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Frazil ice growth and production during katabatic wind events in the Ross Sea, Antarctica

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Section S1: Estimation of frazil ice concentration using temperature anomalies

To measure the amount of the temperature anomaly:

$\Delta T = T_{obs} - T_b$ (S1.1)

T_{obs} =in-situ conservative temperature within the anomaly (°C)
T_b =baseline or far field temperature (10 meter average below anomaly)(°C)
* $\Delta T = ℃$ = degrees K
substituted.

Heat content per volume of water can be quantified as Q and calculated (Talley et al 2011).

$$Q = \rho C_p^W T \qquad \text{(S1.2)}$$

ρ =seawater density (kg m^{-3})
C_p^W= specific heat capacity (J kg^{-1} K^{-1})
T= temperature of the water (degrees K)
Q= heat content per volume (J m^{-3})

To find the heat content in the temperature anomaly, or excess heat, equation S1.1 can be substituted into equation S2.2.

$$Q_{excess} = \rho C_p^W \Delta T \qquad \text{(S1.3)}$$

ΔT=amount of Temperature anomaly (degrees K)
Q_{excess} = excess heat content per volume (J m^{-3})

To find the total mass amount of heat in the water column, the integral of Q_{excess}^{total} is taken over the depth range of the anomaly (z_T).

$$Q_{excess}^{total} = \int_{z=0}^{z=z_T} \rho \ C_p^W \ \Delta T \ dz \qquad \text{(S1.4)}$$

z_T= depth of the temperature Anomaly (m)
Q_{excess}^{total}= total amount of residual heat in the water column (Jm^{-2})

The concentration of frazil ice is estimated by applying the Latent heat of formation as a conversion factor to the calculated internal energy (Q_{excess}^{total}):

$$C_{ice}^T = \frac{Q_{excesl}^{total}}{L_f \ z_T} \qquad \text{(S1.5)}$$

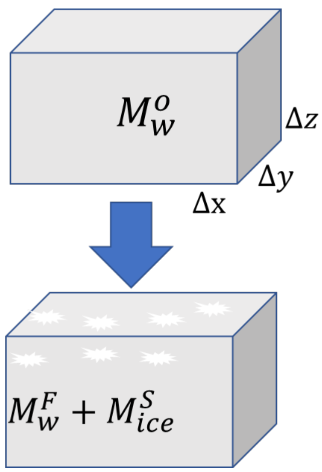
L_f = latent heat of fusion = 3.34 x 10^5 J kg^{-1}
z_T =depth of the temperature anomaly (m)
C^T_{ice}= mass concentration of frazil ice (kg m^{-3}) from temperature derivation

Table S1: Data for frazil ice concentration using temperature anomalies. Includes Baseline Temperature, Depth of the Temperature anomaly, Average Specific Heat Capacity (over the range of the anomaly), Residual heat, and Estimation of Mass concentration of Ice.

Station	T_b (°C)	z_T (m)	C^W_p (J kg^{-1} K^{-1})	Q^{total}_{excess} (KJ m^{-2})	C^T_{ice} (kg m^{-3})
25	-1.910	11.34	3988	183	48 x 10^{-3}
26	-1.912	24.73	3988	122	14 x 10^{-3}
27	-1.914	15.45	3988	115	22 x 10^{-3}
28	-1.915	15.52	3988	92	18 x 10^{-3}
29	-1.906	11.34	3989	82	22 x 10^{-3}
30	-1.916	8.24	3988	68	25 x 10^{-3}
32	-1.914	11.33	3988	121	32 x 10^{-3}
33*	-1.913	---	3988	---	---
34	-1.909	13.40	3988	42	9 x 10^{-3}
35	-1.910	19.58	3988	230	35 x 10^{-3}
40	-1.885	20.61	3991	233	33 x 10^{-3}

* Station 33 does not have a measurable temperature anomaly, but has a measurable salinity anomaly so it was included in this table. The specific heat capacity and density value shown are averages of the values used in the calculation. For each depth step of the integral, an individual value unique to that depth was used.

Section S2: Derivation of Conservation of Mass of Water and Conservation of Mass of Salt

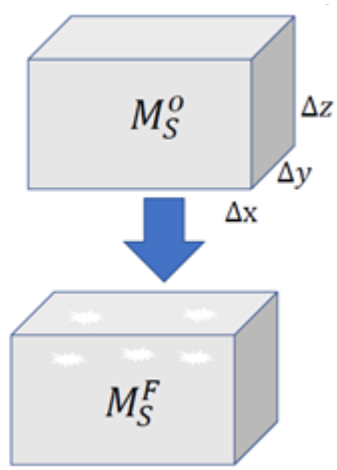


Conservation of Mass of Water:

$$M_W^O = M_W^F + M_{ice}^S \qquad \text{(S2.1)}$$

M_W^O =Mass of Water originally
M_W^F =Mass of Water after freezing
=Mass of Water as Ice

Figure S2.1: 1-D box model of the Conservation of Mass of Water.



Conservation of Mass of Salt:

$$M_S^O = M_S^F \qquad \text{(S2.2)}$$

Salinity Equations:

$$M_S^O = S_b \; M_W^O \qquad \text{(S2.3)}$$
$$M_S^F = S_{obs} \, M_W^F \qquad \text{(S2.4)}$$

M_S^O =Mass of Salt Initially
M_S^F =Mass of Salt, Final
S_b =Original/Baseline Salinity
S_{obs} = Salinity Final/Observed

Figure S2.2: 1-D box model of the Conservation of Mass of Salt.

