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*Supplement of*

**Brief communication: Conventional assumptions involving the speed of radar waves in snow introduce systematic underestimates to sea ice thickness and seasonal growth rate estimates**

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## S1 The Comparable Impacts on SIT of Snow Loading and Slower Radar Wave Propagation in Snow

$$\text{Ice Freeboard} = \text{Radar Freeboard} + \text{Propagation Correction} \quad (\text{Armitage and Ridout, 2015}) \quad (\text{S1})$$

$$h_i = h_r + h_s(c/c_s - 1) \quad (\text{S2})$$

5 The conversion of a given ice freeboard can be combined with a snow depth to estimate sea ice thickness:

$$SIT = h_i \frac{\rho_w}{\rho_w - \rho_i} + h_s \frac{\rho_s}{\rho_w - \rho_i} \quad (\text{Tilling et al. 2018}) \quad (\text{S3})$$

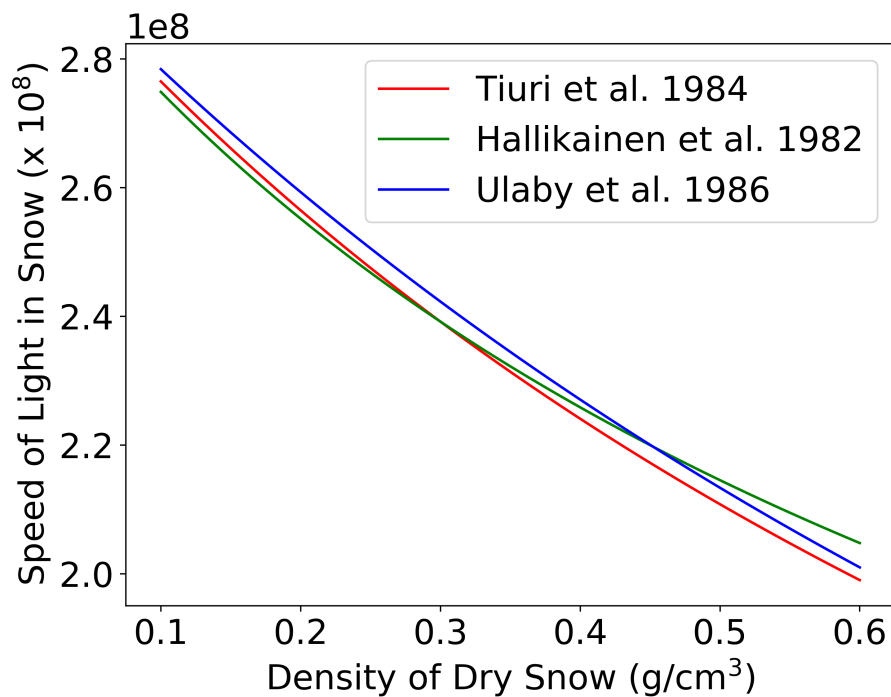
Substituting Eq. (S2) into Eq. (S3)

$$SIT = h_r \frac{\rho_w}{\rho_w - \rho_i} + h_s \frac{\rho_w}{\rho_w - \rho_i} \left[ \frac{c}{c_s} - 1 \right] + h_s \frac{\rho_s}{\rho_w - \rho_i} \quad (\text{S4})$$

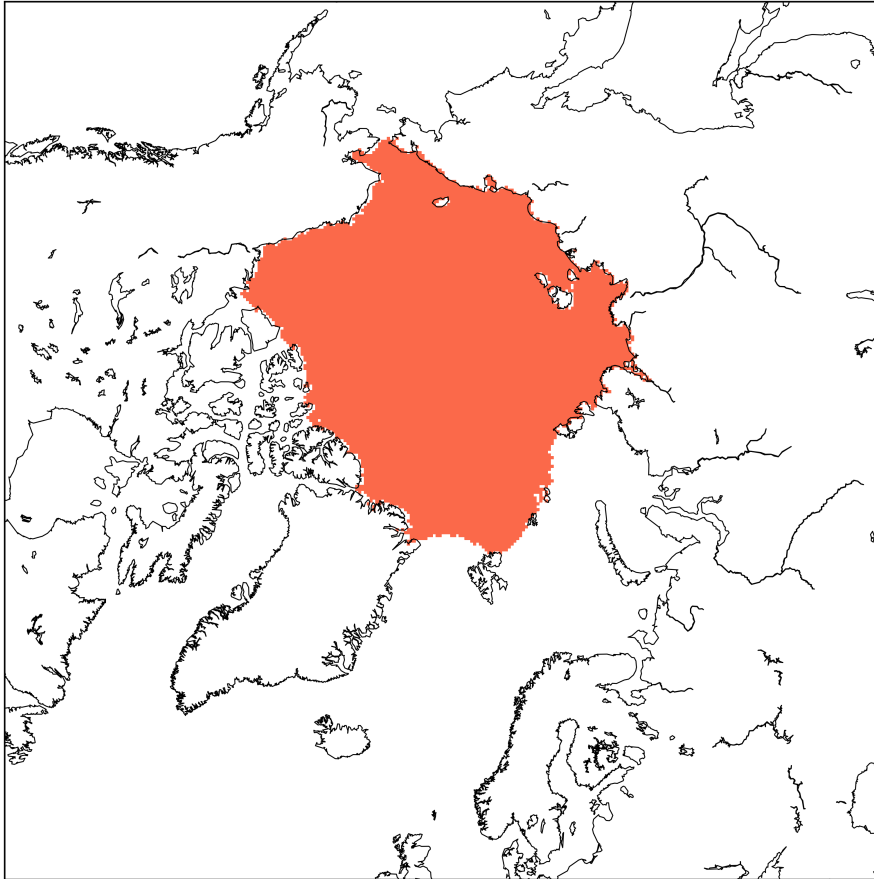
$$10 \text{ Sea Ice Thickness} = \text{Radar Freeboard Component} + \text{Propagation Correction} + \text{Snow Loading} \quad (\text{S5})$$

