



Supplement of

Brief communication: Representation of heat conduction into ice in marine ice shelf melt modelling

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On the validity of the steady state assumption for vertical advection

In a non-steady state, the ice velocity that matters is the velocity with respect to the moving ice–ocean interface, which can be expressed accounting for the ice-shelf floatation as:

$$w'_i = \frac{\rho_i - \rho_w}{\rho_w} m_{\text{steady}} - \frac{\rho_i}{\rho_w} m$$

where ρ_i and ρ_w are the ice and seawater densities, m_{steady} is the steady-state melt rate, i.e., the melt rate that would exactly balance the vertical ice advection, and m the actual (non-steady) melt rate (expressed in meters of ice per time unit). In steady state, this gives $w'_i = -m$, as assumed in approximation (C). The ice shelves of the Amundsen Sea, like Pine Island, Dotson and Getz, are not in steady state and the observational estimates of Davison et al. (2023) indicate $m \approx 3 m_{\text{steady}}$ over 1997–2021. This gives $w'_i \approx -0.93 m$, i.e., an error of $\sim 7\%$ in approximation (C). Obviously, the mismatch is more important in future projections with increasing melt rates, but even with $m \approx 10 m_{\text{steady}}$, which is unlikely for the Amundsen Sea, the error does not exceed 10%. The steady state assumption in approximation (C) therefore seems preferable to approximations (A) and (B) that give near-zero heat flux into the ice (Fig. 2), which is not consistent with the observational temperature profile in the Pine Island ice shelf (Fig. 1).

Things are obviously more complex near the grounding line of warm ice shelf cavities because the ice advected from upstream is not in thermal equilibrium. There are actually two time scales relevant for this: the time scale of vertical ice advection throughout the ice shelf thickness, and the time scale of vertical advection through the basal ice layer with a sharp thermal gradient (Fig. 1c). For typical values of the Amundsen Sea ice shelves*, the first time scale is several decades, which may be longer than the ice life time from the grounding line to the front, while the second is closer to 1 year. The first time scale is relevant for the slow temperature change of the ice interior, which is nearly uniform far from the ice base. This means that instead of temperature T_b in approximation (C), the ice temperature at depth would be more accurate, although it is difficult to estimate without an ice-sheet model that resolves heat advection. The second time scale of ~ 1 year means that approximation (C) is not very good within a few km from the grounding line, even if it depends on the thermal state at the ice base upstream of the grounding line. We also don't see any reason to believe that approximations (A) and (B) would be better than (C) near the grounding line.

* e.g. Pine Island: horizontal velocity of 3 km/yr, vertical velocity of 30 m/yr, ice-shelf thickness of 1000 m, and 30 m thickness for the basal layer of high thermal gradient.

Used Data

Figure 1:

- Amery Ice Shelf: Wang et al. (2022)
- Ross Ice Shelf: MacAyeal and Thomas (1979)

- Pine Island Ice Shelf: Truffer and Stanton (2015)

Figure 2:

- Code and data: <https://doi.org/10.17043/wiskandt-2024-sof-sill-1>

Figure 3:

- Antarctic Melt rates from Rignot et al. (2013)
- Antarctic surface temperature are 40-year (1980-2019) average snow surface temperatures from a regional climate simulation by Kittel et al. (2021)
- Ice shelf thickness from Morlighem et al. (2020)

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