Combine the Conservation of Mass of Salt and Salinity Equations, equations S2.2 and S2.3:

$$M_S^F = S_b \; M_O^W \qquad \text{(S2.5)}$$

Combine S2.5 with Conservation of the Mass of Water S2.1:

$$M_S^F = S_b(M_W^F + M_{ice}^S) = S_b \; M_W^F + S_b \; M_{ice}^S \qquad \text{(S2.6)}$$

Combine the Conservation of Mass of Water and the Conservation of Mass of Salt, equations S2.1 and S2.4:

$$M_s^F = S_{obs}\left(M_W^O - M_{ice}^S\right) = S_{obs}\ M_W^O - S_{obs}\ M_{ice}^S \qquad \text{(S2.7)}$$

Combine equations from S2.6 and S2.7:

$$S_b\ M_W^F + S_b\ M_{ice}^S = S_{obs}\ M_W^O - S_{obs}\ M_{ice}^S \qquad \text{(S2.8)}$$

Combine equations S2.1 and S2.8:

$$S_b\ \left(M_W^O - M_{ice}^S\right) + S_b\ M_{ice}^S = S_{obs}\ M_W^O - S_{obs}\ M_{ice}^S$$

$$S_b\ M_W^O - S_b\ M_{ice}^S + S_b\ M_{ice}^S = S_{obs}\ M_W^O - S_{obs}\ M_{ice}^S \qquad \text{(S2.9)}$$

Rearrange equation S2.9 to isolate, $M_W^O\ and\ M$:

$$M_W^O(S_b - S_{obs}) = M_{ice}^S\ (S_b - S_b - S_{obs}) \qquad \text{(S2.10)}$$

Solved equation S2.10 for M_{ice}^S:

$$M_{ice}^S = \frac{(S_{obs} - S_b)}{S_{obs}}\ M_W^O \qquad \text{(S2.11)}$$

Section S3: Estimation of frazil ice concentration using salinity anomalies

To measure the amount of the salinity anomaly:

$$\Delta S = S_{obs} - S_b \tag{S3.1}$$

$S_b =$ baseline or far field salinity (10 meter average below anomaly)(g kg^{-1})

S_{obs} = in-situ absolute salinity within the anomaly (g kg^{-1})

ΔS= salinity anomaly (g kg^{-1})

Equation S2.11 solves for the mass of water as ice (M_{ice}^{S})at each depth step of the profile.

$$M_{ice}^{S} = \frac{(S_{obs} - S_b)}{S_{obs}} M_W^O \tag{S3.2}$$

Substitute equation S3.1 into equation S3.2:

$$M_{ice}^{S} = \frac{\Delta S}{S_{obs}} M_W^O \tag{S3.2}$$

To find the total mass of frazil ice (M_{ice}^{S}) in the water column, equation S3.2 is solved for each depth step of the anomaly and then the integral is taken to find the total mass of ice. As shown in equation S3.4, the mass of water originally (M_W^O) uses the same assumed baseline density (ρ_b= 1028 kg m^{-3}) at each depth step. This allows for equation the salt ratio is taken at each step of the depth range of the anomaly and multiplied by the mass of water initially at that step. The integral is then taken of the entire depth range of the anomaly.

$$M_{ice}^{S} = \int_{z=0}^{z=z_S} \frac{\Delta S}{S_{obs}} M_W^O \tag{S3.3}$$

$$M_W^O = \rho_b \; dz \tag{S3.4}$$

$$M_{ice}^{S} = \rho_b \text{ x } \int_{z=0}^{z=z_S} \frac{\Delta S}{S_{obs}} dz \tag{S3.5}$$

z_S= depth of the Anomaly (m)

M_{ice}^{S}= total mass of frazil ice (kg m^{-2}) from salinity derivation

M_W^O= Mass of Water initially at each step of the integral (kg m^{-2})

ρ_b= 1028 kg m^{-3}=Assumed baseline/initial density, calculated using S_b.

$$C_{Ice}^{S} = \frac{M_{Ice}^{S}}{z_S} \tag{S3.6}$$

$Conc_{Ice}^{salt}$ = Concentration of frazil ice = kg m^{-3}

M_{ice}^{S}= total mass of frazil ice (kg m^{-2}) from salinity derivation

z_S= depth of the Anomaly (m)

Table S2: Data for frazil ice concentration using salinity anomalies. Includes Baseline Salinity, Depth of the salinity anomaly, mass of water assumed to be initially present, Estimation of Mass of Ice, and Concentration of Ice.

Station	S_b (g kg^{-1})	z_S (m)	M_{ice}^{S} (kg m^{-2})	C_{ice}^{S} (kg m^{-3})
25	34.861	13.40	0.898	67 x 10^{-3}
26	--	--		--
27	34.962	41.22	1.917	46 x 10^{-3}
28	34.867	17.52	0.385	21 x 10^{-3}
29	34.730	21.64	1.106	51 x 10^{-3}
30	34.870	36.07	3.799	105 x 10^{-3}
32	34.849	47.40	5.636	119 x 10^{-3}
33	34.863	22.67	0.646	29 x 10^{-3}
34	34.778	19.58	1.35	89 x 10^{-3}
35	34.798	14.43	3.84	266 x 10^{-3}
40	34.293	18.55	0.245	13 x 10^{-3}

* Station 26 does not have a measurable salinity anomaly, but has a measurable temperature anomaly so it was included in this table.