Comparing the relative impacts of the Propagation Correction term and Snow Loading term is relatively simple given they share a common factor of  $h_s/(\rho_w - \rho_s)$ . The ratio of the two terms is therefore  $c/c_s - 1$  to  $\rho_s/\rho_w$ . For a typical snow density of  $300 \text{ kg m}^{-3}$ , this ratio is 0.25 to 0.29. (Using  $\rho_w = 1023.9$  and Ulaby et al. (1986) to relate  $\rho_s$  to  $c_s$ )

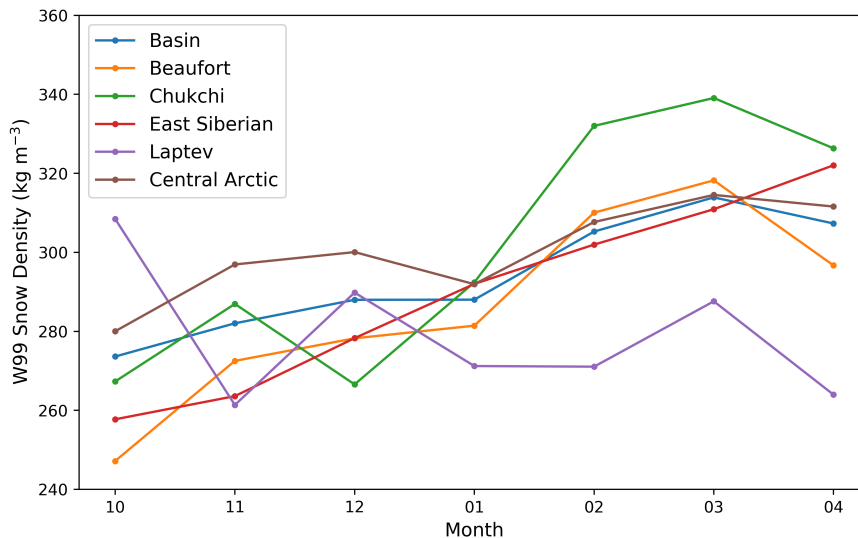
## S2 Supplementary Figures



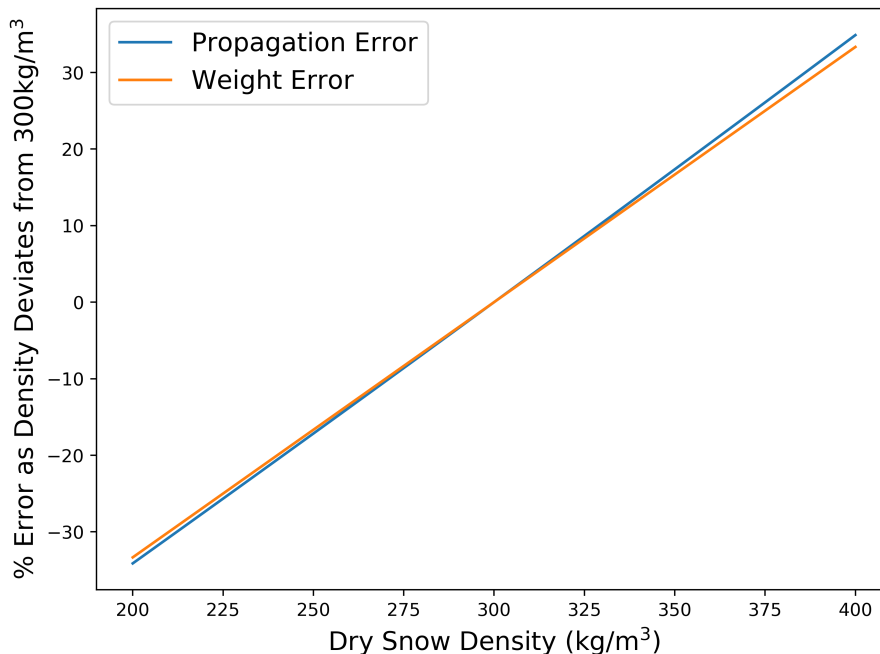
**Figure S1.** Three commonly used relationships between radar wave speed and snow density Hallikainen et al. (1982); Tiuri et al. (1984); Ulaby et al. (1986).



