CISM; dotted: observations), thickness differences (blue crosses) and ice surface velocity differences wrt observations (blue dots) of PIG (left) and TG (right) for the default initialization P05. The integrated grounding line fluxes represented as ice velocity times the ice thickness (modelled/observed) are respectively for PIG 30.4/31.1 km2 yr-1, and for TG 24.5/26.1 km2/yr.

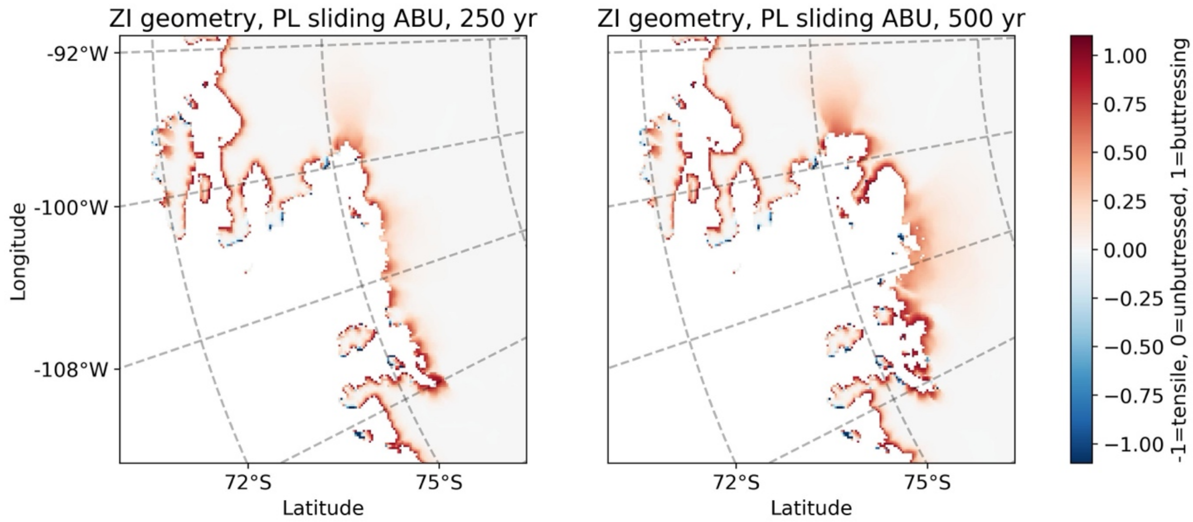


Figure S6 Acceleration number of the ABUMIP experiments with a sudden switch from ZL to PL after 250 (left) and 500 (right) years.