Section S4: Identifying the Length scale

Estimating the maximum dissipation length scale, d_{max} via Monin-Obukhov length (L_{M-O}) (Monin-Obukhov, 1954):

$$L_{M-O} = -\frac{u_*^3}{k\beta g w\overline{\Delta S}} \qquad \text{(S4.1)}$$

u_*=friction velocity, calculated in S.5= m s^{-1}

g= gravitational acceleration= 9.81 m s^{-2}

$w\underline{\Delta S}$=salt flux= m s^{-1} g kg^{-1}

w= 0.015 m s^{-1}, (see Section 5.2.1)

$\overline{\Delta S}$= $\frac{\int_{z=0}^{z=z_S} \Delta S\, dz}{z}$= g kg^{-1}

β= coefficient of haline contraction, calculated from Gibbs Seawater toolbox and averaged over the depth range of the anomaly= 7.87 x 10^{-4}

k= von Karman constant= 0.41

$$L_{M-O} = -\frac{u_*^3}{k\beta g w\overline{\Delta S}} = -\frac{\frac{m^3}{s^3}}{k\,\frac{kg}{g}\,\frac{m}{s^2}\,\frac{m\ g}{s\ kg}} = \frac{\frac{m^3}{s^3}}{\frac{m^2}{s^3}} = m \qquad \text{(S4.2)}$$

Table S3: Data for Monin-Obukhov Length scale calculations.

Station	$\overline{\Delta S}$(g kg^{-1})	u_*(m s^{-1})	L_{M-O}(m)
25	2.2 x 10^{-3}	2.4 x 10^{-2}	141
26	---	2.4 x 10^{-2}	---
27	1.5 x 10^{-3}	2.2 x 10^{-2}	151
28	7.23 x 10^{-4}	1. 2 x 10^{-2}	54
29	1.7x 10^{-3}	1.9 x 10^{-2}	80
30	3.5 x 10^{-3}	2.4 x 10^{-2}	83
32	4.0 x 10^{-3}	3.9 x 10^{-2}	198
33	9.1 x 10^{-4}	1.6 x 10^{-2}	98
34	2.3 x 10^{-3}	1.9 x 10^{-2}	66
35	8.8 x 10^{-3}	1.4 x 10^{-2}	6
40	1.4 x 10^{-3}	2.2 x 10^{-2}	175

Section S5: Wind Analysis

Extrapolation of the wind speed at 10 meters (U_{10}) using the NB Palmer wind speed U_P:

$$U_{10} = U_P \frac{ln(\frac{z}{z_o})}{ln(\frac{z_P}{z_o})} \quad \text{(S5.1)}$$

z_0 =Roughness Class= 0.0002 m
z_P = Reference height= 24 m
z =Desired height = 10 m

Average environmental values from NB Palmer used as inputs for COARE 3 to calculate the Drag Coefficient (C_D):

$average\ U_{10}$ =average wind speed= 9.8 m s^{-1}
$average\ T_{air=}$average air temperature = -18.7 ℃
$average\ RH$ =average relative humidity= 78.3 %
$average\ P$ = average air pressure= 979.4 milli-bar
$average\ T_{water}$ = average water temperature = -1.74 ℃
$average\ R_S$ = average shortwave radiation = -3.56 W m^{-2}
$average\ R_L$ =average longwave radiation = 201.2 W m^{-2}
$average\ Lat$ =average latitude =-75°

Average wave height and wave period of the 04 May SWIFT deployment used the wave as inputs for COARE 3 to calculate the wave dependent Drag Coefficient (C_D):

$average\ Sig_H$ =average significant wave height= 0.58 m
$average\ T$ =average wave period =4.6 seconds

The average phase speed (c_p) was calculated from the wave period (T)using the formula for deep water dispersion:

$$c_p = \frac{g}{2\pi}T \quad \text{(S5.2)}$$

c_p =average phase speed= 7.2 m s^{-1}
g =gravity, 9.81m s^{-2}
$average\ T$ =average wave period =4.6 s

Based on the average values, the Drag Coefficient (C_D)was found to be: C_D=1.525 x 10^{-3}

The wind stress, τ, was calculated for each CTD station based on the extrapolated wind speed at 10 meters, U_{10} , average air density, and average drag coefficient:

$$\tau = C_D\, \rho_{air} U_{10}^2 \quad \text{(S5.3)}$$

ρ_{air} =density of air=1.34 kg m^{-3} calculated using averages from NB Palmer summarized above.

Using wind stress, we derived the friction velocity (u_*) at the air-sea interface using the wind stress and water density, ρ_{water}.

$$u_* = \sqrt{\frac{\tau}{\rho_{water}}} \quad \text{(S5.4)}$$

u_* = friction velocity
ρ_{water} = density of water

Table S4: Data for wind analysis summarized in Supplemental 5.

Station	U_P (m s^{-1})	U_{10} (m s^{-1})	τ (kg m^{-1} s^{-2})	ρ_{water} (kg m^{-3})	u_* (m s^{-1})
25	12.72	11.77	0.622	1028.01	2.5 x 10^{-2}
26	12.31	11.39	0.582	1028.06	2.4 x 10^{-2}
27	11.54	10.68	0.512	1028.14	2.2 x 10^{-2}
28	6.37	5.89	0.156	1028.02	1.2 x 10^{-2}
29	9.62	8.90	0.355	1027.94	1.9 x 10^{-2}
30	12.43	11.50	0.594	1028.12	2.4 x 10^{-2}
32	20.43	18.90	1.603	1028.16	3.9 x 10^{-2}
33	8.37	7.74	0.269	1028.05	1.6 x 10^{-2}
34	9.95	9.21	0.380	1027.97	1.9 x 10^{-2}
35	7.15	6.61	0.196	1027.97	1.4 x 10^{-2}
40	11.59	10.72	0.516	1027.59	2.2 x 10^{-2}

Section S6: Calculating the rate of mixing and production rate

Using the L_{M-O}, turbulent kinetic energy (ε) can be applied to find the minimum time scale for mixing:

$$t = \frac{\pi\, d}{v_*} \approx \frac{d}{(\varepsilon\, d)^{\frac{1}{3}}} \approx \left(\frac{L_{M-O}^2}{\varepsilon}\right)^{\frac{1}{3}} \qquad \text{(S5.1)}$$

t =timescale= s

ε =turbulent kinetic energy dissipation= 1.85×10^{-5} $m^2\ s^{-3}$

L_{M-O}= Monin-Obukhov Length= m

The minimum times scale can be used to calculate an ice production rate (r_{ice}):

$$r_{ice} = \frac{C_{ice}^S\ z_S}{t\ \rho_{ice}} = \text{m d}^{-1} \qquad \text{(S5.2)}$$

C_{ice}^S =mass of frazil ice derived from salinity anomaly per volume= kg m^{-3}

t= timescale= d

$\rho_{ice} = 920$ kg m^{-3}

z_s =depth of the salinity anomaly (m)

Table S5: Calculation of time scale and production rate.