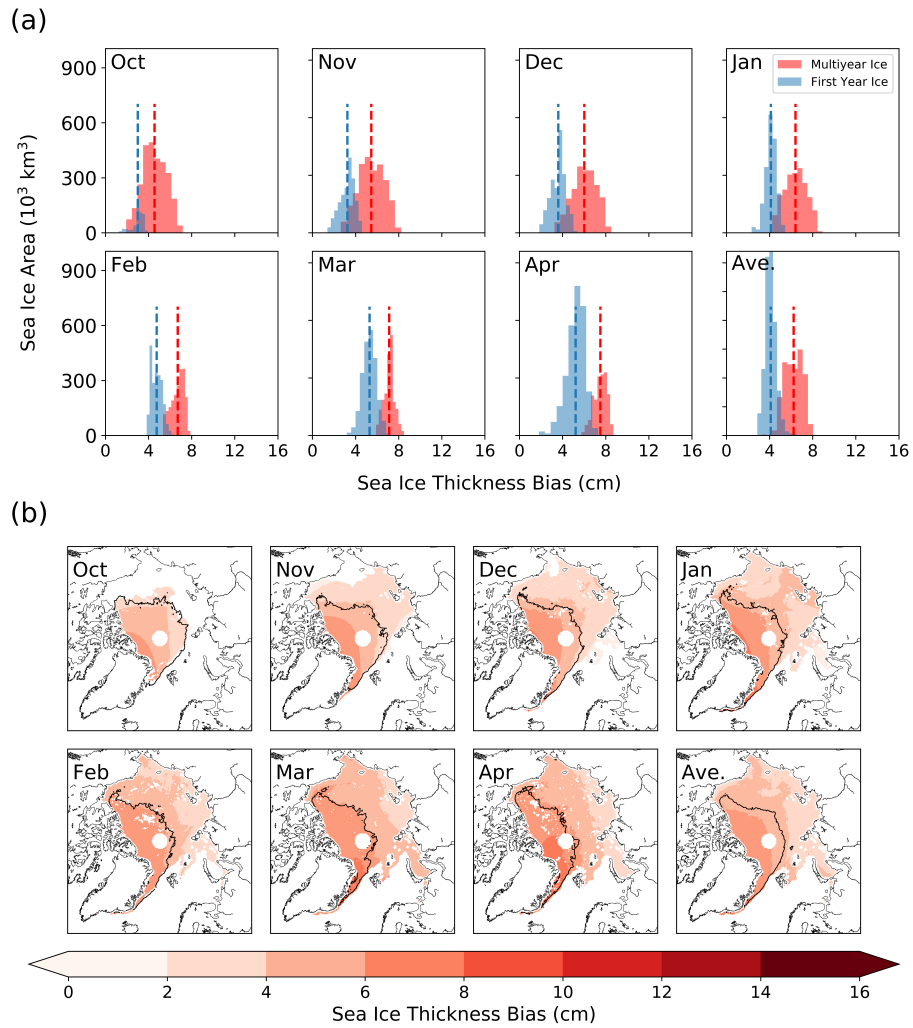
**Figure S2.** The region over which snow depths published in Warren et al. (1999) are generally considered reliable (Laxon et al. (2013); Kwok and Cunningham (2015)), and over which freeboards are considered in this study.



**Figure S3.** Winter snow densification rates for five regions and the basin-wide average. We defined the ‘basin-wide area’ as the shaded area in Fig. (S2). We found the basin-wide densification rate to be roughly representative of its constituent regions apart from the Laptev, which exhibited a small but positive densification.



**Figure S4.** While the functional form and magnitude of expressions for the effect of snow weight and slower radar propagation are different, they have a similar error dependence on snow depth. That is to say, the percentage error introduced to the "weight correction" by snow density uncertainty is the same as that for the "propagation correction".



**Figure S5.** Monthly differences in sea ice thickness from the use of  $\delta h = 0.22Z$  and  $\delta h = 0.25Z$  for the propagation by AWI and CPOM respectively.

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