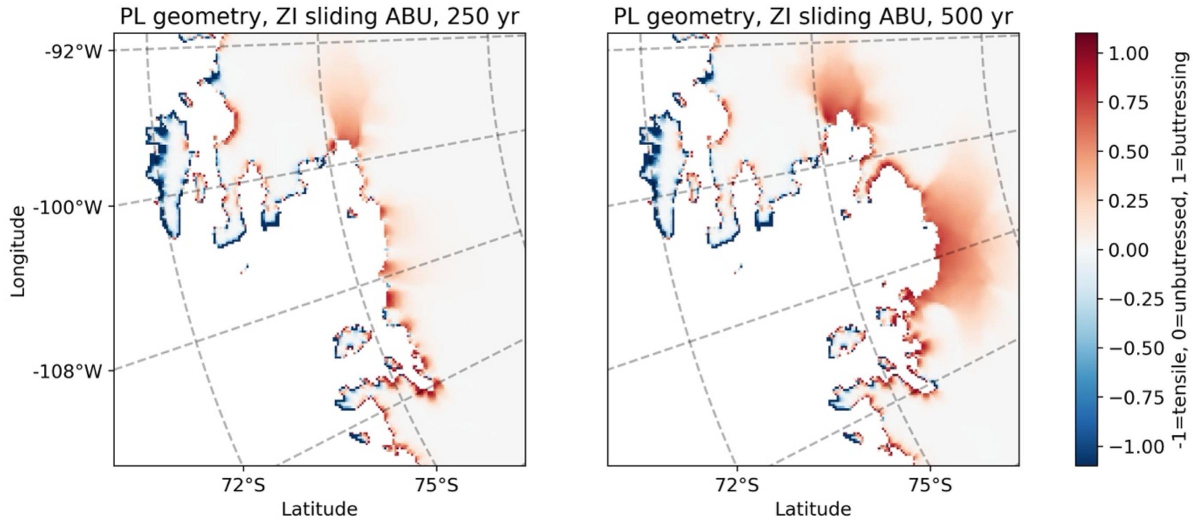


Figure S7 Acceleration number of the ABUMIP experiments with a sudden switch from ZL to PL after 250 (left) and 500 (right) years.

Table S1. Variables used in this study.

Variables	Definition
b	Bedrock height [m asl]
bmlt	Basal melt rate under floating ice [m/yr]
C_c	Coulomb C [unitless]
C_p	Power law C [Pa $m^{1/m}$ $yr^{-1/m}$]

C_r	Basal friction relaxation target [unitless]
E	Flow enhancement factor [unitless]
E_r	Flow enhancement factor relaxation target [unitless]
H	Modelled ice thickness [m]
H_f	Ice thickness above floatation [m]
H_{obs}	Observed ice thickness [m]
N	Effective pressure [Pa]
R	Resistive stress tensor [Pa]
R_n	Normal resistive stress tensor at grounding line [Pa]
R_0	Resistive stress tensor without ice [Pa]
TF_{base}	Thermal forcing applied at the ice shelf draft [K]
u	Ice velocity in the x-direction [m/yr]
v	Ice velocity in the y-direction [m/yr]
u_b	Basal velocity magnitude [m/yr]
$u_{x,b}$	Basal velocity in the x-direction [m/yr]
$u_{y,b}$	Basal velocity in the y-direction [m/yr]
u_s	Surface ice velocity magnitude [m/yr]
$u_{s,obs}$	Observed surface ice velocity magnitude [m/yr]
δT	Ocean temperature correction [K]
s	Surface elevation [m]
β	Basal traction parameter [Pa yr m^{-1}]
ϕ	Till friction angle [degrees]
η	Effective viscosity [Pa yr]
θ	Buttressing fraction [unitless]
τ_b	Basal shear stress [Pa]
τ_{ij}	Deviatoric stress tensor [Pa]
χ	Buttressing number [unitless]
χ_f	Stress boundary condition at the calving front [Pa]

Table S2. Parameters and their units and values used in this study

Parameters	Values	Units	Definition
c_{pw}	3974	J kg^{-1} K^{-1}	Specific heat of seawater
g	9.81	m s^{-2}	Gravitational acceleration
H_0	100	m	Ice thickness scale in the inversion
L_f	3.34 x 105	J kg^{-1}	Latent heat of fusion
m	3	-	Basal friction exponent
pp	5	-	Basal friction exponent, pseudoplastic law
T_r	0	K	Relaxation target of the ocean temperature inversion

u_0	200	m yr^{-1}	Constant ice speed in the Zoet-Iverson and pseudoplastic laws
p	0.5	-	Exponent in effective pressure relation
r	0.5	-	Strength of inversion relaxation
ρ_i	917	kg m^{-3}	Density of ice
ρ_w	1027	kg m^{-3}	Density of ocean water
κ	100	yr	Time scale in the inversion
γ_0	30000	m yr^{-1}	Basal melt rate coefficient
A_0	1.733 x 10^{3} 3.613 x 10^{-13}	Pa^{-3} s^{-1}	Constant of proportionality for the rate factor (above 263 K),(below 263 K)
Q	139 x 10^{3}, 60 x 10^{3}	J mol^{-1}	Activation energy for ice creep (above 263 K), (below 263 K)
R	8.314	J mol^{-1} K^{-1}	Universal gas constant
Φ	9.8 x 10^{-8}	K Pa^{-1}	Pressure-dependent melt constant