Station	C_{ice}^{S} (kg m^{-3})	L_{M-O} (m)	ε (m^2 s^{-3})	t (min)	r_{ice} (cm d^{-1})	r_{ice} 95% CI (cm d^{-1})
25	67 x 10^{-3}	141	9.648 x 10^{-05}	9.8	14	[10 - 20]
26	--	---	7.191 x 10^{-05}	---	---	--
27	46 x 10^{-3}	151	8.188 x 10^{-05}	10.9	28	[20- 37]
28	21 x 10^{-3}	54	1.622 x 10^{-05}	9.4	6	[4- 10]
29	51 x 10^{-3}	80	5.375 x 10^{-05}	8.2	21	[15 - 28]
30	105 x 10^{-3}	83	3.771 x 10^{-05}	9.5	63	[45- 88]
32	119 x 10^{-3}	197	3.466 x 10^{-04}	8.0	110	[67-181]
33	29 x 10^{-3}	98	2.844 x 10^{-05}	11.6	9	[5- 13]
34	89 x 10^{-3}	66	6.397 x 10^{-05}	6.8	31	[23 - 42]
35	266 x 10^{-3}	6	2.343 x 10^{-05}	2.0	302	[200- 456]
40	13 x 10^{-3}	175	9.603 x 10^{-05}	11.7	3	[2- 5]

Section S7: Seasonal Ice Production Estimate and Comparison

$$Q_s = c_p^A \rho_a C_s u_{10} (T_b - T_a) \quad \text{(S.6.1)}$$

c_p^A= 1.003 kJ kg^{-1} K^{-1}, the specific heat capacity of air at -23 ◦C

$\rho_{a=}$ =density of air=1.34 kg m^{-3} calculated using averages from NB Palmer summarized in supplemental 5

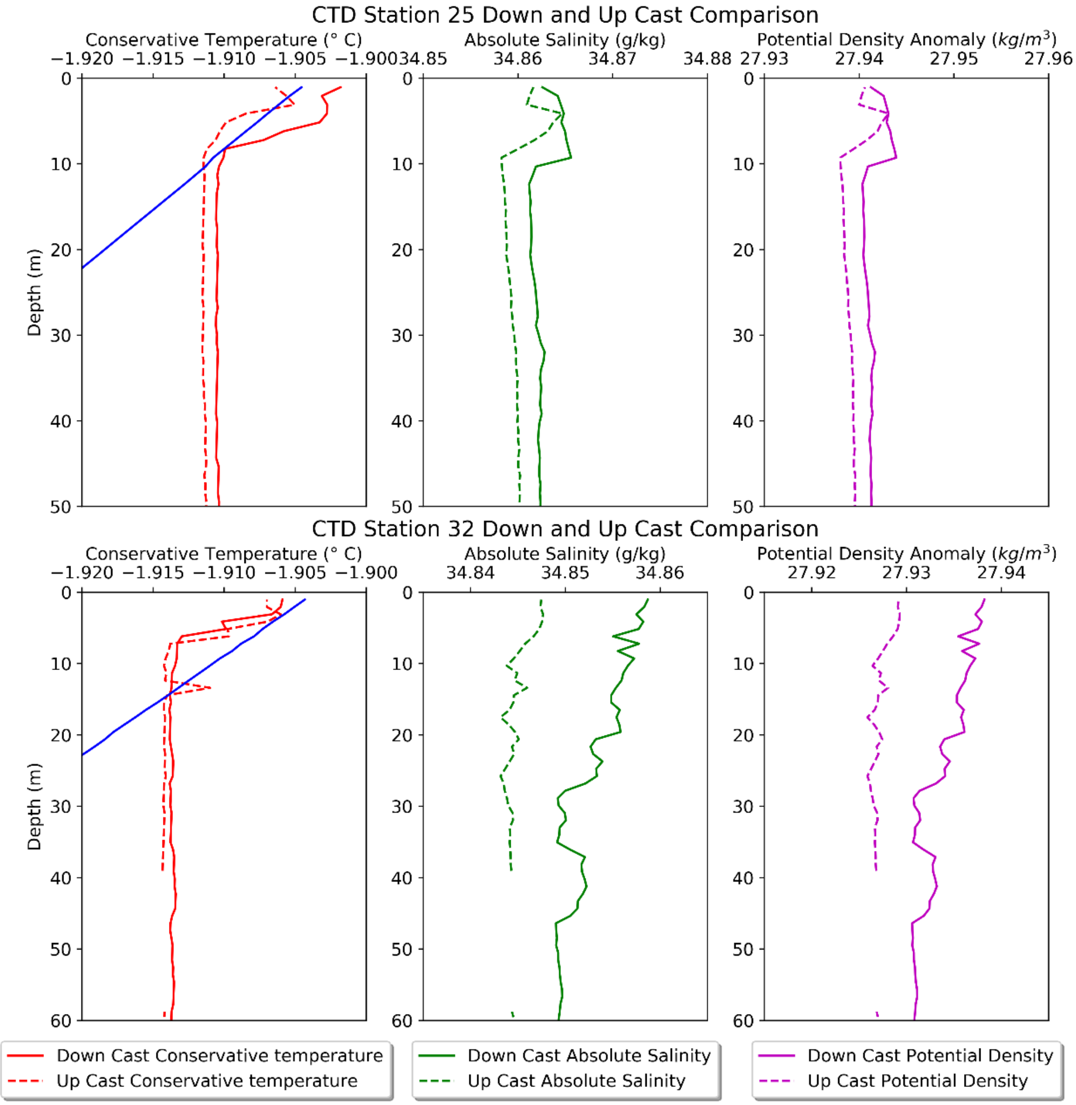
C_s= 1.297 X 10^{-3}, the heat transfer coefficient over snow, ice, water, calculated using the COARE 3.0 code (Fairall et al, 2003)

