



Supplement of

The current state and 125 kyr history of permafrost on the Kara Sea shelf: modeling constraints

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Supplement. Extrapolation of the modeling results of permafrost thickness to real geological

sections.

Carrying out such a procedure is necessary to reduce the number of simulated thermal tasks to a reasonable limit. There are several approaches.

A. In the first case, we consider the uniform alternation of homogeneous layers with a relatively low thickness (relative to the total thickness of the permafrost). The linear interpolation is simply performed in accordance with the percentage of the thickness of frozen ground obtained during modeling for two "pure" soils at a given moment.

There are 2 options here:

1. We have the thicknesses of frozen layers in sand (T_s) and loam (T_l) (clay) sediments from the simulation in similar conditions (scenario, heat flux). Then the formula will be:

$$T_{al} = T_l + n_s * (T_s - T_l), (1)$$

where n_s is the relative content of sand layers in the section.

The same formula can also be used to evaluate the thawing of permafrost from above, if this occurs in accordance with a specific scenario.

2. More complicated is the very common case when, for example, relict permafrost is preserved in sandy sediments within the water area, and as for the loam it thaws completely long before modern times.

In this case, we apply the following approximate approach.

For a reference soil in which permafrost is not preserved up to the present (and this is always loam (clay) in the considered pair of reference soils), the last point of the existence of permafrost at the τ_{deg} (time of degradation) is found at the corresponding graph. This point, in the absence of thawing from above, is located on the surface, otherwise, at a certain depth from the bottom z_{deg} , where the upper and lower fronts of thawing of permafrost in the loams meet.

At this point, according to the slope of curve representing the movement of the base of permafrost, the rate of thawing v_{th} from below is determined right before the complete disappearance of the frozen layer. Based on this rate of the upward movement of the lower boundary of the permafrost, the value of the potential thawing of loams from the moment τ_{deg} is calculated:

$$\xi_{lb} = v_{\rm th} \times \tau_{\rm deg} , \qquad (2)$$

and the estimated (fictitious) position of the modern lower permafrost boundary in the loamy rocks relative to the bottom surface will be equal to

$$T_{lb}^f = \mathbf{z}_{deg} - \boldsymbol{\xi}_{lb} \tag{3}$$

The value T_{lb}^{f} can be either positive (below the bottom) or negative (above the bottom).

Putting T_{lb}^f in (1) instead of Tc allows to find the position of the lower boundary T_{al} of frozen soils in a layered sandy-loamy section with a given relative content of sand layers (n_s) . A negative value clearly indicates the complete degradation of permafrost so far even in the absence of thawing from above.

To obtain the thawing from above for the alternating layers of sediments, the following approximate approach is used. At the moment of the complete degradation of frozen loams τ_{deg} , according to the graph of permafrost dynamics, the thawing value from above ξ_{ub} (τ_{deg}) is found for sandy soil and the ratio

 $p = z_{deg}/\xi_{ub}$ (τ_{deg}) is calculated. It is assumed that the indicated ratio of thawing depths from above for soils of different compositions at the time of permafrost degradation in loamy sediments is preserved up to the present. Then the potential thawing on top of the loams at the moment will be

$$\xi_{ub}^m = p \times \xi_{ub} \quad (4),$$

where ξ_{ub}^m and ξ_{ub} are the calculated depth of thawing from the top for the loams and the model depth of thawing from the top for the sand accordingly, to the current time

The current position of the permafrost top under the sea floor ξ_m for a layered section of bottom sediments with a given relative content of sand (n_s) is found from a dependence of type (1), which in this case has the next form:

$$\xi_m = \xi_{ub}^m + n_s \times (\xi_{ub} - \xi_{ub}^m) \tag{5}$$

Next, the residual thickness of the layered permafrost under the sea bottom should be found.

$$th_{res} = T_{al} - \xi_m$$
 (6)
If the value $th_{res} < 0$ then the relict permafrost do not persist at a specific point in the water area.

B. Extrapolation of simulation results of submarine permafrost in homogeneous soils into a two-layer geological section.

The case is considered when the thickness of the sediments is relatively small and bedrock lies from a shallow depth under the bottom. An approximate approach is known to estimate the freezing depth of a two-layer strata with known freezing depths of the rocks of the upper and lower layers separately, all other things being equal.

This approach is based on obvious points. At zero thickness of the upper layer of sediments (th₁=0), freezing of a two-layer system is equal to freezing in the lower layer $\xi_{2l} = \xi_l$, and at a thickness of the upper layer equal to the depth of freezing of the upper layer (th₁ = ξ_u) freezing of the system is equal to freezing of the upper layer (th₁ = ξ_u) freezing of the system is equal to freezing of the upper layer $\xi_{2l} = \xi_u$.

Intermediate freezing depths of this two-layered system for random values of the upper layer thickness are considered varying linearly between these extreme values, where the following relationship is obtained.

$$\xi_{2l} = \xi_l + (1 - \frac{\xi_l}{\xi_u}) \times th_1$$
⁽⁷⁾

It is clear that the limits of variation in the thickness of the upper layer of ground are limited – when $th_1 > \xi_u$ the system becomes single-layer. Equation (7) is valid as well for thawing of the permafrost for two-layer structure section.

There are also two cases of interpolation.

1. The modern thickness of permafrost in sediments (Th_{sed}) and bedrocks (Th_{br}) , is obtained under the same other conditions (scenario, heat flow) from the simulation results. Then the formula for determining the thickness of permafrost in a two-layer system will obviously have the next form:

$$Th_{2l} = Th_{br} + \left(1 - \frac{Th_{br}}{Th_{sed}}\right) \times th_1$$

where th_1 is the thickness of the upper layer of sediments or the depth of the top of the bedrocks.

We consider the sediments as the strata of alternating sand and loam with a given ratio in the section. The extrapolation of modeling results of homogeneous reference soils to a layered stratum was considered in the previous section.

The same dependence can also be used to evaluate thawing of two-layer permafrost from above.

2. Often there is a situation when the relict permafrost is preserved in sediments but in rocky deposits it is completely degraded to the current moment. In this situation, the same approach is used as previously considered (see section A). Using data on the depth and period of complete thawing of permafrost in bedrocks as well as on the dynamics of the movement of thawing fronts immediately before the disappearance of frozen rocks , the calculated (fictitious) position of the boundaries of frozen rocks at the modern time is determined using the dependencies (2-4), and the prediction of thawing of frozen rock from above is carried out according to (4) using the dynamics of sandy permafrost.

Using the same methodology, the fictitious positions of the upper and lower boundaries of the permafrost in sediments are calculated - i.e. layered strata with a given relative sand content. Further, substituting the obtained fictitious positions of the permafrost boundaries in sediments and rocks in (8), we find the position of these boundaries in a two-layer section. The current residual thickness of the permafrost is necessarily calculated as the difference in the depths of the positions of their lower and upper boundaries - a negative sign of this value indicates degradation of shelf permafrost.



Supplementary Figure 1. The influence of various environmental factors on the dynamics of shelf permafrost over the past 125 kyr according to the results of mathematical modeling

a) the influence of latitudinal climatic zonation and division into sectors : southwestern (SW) and northeastern (NE) shelf parts (the loam, 50 m isobaths, q=50 mW/m2)

b) the influence of heat flux: 50 mW/m2 and 75 mW/m2 (the loam, 50 m isobaths, SW)

c) the influence of lithology and properties of deposits : sand and loam (50 m isobath, q=50 mW/m2, SW)

d) the influence of sea depths (bottom isobaths): 5 and 50 m (the sand, q=50 mW/m2, SW)