T_b = sea surface temperature/ baseline or far field temperature (10 meter average below anomaly)(°C)

T_a= air temperature from NB Palmer(°C)

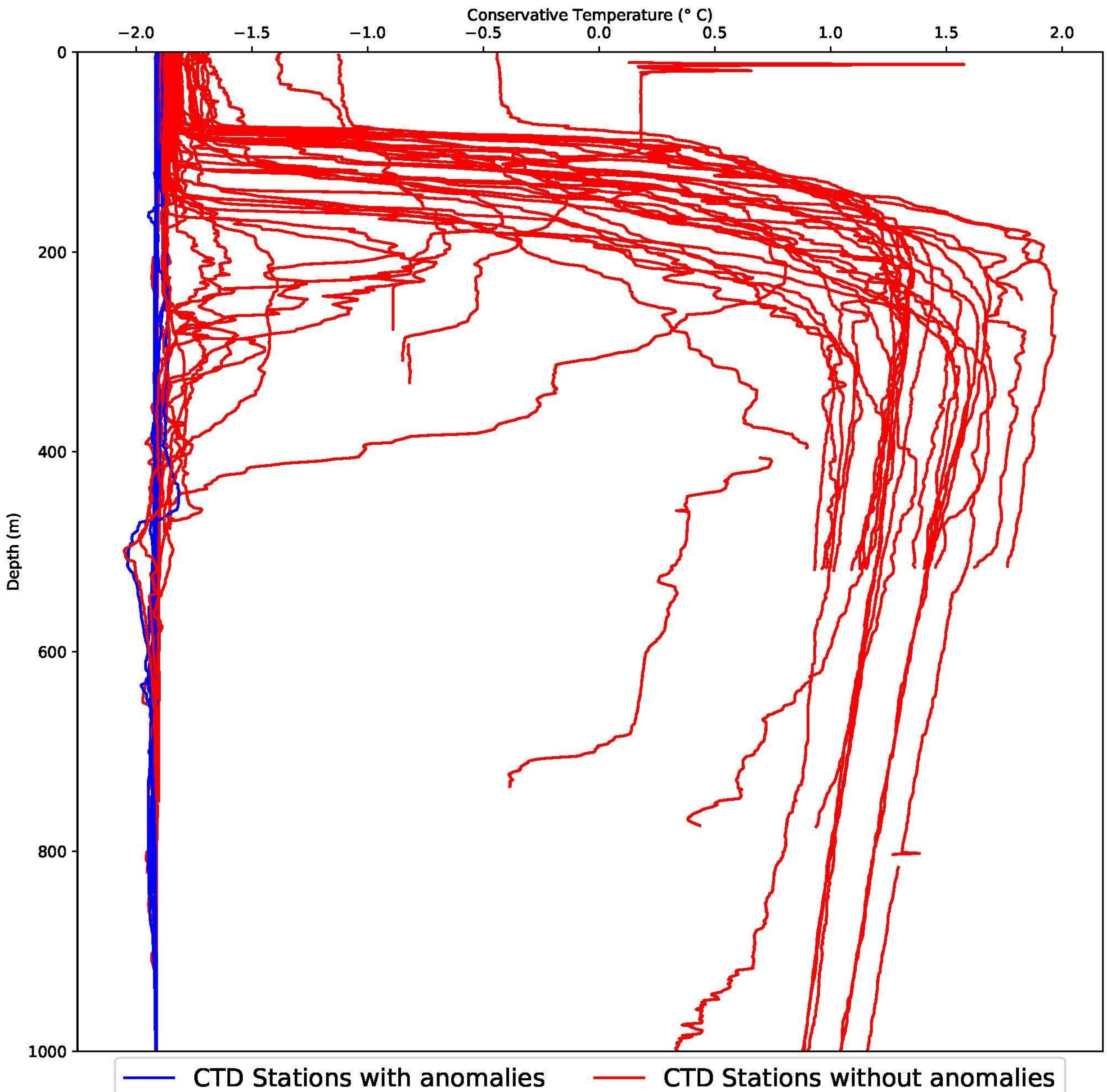
Table S6: Baseline Sea Surface Temperature, Air Temperature, Wind Speed (10m), Calculated Sensible Heat Flux, and Production Rate

Station	T_b (°C)	T_a (°C)	U_{10} (m s^{-1})	Q_S (W m^{-2})	r_{ice} (cm d^{-1})
25	-1.910	-16.58	11.77	301	14
27	-1.914	-15.83	10.68	259	28
28	-1.915	-15.93	5.89	144	6
29	-1.906	-24.71	8.90	354	21
30	-1.916	-25.6	11.50	475	63
32	-1.914	-24.95	18.90	759	110
33	-1.913	-21.56	7.74	265	9
34	-1.909	-19.39	9.21	281	31

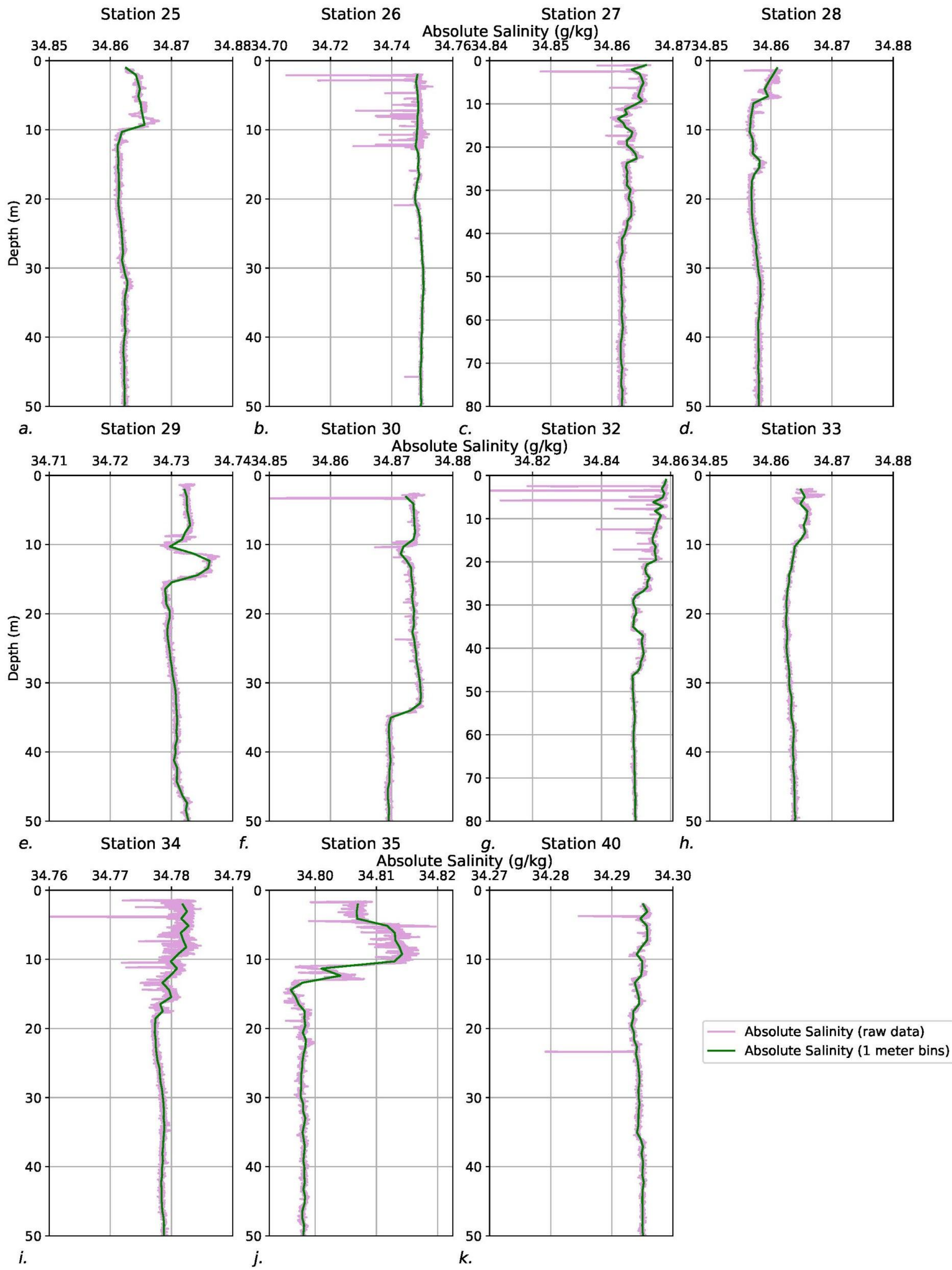


Supplemental Figure 1: Comparison of Down and Up Cast Profiles from CTD Station 25 and Station 32. The Down Cast Conservative Temperature, red solid line, is slightly warmer than the Up Cast Conservative Temperature (red dashed line) for Station 25, resulting in a smaller up cast anomaly. For Station 25 the same trend is seen in salinity and attributed to the wake of the CTD.

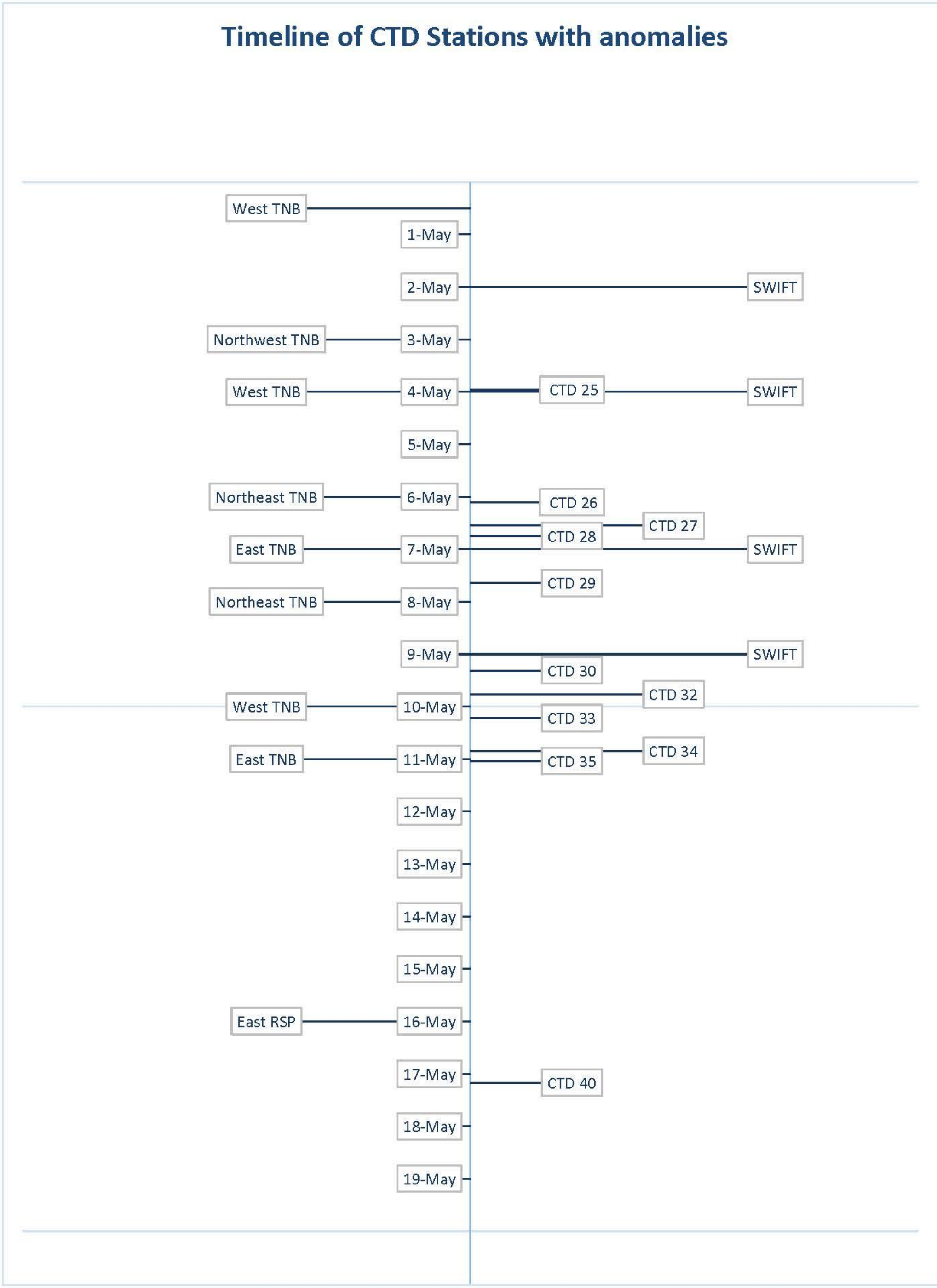
For Station 32, the Conservative Temperature profiles are very similar, however of note there is missing data between 40-60 meters that is attributed to the wake of the CTD. There is a notable difference in the Absolute salinity, however there is still presence of an anomaly/



Supplemental Figure 2: Conservative Temperature profiles of all 57 (of 58) PIPERS CTD stations. One CTD profile acquired north of the Polar Front was not included here, since its temperature range lie outside the range chosen here. Those CTD profiles from TNBP and RSP that indicated frazil ice production are plotted in blue, while profiles without frazil ice anomalies are plotted in red. In addition to large mixed layers, the polynya profiles also show the coldest temperatures.

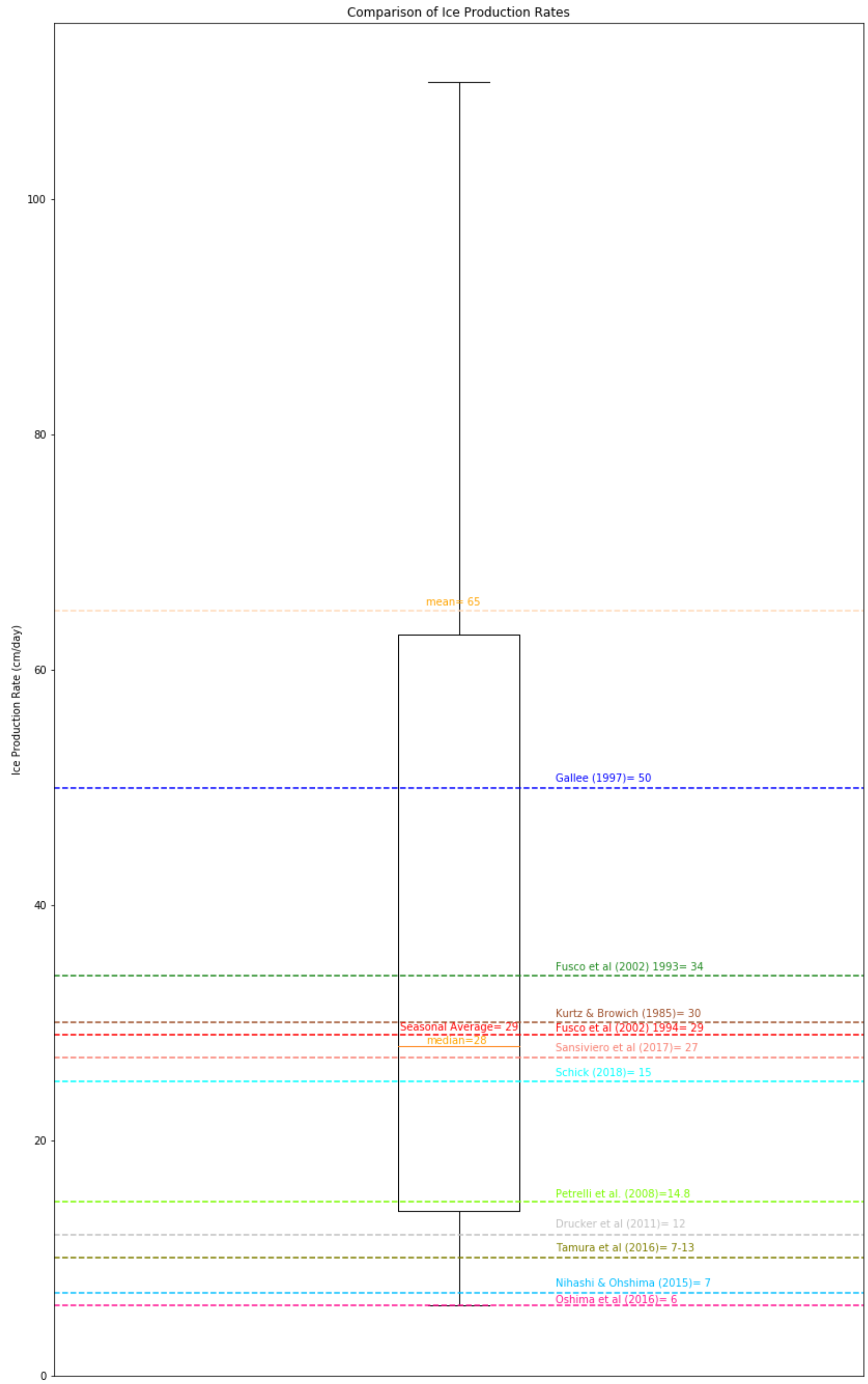


Supplemental Figure 3: Absolute Salinity plotted from raw conductivity data and from 1 meter binned data for the CTD Stations with anomalies. The x-axis for a, c, d-f, h-k are all 0.03 g kg^{-1}; b and g 0.06 g kg^{-1}. The raw data, plotted in purple, shows varying levels of noise in the signal and spikes of lesser magnitude values. This noise and the spikes in the data likely due to frazil ice crystal interference. Values of spikes extending off the plot: f: 34.670 g kg^{-1}; g: 34.800 g kg^{-1} ; i: 34.740 g kg^{-1}. Plots b, c, i, j display more noise than the other plots. The 1-meter bin data, plotted in green, does not follow the spike excursions, indicating that binning the minimizes or removes the effects of the noise and spikes.

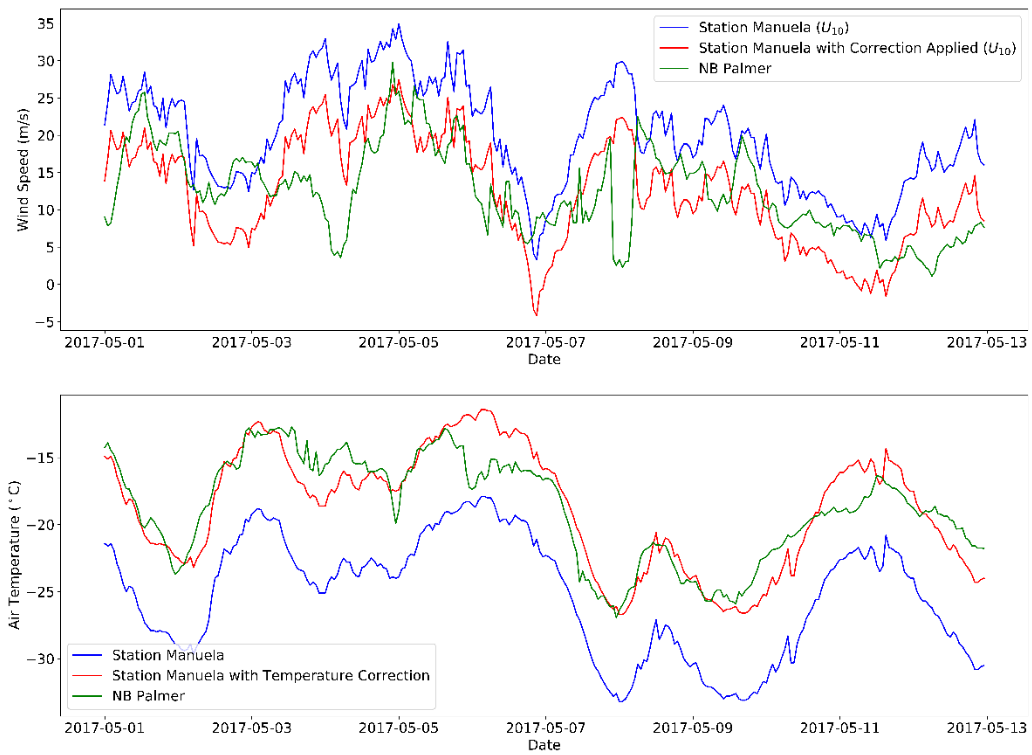


Supplemental Figure 4: Timeline of TNBP and RSP CTD casts and SWIFT deployments. A timeline of CTD and SWIFT deployments while in TNBP and RSP. To the left of the date, the geographic region is noted. This indicates when NB Palmer entered that portion of each polynya. The NB Palmer was in TNBP from May 1 to May 13. The

NB Palmer was in the RSP from May 16 to May 18. To the right of the date the CTD stations with anomalies and SWIFT deployments are shown. All of the SWIFT deployments where in TNBP.



Supplemental Figure 5: Comparison of Ice production rates. This box and whisker plot shows the production rates calculated in this study. Station 35, marked as an outlier is not shown, but was included in the mean and median calculations.



Supplemental Figure S6: The Station Manuela (blue), NB Palmer (green), and Station Manuela corrected (red) 10-meter wind speed and air temperature for the 13 days that NB Palmer was in TNBP. The air temperature correction fits the NB Palmer weather well. The wind speed correction varies between being an over and underestimate, however both the NB Palmer data and the corrected Station Manuela data average to 12.4 m s^{-1} indicating that in the context of a long term seasonal average, the wind correction is accurate.